OBJECTIVE ANALYSIS
Numerical exercise instructions

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1 Introduction

In order to get a better understanding of data assimilation, in particular objective analysis, we have made an analysis system using FORTRAN. The system is called ANALAB. It has been implemented on SUN workstations at MISU (it has also been tested on SGI, NEC, Linux-PC). The exercises are to use ANALAB to view different aspects of the analysis schemes. The results are presented using a simplified Metgraf graphics package. A written report is required from each student.

We have chosen to analyze mean sea level pressure and 2 meter temperature using real SYNOP observations. Artificial observations can also be introduced for special purposes.

This note is organized as follows. In section 2, the theory is briefly reviewed. In section 3, the analysis system ANALAB is described. In section 4, exercises and the motivations for them are given.

2 Theory

2.1 Optimum Interpolation

Assume we have a regular analysis (model) grid with N gridpoints. We have some background information, for example, from a short range forecast. We denote this field as $\mathbf{x}^b$, which is a vector with elements $x_n^b, n = 1, N$.

Assume we have a irregular distribution of M observations. We denote them as $\mathbf{y}$, which is also a vector having elements $y_m, m = 1, M$.

If the observation locations do not coincide with the analysis grid points an observation operator is needed to relate a variable on the analysis grid to observation locations. Here we assume the operator is linear and can be expressed as a matrix $\mathbf{H}$ which has the dimension $M \times N$. For example, the difference between the background and observation, evaluated at the observation locations, is expressed as $\mathbf{H}\mathbf{x}^b - \mathbf{y}$.

What we want to get is an analysis, $\mathbf{x}^a(x_n^a, n = 1, N)$, using both background and observations.

There are different ways to combine $\mathbf{x}^b$ and $\mathbf{y}$. The Optimum Interpolation (OI) method produces a combination which is statistically optimal, provided we can assume that background errors and observation errors have Gaussian distributions (Normal distributions) and provided we know the error covariance matrices $\mathbf{B}$ and $\mathbf{R}$.

$\mathbf{B}$ is called the background error covariance matrix. It has $N \times N$ elements, b_{ij}, $i = 1, N;\ j = 1, N$, which are defined as

$$b_{ij} = \left\langle (x_i^b - x_i^t)(x_j^b - x_j^t) \right\rangle$$

where $\mathbf{x}^t$ is the true value on analysis grid and $\langle . \rangle$ denotes statistical expectation (or average).

$\mathbf{R}$ is called the observation error covariance matrix. It has $M \times M$ elements, $r_{ij}, i = 1, M;\ j = 1, M$, which are defined as

$$r_{ij} = \left\langle (y_i - y_i^t)(y_j - y_j^t) \right\rangle$$

where $\mathbf{y}^t$ is the true value at the observation locations.

The OI analysis is a minimum variance solution, which can be expressed as

$$\mathbf{x}^a = \mathbf{x}^b + \mathbf{B}\mathbf{H}^T \left(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R}\right)^{-1} \left(\mathbf{y} - \mathbf{H}\mathbf{x}^b\right). \tag{1}$$

and it can be shown that the analysis error covariance matrix is

$$\mathbf{A}^{-1} = \mathbf{B}^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H} \tag{2}$$

2.2 Simple examples

In order to gain some insight of the OI solution, let us first consider a few simple examples.

Example I. Consider a scalar problem. We have one observation y which is located at the analysis point. We also have a background estimate x^b. In addition, we assume we know the error statistics of y and x^b:

$$< x^b - x^t >= 0, < (x^b - x^t)^2 >= \sigma_b^2,$$

$$< (y - y^t)(x^b - x^t >= 0,$$

$$< y - y^t >= 0, < (y - y^t)^2 >= \sigma_r^2.$$

Using the notation for the OI solution,

$$N = 1$$

$$M = 1$$

$$\mathbf{H} = (1)$$

$$\mathbf{R} = \left(\sigma_r^2\right)$$

$$\mathbf{B} = \left(\sigma_b^2\right)$$

$$(x^a) = (x^b) + \frac{(1)(\sigma_r^2)^{-1}\left[(y) - (1)(x^b)\right]}{(\sigma_b^2)^{-1} + (1)(\sigma_r^2)^{-1}(1)} = (x^b) + \frac{\sigma_b^2}{\sigma_b^2 + \sigma_r^2}(y - x^b)$$

$$(\sigma_a^2)^{-1} = (\sigma_b^2)^{-1} + (\sigma_r^2)^{-1}.$$

We can re-write the OI solution as

$$\frac{x^a}{\sigma_a^2} = \frac{x^b}{\sigma_b^2} + \frac{y}{\sigma_r^2} \tag{3}$$

From equation (3) we see that the analysis x^a is a linear combination of the background x^b and the observation y, with the weighting coefficients proportional to the inverse of variances. The variance of the analysis is smaller than that of background and observation, $\sigma_a^2 < min\,[\sigma_b^2, \sigma_r^2]$.

Now assume the analysis is for temperature and we have:

$$y = 22.0, R = (\sigma^o)^2 = 1.0,$$

$$x^b = 20.5, B = \left(\sigma^b\right)^2 = 2.0,$$

and

$$x^a = 21.5, A \approx 0.7\,(\sigma^a \approx 0.8)\,.$$

In Figure 1, we show the background, observation and analysis probabilities assuming the errors are Gaussian. It is clear that the analysis is between the background and observation, and it is, in this particular case, closer to the observation as it has smaller expected errors. It is also shown that the analysis has a higher probability and smaller expected errors compared to the background and the observation.

Example II. We have one observation, y, located between two analysis points. We have background information on the two analysis points, denoted by x_1^b and x_2^b, and we

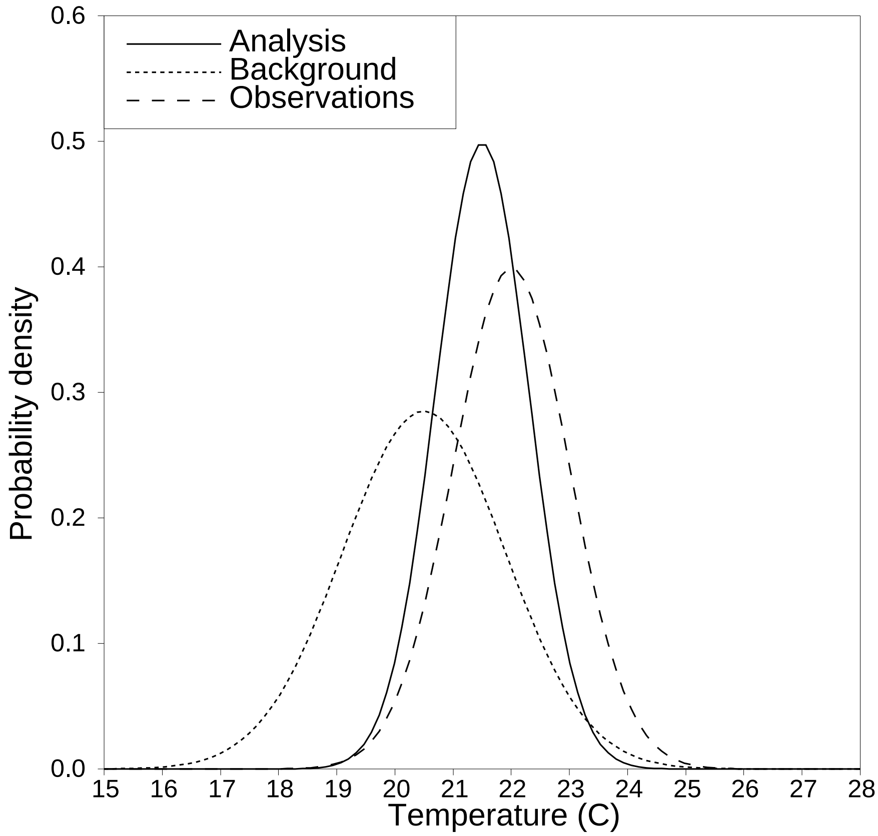

Figure 1: Probability density for analysis, background and observation.

can linearly interpolate the background information to the observation location as

$$\mathbf{Hx}^b = \alpha x_1^b + (1-\alpha)x_2^b$$

where $0 \leq \alpha \leq 1$ ($\alpha = 1$ if the observation is co-located with the analysis point 1; $\alpha = 0$ if the observation is co-located with the analysis point 2).

The assumed observation error is as in the previous example,

$$\mathbf{R} = (\sigma_r^2),$$

while the background error covariance matrix now takes the form

$$\mathbf{B} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \sigma_b^2 \begin{pmatrix} 1 & \gamma \\ \gamma & 1 \end{pmatrix}.$$

Here we have made the assumption that

$$b_{11} = b_{22} = \sigma_b^2$$

$$b_{12} = b_{21} = \sigma_b^2 \gamma$$

The OI solution (1) to this problem is

$$\begin{pmatrix} x_1^a \\ x_2^a \end{pmatrix} = \begin{pmatrix} x_1^b \\ x_2^b \end{pmatrix} + \sigma_b^2 \begin{pmatrix} \alpha + \gamma(1-\alpha) \\ \gamma\alpha + (1-\alpha) \end{pmatrix} \frac{y - \left(\alpha x_1^b + (1-\alpha) x_2^b\right)}{\left[\alpha^2 + 2\alpha(1-\alpha)\gamma + (1-\alpha)^2\right]\sigma_b^2 + \sigma_r^2} \tag{4}$$

Consider three cases:

Case 1. The observation is co-located with analysis grid point 1 ($\alpha = 1$) and the background errors are not correlated between point 1 and point 2 ($\gamma = 0$). Equation (4) reduces to:

$$\begin{pmatrix} x_1^a \\ x_2^a \end{pmatrix} = \begin{pmatrix} x_1^b \\ x_2^b \end{pmatrix} + \sigma_b^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \frac{y - x_1^b}{\sigma_b^2 + \sigma_r^2}$$

In this case, the solution at point 1 is identical to that in Example I. The solution at point 2 is equal to the background and no information from the observation is added there.

Case 2. The observation is co-located with analysis grid 1 ($\alpha = 1$) as in case 1. However, the background errors are correlated between point 1 and point 2 ($\gamma \neq 0$). Equation (4) reduces to:

$$\begin{pmatrix} x_1^a \\ x_2^a \end{pmatrix} = \begin{pmatrix} x_1^b \\ x_2^b \end{pmatrix} + \sigma_b^2 \begin{pmatrix} 1 \\ \gamma \end{pmatrix} \frac{y - x_1^b}{\sigma_b^2 + \sigma_r^2}$$

In this case, the solution at point 1 is unchanged. The solution at point 2 is equal to the background plus γ times the analysis increment added to point 1.

Case 3. The observation is located between the analysis points ($\alpha \neq 1$) but the background errors are not correlated between point 1 and point 2 ($\gamma = 0$). Equation (4) reduces to:

$$\begin{pmatrix} x_1^a \\ x_2^a \end{pmatrix} = \begin{pmatrix} x_1^b \\ x_2^b \end{pmatrix} + \sigma_b^2 \begin{pmatrix} \alpha \\ 1-\alpha \end{pmatrix} \frac{y - \left(\alpha x_1^b + (1-\alpha) x_2^b\right)}{\left[\alpha^2 + (1-\alpha)^2\right] \sigma_b^2 + \sigma_r^2}$$

In this case, the analysis increments for point 1 and point 2 are proportional to α and $1-\alpha$, respectively.

Finally, for the full solution (4), we can see that both the observation operator and error correlation have made contributions. However, when generalize the solution (4) from two analysis points to N points, the linear interpolation operator only influence the the analysis points around the observation, while the error correlations may spread information to all analysis points as long as the error correlation between the points is non-zero.

2.3 Common assumptions on R and B

For realistic atmospheric applications, **R** and **B** are very large matrices. Assumptions are often made to simplify them.

It is a common assumption that observation errors are not correlated and error variances are the same for the same observation type:

$$\mathbf{R} = \sigma_r^2 \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

It is not realistic to assume the background errors to be uncorrelated. The simplifications on **B** often been used are that they are homogeneous and isotropic, i.e., the correlation between two analysis points is only a function of distance:

$$\mathbf{B} = \sigma_b^2 \begin{pmatrix} 1 & \gamma_{12} & \cdots & \gamma_{1N} \\ \gamma_{21} & 1 & \cdots & \gamma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N1} & \gamma_{N2} & \cdots & 1 \end{pmatrix}.$$

Further the correlations are often modeled by some analytical functions. For example, as the correlation model we will use in the exercises, the Gaussian function can be used to model the correlation between analysis point i and analysis point j:

$$\gamma_{ij} = exp\left\{-\left(\frac{r_{ij}}{L}\right)^2\right\}$$

where r_{ij} is the distance between i and j, L is a length scale which could be determined theoretically or by observations.

In the implementation of (1), $\mathbf{B}\ \mathbf{H}^T$ and $\mathbf{H}\ \mathbf{B}\ \mathbf{H}^T$ are actually used, not $\mathbf{B}$ itself. $\mathbf{B}\ \mathbf{H}^T$ is a $N \times M$ matrix evaluated as:

$$\mathbf{BH}^T = \sigma_b^2 \begin{pmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1M} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N1} & \gamma_{N2} & \cdots & \gamma_{NM} \end{pmatrix},$$

which is the *background* error covariance between analysis points and observation locations. The index pair ij corresponds to the ith analysis point and the jth observation location. $\mathbf{H}\ \mathbf{B}\ \mathbf{H}^T$ is a $M \times M$ matrix evaluated as:

$$\mathbf{HBH}^T = \sigma_b^2 \begin{pmatrix} 1 & \gamma_{12} & \cdots & \gamma_{1M} \\ \gamma_{21} & 1 & \cdots & \gamma_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{M1} & \gamma_{M2} & \cdots & 1 \end{pmatrix}$$

which is the *background* error covariance between the observation locations.

2.4 Quality control (QC)

In practice, observations can have unrealistic errors occasionally or even over longer periods at certain locations. Some kind of quality control is needed. The quality control method we will use in the exercises is to check each observation by comparing it to the OI analysis which is obtained without the observation in concern. This is an expensive procedure as we have to compute one additional OI analysis for each observation. The difference is then compared to the analysis standard deviation:

$$\frac{y - x^a_{-y}}{\sigma_a} \leq \tau_1 \quad \text{the observation is correct;}$$

$$\tau_1 < \frac{y - x^a_{-y}}{\sigma_a} \leq \tau_2 \quad \text{the observation is probably correct;}$$

$$\frac{y - x^a_{-y}}{\sigma_a} > \tau_2 \quad \text{the observation is wrong and should be rejected.}$$

where τ_1 and τ_2 are empirically determined criteria. You will also do experiments to see the effect of changing them.

The quality control process is in practical applications more complicated, since observations used to obtain the interpolated value x^a_{-y} may also be in error. This is generally solved by rejecting observations one-by-one, starting with the observations that have the largest deviations from the interpolated values. This approach has been implemented in ANALAB to be described below.

2.5 Data selection

Mainly due to the large size of $\mathbf{HBH}^T + \mathbf{R}$, which is often difficult to invert with current computer power, a data selection method is needed. In the current HIRLAM OI, the so-called box-method is used. It divides the analysis domain into a number of boxes. In each box, the same type of OI analysis is performed, but with a reduced number of observations. At the end, the final analysis is obtained by putting all the box analyses together. In order to have smooth transitions at the box borders, the boxes are often made to overlap.

3 ANALAB

An analysis system, ANALAB, has been developed using Fortran. OI is among the analysis schemes included in ANALAB. The other schemes are all of variational type. Here we only concentrate on the OI scheme.

In order to use the lab, you need some very basic knowledge about Unix. I assume that all Unix commands in the following text are known.

```
login to rossby ...
```

Copy the complete ANALAB:

```
cp /home/guestdm/hans/analab9909.tar.gz
gunzip analab9909.tar.gz
tar xf analab9909.tar
cd analab
```

Take a look of the directory:

```
README -------- a short description of ANALAB
anaclean ------ script to clean ANALAB
anainstall ---- script to install ANALAB
exp ----------- experiment directory
map ----------- source code for graphics
src ----------- source code for analysis
```

Now you are in the analab directory. Check and modify anainstall so that ROOT points to the current directory. The current OPTION is set to sun, assuming you are using a sun workstation at MISU. If you want to install ANALAB on other computers, select other OPTIONS or create a new OPTION. Then install ANALAB by:

```
anainstall
```

If the installation is successful, you should see the following on your screen:

```
Installation is done. Go to exp to run experiments...
```

Otherwise you need help to re-install.

Go to exp directory, you should have the following:

```
nn060000.dat --- coastline data (for graphics)
params.dat ----- namelist for analysis
92030303.obs --- observations
map ------------ a link to ../map/map1.x (graphics)
analysis ------- a link to ../src/analysis.x (analysis)
namexp.dat ----- namelist for graphics (contour)
namgeo.dat ----- namelist for graphics (geometry)
92030303.fgs --- background (firstguess)
README
```

The experiments are performed by changing params.dat

```
 &PARAMS
   iy=92,im=03,id=03,ih=03 ------ date of the observation data
   ischeme=4, ------------------- analysis scheme, 4 - OI
   gscale=200000.,200000. ------- the length scale (m) for P and T
   sdevfg=1.,1. ----------------- sigma_b for P (mb) and T (C)
   sdevob=1.,1. ----------------- sigma_r for P (mb) and T (C)
   tolera1=2,2 ------------------ tau_1 for P (mb) and T (C)
   tolera2=3,4 ------------------ tau_2 for P (mb) and T (C)
   idim=21,jdim=21 -------------- dimension of the analysis grid
   latm=56.5,lonm=14. ----------- analysis domain center (lat,lon)
   dlon=0.6,dlat=0.3 ------------ analysis resolution
   lfgs=.t. --------------------- using the provided background
   lplot=.t. -------------------- plotting the results
 /
```

After selecting parameters, simply run

```
analysis
```

and you will see two maps on your screen, one for pressure and one for temperature. Finish the analysis by

```
quit
```

You have an ascii file, 92030303.E01, and a postscript file, POST, as results. You can view the results again or print it out:

```
gs POST
lpr POST
```

In the result file, 92030303.E01, the observations are included together with quality control flags (0=correct, 1=probably correct, 2=rejected, 3=missing).

```
COUNTRY STN LAT LON P T FLAG_P FLAG_T
```

By using different quality control values, we could get different flags.

The original observation file, 92030303.obs, has the following format:

```
COUNTRY STN LAT LON P T WIND_DIRECTION WIND_SPEED
```

where the wind data have not been used by ANALAB.

4 Exercises

4.1 Change the length scale

Run a series of experiments, varying only the length scales. Give a summary of your results.

4.2 Select a background field

In the default parameter settings, lfgs=.true., we use the provided background field, 92030303.fgs. You can set lfgs=.false., in this case the average value of all observations is used as the background field.

4.3 Analysis domain size and resolution

Using lfgs=.false., the parameters related to domain and resolution can be varied freely.

4.4 Box structures

Using lfgs=.false., idim=21, jdim=21, you can select nbox=9. In this case, we are using 9 analysis boxes. What are the differences between this analysis and the one with just one box (nbox=1)? What is the reason for using box structures? What are the disadvantages of using boxes?

4.5 Quality Control

1. Use different tolera1 and tolera2.
2. Modify 92030303.dat by introducing large errors to some observations.

4.6 Single observation experiment

Change 92030303.dat, to have only one observation. Run analysis with and without background.

4.7 Variational analysis (optional)

You can select the follow for ischeme:

```
1 - Standard Variational Analysis.
2 - Variational Analysis with No inversion of B.
3 - Variational Analysis using a Filter
4 - OI (done above)
5 - Physical-space Statistical Analysis
```

In these experiments, more parameters need to be included in params.dat. You can read the source code, src/analysis.f, or contact xyh@dmi.dk.