12.4 **NEW DEVELOPMENTS OF A 3DVAR SYSTEM FOR A NONHYDROSTATIC NWP MODEL**

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1. INTRODUCTION

An important advantage of variational data assimilation methods is that observations that differ from the analysis variables can be directly analyzed as long as they can be expressed in terms of the model state variables. Examples include radar radial velocity and reflectivity, the GPS precipitable water, and satellite radiances. A variational method produces an analysis that minimizes a cost function that measures the fit of this function to the observations while subjecting to the background and other dynamical constraints. Three dimensional variational (3DVAR) analysis systems, thanks to their relatively low cost compared to 4DVAR, have been developed and used operationally for large-scale NWP at a number of operational centers in recent years (e.g., Parrish and Derber 1992; Courtier 1998) and progress is also being made in developing systems for mesoscale models (e.g., Wu et al, 2001).

In the 14th NWP conference, we reported an incremental 3DVAR data assimilation system developed for the ARPS model (Gao et al. 2001;Xue et al. 2000, 2001). The system is developed based on the existing infrastructure of the ARPS Data Analysis System (ADAS, Brewster 1996). The 3DVAR system is preconditioned by the background error covariance matrix (Courtier 1997) and uses recursive filter (Hayden and Purser 1995) to model the background covariances. Numerical experiments show that a reasonable reduction in the cost function is achieved in the minimization process and the quality of the analysis is good. The method is flexible and computationally efficient.

In this paper, we report further development of this system, in particular, the addition of two dynamic constraints based on the ARPS equations and the inclusion of the anelastic mass continuity equation as the third constraint. We consider such features very important for the assimilation of data at the convective scales. The inclusion of these equation constraints couple together the analysis variables and make the analysis of variables not directly observed (e.g., temperature and pressure by radar) possible.

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The latter process is often referred to as parameter retrieval.

2. THE 3DVAR FORMULATION

The basic cost function J, may be written as the sum of two quadratic terms plus a penalty term:

$$J(x) = \frac{1}{2}(x - x^b)^T B^{-1}(x - x^b)$$
$$+ \frac{1}{2}(H(x) - y^o)^T R^{-1}(H(x) - y^o) + J_c. \tag{1}$$

The first term measures the departure of the analysis vector, x from the background x^b, which is weighted by the inverse of the background error covariance matrix B^{-1}; the second term measures the departure of the projection of the analysis to the observation space, $H(x)$, from the observations themselves (y^o), which is weighted by the inverse of the combined observation and observation-operator error covariance matrix, R^{-1}. In our scheme, the background field can be provided by a single sounding, a previous ARPS forecast, or another operational forecast model. Observations currently tested include: single-level surface data (including Oklahoma Mesonet), multiple-level observations (such as rawinsondes and wind profilers), as well as Doppler radar observations. The first two terms have been well discussed in our previous report (Gao et al. 2001). The last part, J_c, include any penalty terms that may be added to the system and play important roles in correlating the desired analysis variables. In the following part of this section, we will focus our discussion on the J_C term.

In the case of radar observation of convective storms, in order to initialize such storms in a numerical model, we need to analyze all state variables (include all wind components and thermodynamic fields) from mere radial velocity and reflectivity observations, dynamic constraints that relate all these variables are critical in the cost function and in the analysis procedure. We assume that we know radial velocity and its time tendency (from successive radar scans) as is the case for real radar observations, and we will analyze u, v, w, θ', p', and qv' i.e., the three wind components and the perturbation potential temperature and pressure and relative humidity. The dynamic constraint term, J_c, is given as

$$J_c = \sum_{ijk} l_P P^2 + \sum_{ijk} l_Q Q^2 + \sum_{ijk} l_D D^2 . \qquad (2)$$

The first term in (2) is the pressure diagnostic equation constraint in which,

$$P = \nabla \cdot (\vec{E}) = -\nabla^2 p' + \frac{\partial}{\partial z}\left(\frac{\overline{r} g q'}{\overline{q}} - \frac{\overline{r} g p'}{g \overline{p}} \right) + \nabla \cdot \vec{G} , \qquad (3)$$

$$\vec{G} = -\overline{r}\vec{V} \cdot \nabla \vec{V} + \vec{k}\, \overline{r}\, g \left[-q_{li} + 0.628 q_v' \right] , \qquad (4)$$

where $\vec{E}$ is the forcing term of the vector Euclidian momentum equation. Here, $P = 0$ gives the elliptic diagnostic equation for p' found in anelastic nonhydrostatic models. Minimizing P provides an important coupling between p' and other state variables.

In the second term, $Q = \frac{\vec{r} \cdot \vec{E}}{|\vec{r}|} - \left(\frac{\partial \overline{r} V_r}{\partial t} \right)_{ob}$ is basically the difference between the analysis and observed time tendencies of radial velocity (or, more accurately, radial momentum). This constraint provides additional coupling among analysis variables. Currently we have not included this constraint in our following test but are working on adding this constraint to the system.

Another important dynamic constraint that couples the three velocity components is the 3D mass divergence constraint (the third term in Eq.(2)) in which

$$D = \nabla \cdot (\overline{r}\vec{V}) . \qquad (5)$$

This constraint provides the key coupling among three velocity components. The three l parameters in Eq (2) determine the relative importance of each constraint. They can be determined by experience and through experimentations.

The scheme outlined above emphasizes the use of dynamic constraints that are important for small-scale nonhydrostatic flows, with particular suitability for WSR-88D Doppler radar data. This need arises from the very different nature of small scale, especially convective flows. Weather features at these scales are often highly intermittent in both space and time and tend to have much shorter life times than large-scale ones therefore stationary, spatially homogeneous correlations and balance constraints typically employed in large-scale 3DVAR systems become unsuitable. In contrast to procedures that perform retrieval and analysis in stepwise manner (e.g., Weygant et al 2001), in our current scheme, data and dynamic equation constraints are incorporated into a single cost function and the analysis of all data is performed in a single step. In this case, high analysis resolution is typically needed for the entire analysis domain.

3. TEST RESULTS

We present here preliminary results from the 3DVAR analysis of the May 3, 1999 central Oklahoma tornado case. In this day, tornadoes with up to F5 intensity caused server damages to the southwest through southeast Oklahoma City (OKC) areas. The analysis grid was 43x43x43 in size and the grid interval is 3 km in the horizontal. The grid is stretched in the vertical with average grid spacing of 500m and a minimum grid spacing of 20m at the low levels. The analysis background was from a previous 2h ARPS forecast with total grid points 43x43x43 and 12 km resolution. The radar observation used was from the OKC (KTLX) WSR-88D radar at 22:00 UTC, May 3 of 1999. At this time, an active supercell storm was located in Comanche country, southwest of Oklahoma City. In the test, since the control variables are incremental variables, the first guess values were zero. Doppler Radar data (both radial velocity and reflectivity) were used in the analysis.

The quality of variational analysis can be examined, qualitatively for now, by looking at the analysis increment fields. The background field (not shown) was smooth and contained no clear sign of convection inside this small analysis domain. After the analysis using single radar data only (other data were purposed excluded to highlight the impact of radar data), structures associated with the tornadic thunderstorm are obtained (Fig. 1). Figs.1a and 1b show the u and v components of wind field. It is clear from the u field that there exists horizontal convergence at the low levels and outflow (strong divergence) at the high levels at around x=30 km. Figure 1c shows an updraft core at about the same location. The updraft intensity is obviously underestimated significantly, however, with the maximum value being only 3.5 m/s. Figure 1d shows analyzed water vapor and Fig. 1e shows the analysis increment of perturbation pressure. The structures found in the pressure field are, in a sense, pure retrievals, because no direct observations of pressure were used in the analysis. The magnitude of pressure perturbation and potential temperature (not shown) appears small, however, and we are investigating the cause of it. This preliminary example shows that the afore-described scheme is able to analyze the internal structure of a supercell thunderstorm with reasonable success although quantitative errors still appear significant. Further tuning of the procedure and the incorporation of the second term in Eq.(2) will be performed and numerical forecast experiments will be conducted to further examine the quality of the analysis. Further results will be reported at the conference.

4. SUMMARY

In this paper, we described new developments of a 3DVAR system developed in the ARPS model framework. In the updated system, two dynamic constraints based on the ARPS equations plus an anelastic mass continuity equation forms the three (weak) constraints in the cost function. We consider this a very important feature for storm-scale data assimilation. In this way, it may be possible to include the retrieval process for wind field and thermodynamic field directly in a three-dimensional variational data assimilation system. Preliminary results of the analysis of a tornadic supercell storm were presented that showed reasonable success though improvements are needed. Further analysis will incorporate other data types, including the Oklahoma Mesonet surface measurements, forecast experiments will be performed to further test the quality of analysis and the results will be reported at the conference.

Finally, we note that when 3DVAR scheme is applied to the analysis of data of vastly different data densities and when the data may be representative of flow structures of very different scales, such as the rawinsonde network data versus the radar observations, it will be difficult to perform the analysis in a single step unless the background error estimate is very accurate. At least the error structure has to be flow dependent and be aware of the existence of localized features. While ensemble Kalman filter technique appears to hold promises, we will also experiment with a multi-step approach in which different passes of analysis incorporate difference data sets, using different background error structures. Computationally this may also be a more viable solution.

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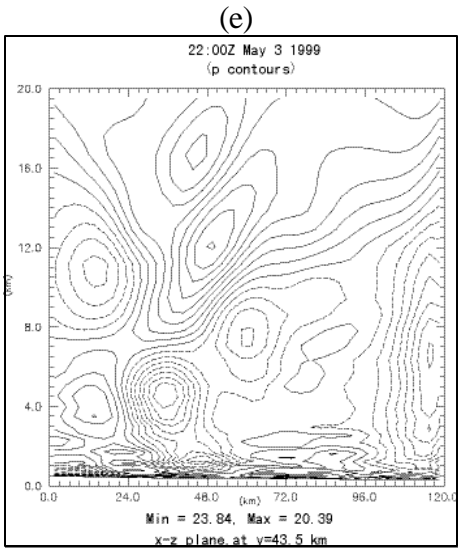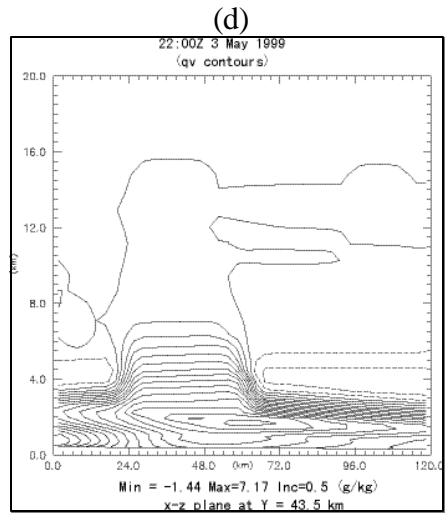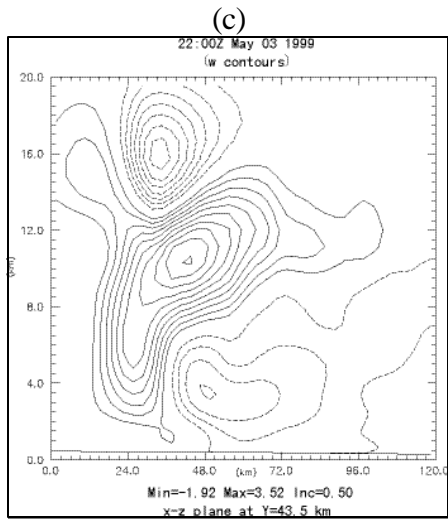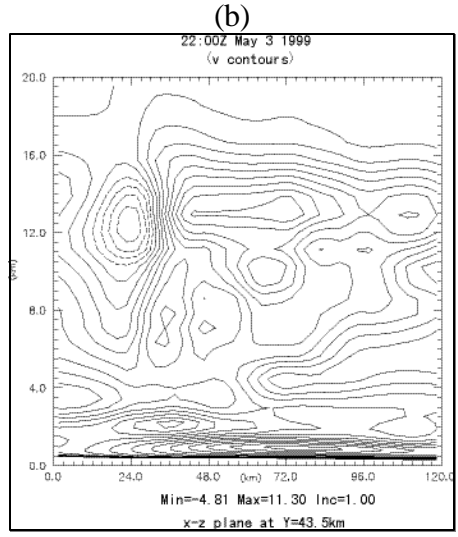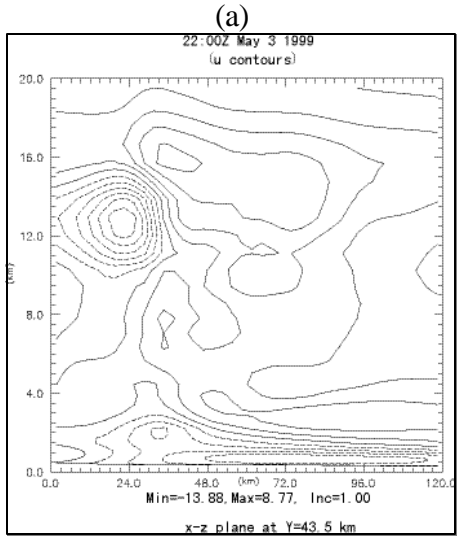
(a)

(b)

(c)

(d)

(e)

Fig. 1. The vertical cross section of 3DVAR analysis increment for the u component (a), v component (b), w component (c), the water vapor mixing ratio (d), and the perturbation pressure (Pascal) (e), at 10:00Z 3 May, 1999.