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AN OPERATIONAL OBJECTIVE ANALYSIS SYSTEM

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ABSTRACT

The system of objective weather map anaylsis used at the Joint Numerical Weather Prediction Unit is described. It is an integral part of the automatic data processing system, and is designed to operate with a minimum of manual supervision. The analysis method, based mainly on the method of Bergthórssen and Döös, is essentially a method of applying corrections to a first guess field. The corrections are determined from a comparison of the data with the interpolated value of the guess field at the observation point. For the analysis of the heights of a pressure surface the reported wind is taken into account in determining the lateral gradient of the correction to be applied. A series of scans of the field is made, each scan consisting of application of corrections on a smaller lateral scale than during the previous scan.

The analysis system is very flexible, and has been used to analyze many different types of variables. An example of horizontal divergence computed from a direct wind analysis is shown.

1. INTRODUCTION

The process of transforming data from observations at irregularly spaced points into data at the points of a regularly arranged grid has often been referred to as "objective analysis." An objective analysis scheme must perform several functions, namely, interpolation, removal of data errors, smoothing, and, in most applications, should contain some method of insuring internal consistency.

The method of analysis originally suggested by Panofsky [9] and modified to the system described by Gilchrist and Cressman [5], consisted of fitting some kind of polynomial by least squares to the data. Further efforts with this method were described by Cressman [4] and by Johnson [7]. This method, originally designed for areas of relatively dense and redundant data, did not prove too practical for use over a hemispheric area. Much of the hemispheric area is characterized by a few widely scattered observations, which one desires to fit as exactly as possible. The least squares polynomial scheme tended to develop instabilities of a certain type over such an area. Considerable effort was necessary to control these (Cressman [4]). Furthermore, the formation of the matrix elements and the matrix triangularization required at each grid point involved considerable computation. As a result of these considerations, the Joint Numerical Weather Prediction (JNWP) Unit, after extensive trials of the above-mentioned scheme, changed its analysis procedure to the one described in the following sections. The system described below is based essentially on the general method described by Bergthórssen and Döös [2] and resembles to some extent the method recently reported by Haug [6].

2. PREPARATION OF THE DATA

One of the major problems in the practical application of any objective analysis scheme is the problem of data reliability and the detection and elimination of errors in the data. For this reason, a short description of the data preparation method follows.

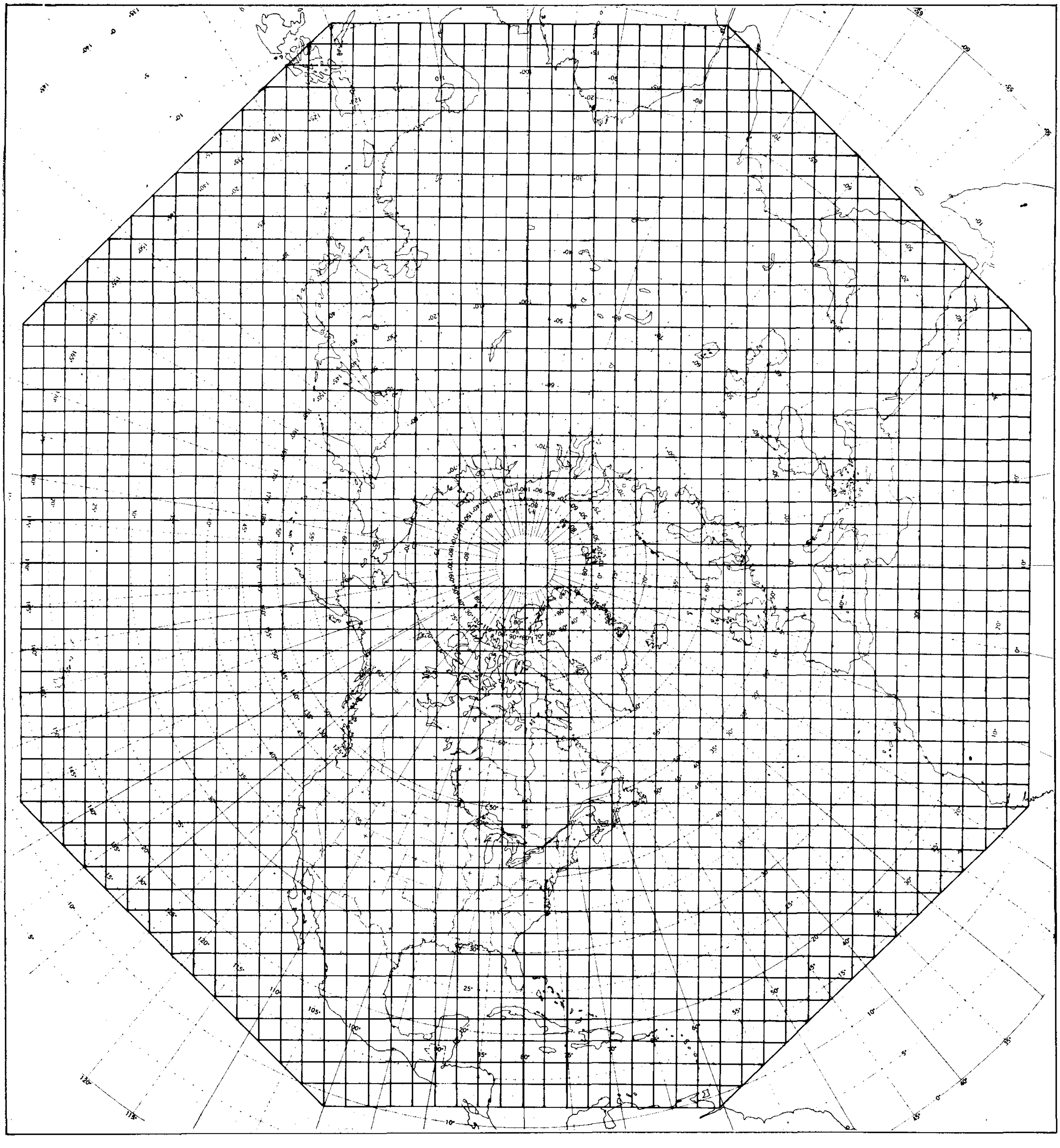

Figure 1.—The JNWP grid.

Data are received via teletypewriter from the Northern Hemisphere. The teletypewriter reports are processed automatically by an improved version of the automatic data processing system described by Bedient and Cressman [1], using the JNWP Unit's IBM 704 computer. During this process the temperature soundings are subjected to a hydrostatic check and corrected where possible. The erroneous data which cannot be corrected are deleted. All wind soundings are checked for vertical consistency, with the erroneous parts of the reports being deleted. The elements to be analyzed are then selected and sorted into a geographical order.

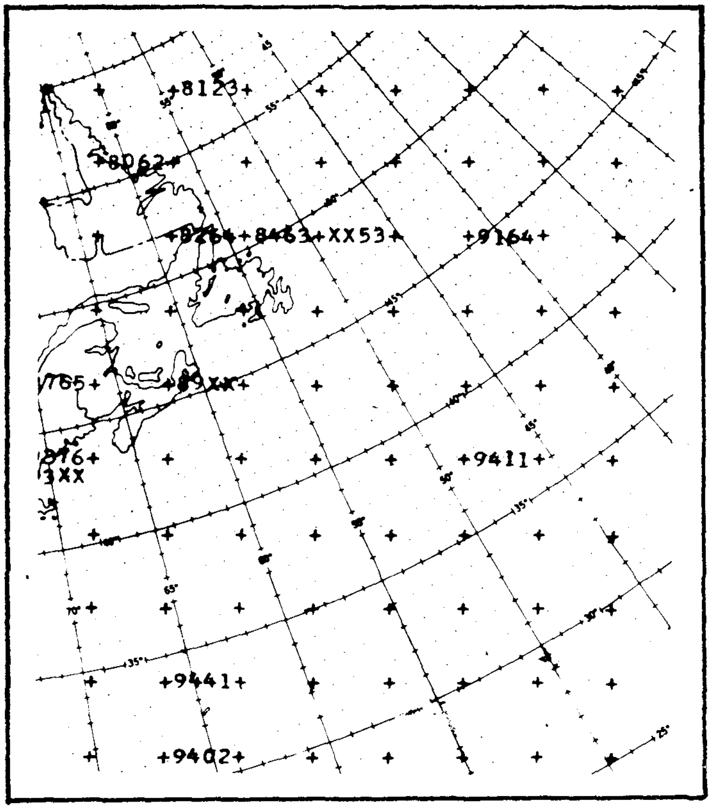

Figure 2.—Section of a plotted chart. The two left-hand digits are the thousands and hundreds digits of the 500-mb. height. The third digit is the wind direction on an eight point scale (0 = North), and the fourth digit is the wind speed in tens of knots. Missing data are indicated by X's.

At this point a control on the process is introduced by means of the production of a plotted chart. Here, the digits of highest significance are printed in their approximate geographical location (within the nearest grid square). The grid currently used for analysis and forecasting is shown in figure 1. A section of the plotted map is shown in figure 2. This plotted map gives the monitoring analyst a picture of the data coverage. The listing of each report on the map is also helpful for subsequent monitoring of the automatic error rejection process. The plotted map is quite cheap to obtain, requiring 10 seconds for the calculations and the preparation of the tape, which is then used to print the map on the off-line printer. The printing requires about 40 seconds.

Table 1.—*The type of first guess used for each type of analysis*

Field analyzed	First guess
500-mb. height	12-hr. forecast.
850-mb. height	Current 500-mb. chart minus forecast 850–500-mb. thickness.
300-mb. height	Estimated 300-mb. chart obtained by vertical extrapolation from 850 mb. and 500 mb.
700-mb. temperature	700-mb. temperature estimate obtained by relabeling current 850–500-mb. thickness chart.
700-mb. dewpoint depression	24-hr. forecast, if available, otherwise a constant average value.
u and v components of the wind	Geostrophic u and v components.

(All forecasts are from numerical models now in use at JNWP)

3. THE ANALYSIS SCHEME

The analysis scheme has been designed to have a maximum of flexibility, and has been used for the analysis of many different elements. The method consists essentially of using the reported data to make successive corrections to an initial guess. Suitable smoothing routines are interspersed.

In order to obtain the greatest amount of accuracy with the least amount of computation, it is essential that the first guess be as accurate an approximation to the final field as possible. Table 1 lists some of the various types of analysis which have been performed together with the first guess which was used. The choice of a first guess is in some cases arbitrary, but can easily be changed if a better guess can be obtained by some other means. The use of a forecast of any element as a first guess for the analysis at a subsequent time would involve complications from boundary errors if the forecast and analysis grid were of much smaller size than that shown in figure 1.

The analysis is then performed in a series of "scans"; i.e., passes through the field. In a given scan, the grid points are considered successively. The values of the first guess of the element to be analyzed are corrected according to the reported data. In order to illustrate the correction process, the analysis of 500-mb. heights from mixed wind and height data will be considered.

If height only is reported by an observing station, a correction C_h is computed for a nearby grid point, as given by the relation

$$C_h = -WE_h \tag{1}$$

where E_h is the error of the interpolated value of the first guess field at the location of the observation, and W is a weighting factor given by

$$W = \frac{N^2 - d^2}{N^2 + d^2}. \tag{2}$$

In the expression for W, d is the distance between the grid point and the observation, and N is the distance at which W goes to zero (only positive values of W are used). The curve of W vs. d is shown in figure 3. The simplified form for W, as compared to that used by Bergthórssen and Döös, or as compared to some suggestions received, was chosen in order to minimize the amount of computing required. The weighting factor, depending on the distance d, must be computed a very large number of times in each scan, and the computation time must be kept short. Since N^2 is fixed for a given scan, the computation of a value of W involves one each multiplication, subtraction, addition, and division, and is economical.

If both a height and a wind are reported, a correction C_v is computed for a nearby grid point, as given by

$$C_v = W\left[D_o + \frac{kf}{mg}(v\Delta x - u\Delta y) - D_g\right] \tag{3}$$

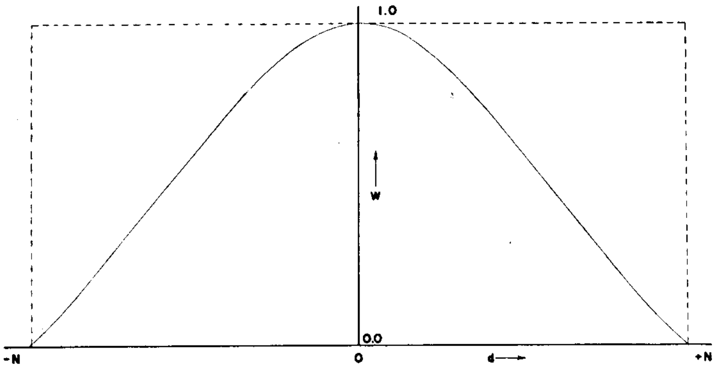

FIGURE 3.—Curve of the weighting function W vs. distance d. Solid line refers to equation (2). Dashed line refers to recent changes for scan 4 (see text).

Where D_o is the observed height at the location of the observation, u and v are the observed wind components in the x and y directions at the same point, f is the Coriolis parameter, g is the acceleration of gravity, m is the map scale factor, Δx and Δy are the components of the map distance from the observing point to the grid point, and D_g is the value of the height at the grid point in the first guess. The factor k represents the average ratio of the geostrophic to the actual wind, and is set at 1.08 (see [8]). This amounts to passing a plane in the xyz coordinates through the observation, multiplying the increment between this plane and the first guess of the pressure surface by W, and adding the resulting increment to the first guess.

If only a wind is reported, C_v is determined by equation (3) except that D_o is then the height interpolated from the first guess at the location of the observation.

All the data within the radius N of the grid point under consideration are considered. Each observation reporting a height is used in the computation of a value of C_h, and each observation reporting a wind is used in computing a value of C_v. Thus for a station reporting both height and wind there is a value of C_h and a value of C_v. After the values of C_h and C_v have been computed, the correction C to be applied is determined as a weighted mean; i.e.,

$$C=\frac{A\Sigma C_h+\Sigma C_v}{An_h+n_v}, \tag{4}$$

where n_h and n_v are the numbers of C_h and C_v values respectively, and A is a weighting factor. The factor A can be regarded as the weight given to the lateral gradients of the first guess as compared to the observed winds, which have a weight of one, and is given the value of ¼. The value given to A naturally depends on the quality of the lateral gradients of the first guess. For example, in the computation of 100-mb. analyses, the first guess is taken as the chart for the previous time. Since it is desired to fit the observed winds quite closely, A is taken to be 2^{-5}. A short series of tests with various values is desirable in order to produce the best value of A for a given type of analysis.

In a given scan, a correction is computed for each grid point within radius N of any reporting station.

The feature of this analysis scheme which differs most from others referred to earlier is that a series of scans is made using successively decreasing values of N. The fact that the various corrections (computed for a grid point from data with the circle of radius N) are averaged to obtain a single correction (equation (4)) means that a type of smoothing has taken place over this circle. The use of a series of scans with decreasing N values allows the analysis of a spectrum of scales. The largest value of N used can be set to permit the correction of the largest-scale errors in the first guess, while the smallest value sets a lower limit to the scale that can be analyzed.

The machine program has been written to permit a large flexibility in the number of scans and in the size of the values of N. Table 2 shows the values used for 500-mb. analysis up to this time. The size of the area considered during the first scan is so large that heights only are considered during this scan. On all subsequent scans both heights and winds are used.

Quite recently it has been determined that a better horizontal resolution of the smaller-scale components of the 500-mb. height field can be obtained by modification of the weighting factor used during the last scan to a constant value of 1.00 from the observation location out to the distance N, where it becomes zero. The value of N for the fourth scan is set at 1.00 grid lengths. This revised weighting factor is indicated by the dashed line in figure 3.

Early trials with the analysis indicated that in an area where the first guess was very poor, slight discontinuities tended to develop at a distance of 3 to 4 grid lengths from an isolated observation; i.e., at the distance where the weighting factor is rapidly approaching zero. This was corrected economically by the introduction of a smoothing program. This program makes passes of the field D using the smoothing function

$$\overline{D}=\tfrac{1}{2}D+\tfrac{1}{8}\Sigma D \tag{5}$$

where ΣD refers to the sum of the values of D at the four nearest grid points. During a 500-mb. analysis the smoothing is done one time between scans 2 and 3 and one time between scans 3 and 4.

In an operational system where a forecast from a previous analysis is used as a first guess for the next anal-

TABLE 2.—*Values of N corresponding to successive scans.*

Scan No.	N (grid lengths)
1	4.75
2	3.60
3	2.20
4	1.80

ysis, the above smoothing has the additional function of gradually reducing the amplitude of smaller-scale systems which continually remain in areas of very little or no data. This is believed to be a desirable attribute of the system, since there is no known method of forecasting which permits an accurate description of small-scale systems a number of days in advance.

Since the 500-mb. height field is used as input to the balance equation (Shuman [11]) the conclusion of the analysis is used as an opportunity to apply the restriction $\zeta_g > -f/2$, where ζ_g is the geostrophic relative vorticity. This is done by making successive modifications of the height field until the required criterion exists. When applied at 500 mb. and at lower levels this usually results in modifications of the height field which are smaller than the probable errors of the observations. At higher levels this seems at times to be an unrealistic criterion to apply, since the required height changes can greatly exceed the probable data errors.

Finally, a smoothing is applied to the height field with the use of a smoothing operator (Shuman [12]) which eliminates components of wavelength of 2 grid units, while leaving those of 5 or more grid units wavelength practically unchanged.

The results are printed in two forms. First is the printed 500-mb. chart. Following this, a chart is printed showing the difference between the final result and the first guess. This latter chart is useful in monitoring, and for detection of special errors and any unusual problems which may arise.

4. AUTOMATIC ERROR DETECTION AND REMOVAL

Errors of all sizes can still occur in the data furnished to the analysis program. The checks for vertical and internal consistency as well as the comparison of duplicating reports made in the pre-analysis processing do not remove all errors. For example, there may be a partially garbled report which was too damaged to permit the usual checks. The remaining data could contain an error.

As mentioned earlier, it is necessary to obtain an interpolated value of the height field of the previous scan or guess field at the location of an observation before making the next scan. At the same time an interpolated value of the geostrophic wind is computed. Before each scan, a comparison is made between the interpolated and reported winds and heights. If the difference exceeds a certain value, the reported value is rejected and erased from the memory of the computer. The limits set as rejection criteria are listed in table 3. As a last check the stations whose reports deviate from the final analysis by more than 200 ft. or 40 knots are listed for the information of the monitoring analyst.

When data are selected for rejection by the above process, the location of the reporting station is listed on the printer, together with the type of rejection. If the monitoring analyst chooses, he can cause the computer to stop after each data listing. By the use of certain switches, he can cause the listed data to be rejected or retained and used, as he wishes. However, ordinarily all listed data are rejected.

Table 3.—*Maximum permissible difference between data and the interpolated field values of the previous scan for 500 mb.*

Prior to scan No.—	Height difference	Vector wind difference
	Feet	*Knots*
1	700	90
2	600	70
3	500	70
4	350	60

The use of successively decreasing error limits as shown in table 3 represents a gradual tightening up of the error toleration as the analysis is refined by successive iterations. Observations differing unreasonably from the first guess are rejected immediately before scan 1. Observations differing excessively from their neighbors are rejected later. As an average number, about two or three reports are rejected during the course of a hemispheric analysis, in which 400 to 500 reporting stations are used. Naturally, there are many more errors in the data than two or three, but the majority are removed or corrected in the pre-analysis processing.

5. OPERATIONAL USE OF THE ANALYSIS PROGRAM

The total time required on the IBM 704 computer for a 500-mb. analysis over the grid of figure 1 averages about 15 minutes, with some variation depending on the amount of data received. All maps are printed on the off-line printer, requiring the additional time of 3 minutes each for the last two maps. The time required for the 850-mb. analysis is about the same. An analysis of dewpoint depression has been done 3 times weekly for experimental quantitative precipitation forecasts and requires only about 9 minutes, since experience has shown that fewer iterations of this analysis are required. Maps of any single field for the United States area require a total of 6 minutes, since this occupies only a small fraction of the total grid area. The time of 6 minutes represents almost an irreducible minimum with the present program due to the required overhead of sorting, tape handling, etc.

The 500-mb. analysis has been done by computer at the JNWP Unit every 12 hours for 1½ years, starting in early April 1958. Since that time no hand analyses have been made. The 850-mb. analysis has been done by computer for over a year, starting in late summer of 1958. The only manual interventions required come in the form of additional reports, sometimes synthetic, which are introduced by the monitoring analyst.

For the 500-mb. analysis, the additional reports are of four types. The first represents a hand-editing of reports from non-reconnaissance aircraft. These reports, in so-called plain language, are at the present time too unsystematic in form and doubtful in accuracy to be edited automatically. The second type represents a vertical extrapolation of surface data to 500 mb. This is done occasionally in areas of surface but no upper air data, especially when it is thought that a failure might occur in the 12-hr. forecast because of sudden cyclogenesis. The third type is entered in certain areas where the data are always received too late for current use (more than 6¼ hr. after observation time). In such areas selected pieces of the late data are modified by the 12-hr. forecast and entered as current data on the next map. The Siberian Arctic coast is one area where such steps are necessary. Finally, there are always some reports which are received too badly garbled for machine recognition. These are indicated on the monitoring list provided by the automatic data processing program. In some of these cases, the monitoring analyst can retrieve the 500-mb. data and enter them by hand.

For the 850-mb. analysis a number of surface ship reports are entered by hand for oceanic areas of sparse upper air data. These are automatically extrapolated by the computer to 850 mb. These must be entered by hand due to the large frequency of errors in the ship position reports, which makes automatic editing risky. In principle one can handle this problem automatically by simulating the manual procedures; i.e., by keeping a running log of several days on the position of the ships and by correcting the erroneous position reports. However, the problem is complicated by the lack of identification of many ships and by errors in the identification of others. Nevertheless, we plan to attempt to program a system of this type before long.

It can be seen that most of the hand entering of data is necessitated by imperfections of the meteorological reporting and communication methods; i.e., by slow or inaccurate communications from remote areas and by insufficient checking information in the reports. In time these situations may improve, permitting increased automation.

6. GEOSTROPHIC AND NON-GEOSTROPHIC ANALYSIS

The analysis scheme described above, when used for analysis of the height of a pressure surface, needs a specification of the relation between the wind and the height gradient. In equation (3), this is specified as being essentially geostrophic. However, it is important to note that this specification applies less and less strictly as one goes farther from an observing station, with the result that the final analysis is not necessarily very geostrophic. The departures from geostrophic winds shown in the analysis can be considerable in an area of high observation density where the reported winds and heights do not agree well with the geostrophic approximation.

In [4] an analysis experiment was described in which the wind relation used was given by the balance equation; i.e.,

$$\nabla^2\psi=\frac{1}{f}\left\{\nabla^2\phi+2\left[\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2-\frac{\partial^2\psi}{\partial x^2}\frac{\partial^2\psi}{\partial y^2}\right]-\nabla\psi\cdot\nabla f\right\}, \tag{6}$$

and

$$u=-\frac{\partial\psi}{\partial y},\quad v=\frac{\partial\psi}{\partial x}, \tag{7}$$

where ϕ is the height of the pressure surface and ψ is the stream function.

An analogous experiment was performed with the analysis program described in this paper, with the use of the following procedure:

(a) Using a 24-hour forecast as first guess (for consistency with the other examples shown below) the 500-mb. height analysis was made using heights only. Reported winds were not considered.

(b) Equation (6) was solved for a stream function map, using the heights of (a) as input data.

(c) Using the stream function map of (b) as first guess, an analysis was made using only the reported winds. For this purpose equation (3) was changed to

$$C_v=W\left[\psi_o+\frac{1}{m}\,(v\Delta x-u\Delta y)-\psi_g\right] \tag{8}$$

The symbols have the meanings given previously except that ψ_o and ψ_g are the stream functions of the first guess at the observation point and the grid point, respectively.

(d) Given values of ψ, equation (6) was solved for the height field.

Insufficient analyses of this type have been done to show any pronounced differences between nongeostrophic and geostrophic analyses. However, it is probable that the inaccuracies arising in the solution of a Poisson equation over a large (1,977 points) grid make the accurate representation of the final heights of the pressure surface extremely difficult. This conclusion is supported by the examples given in the next section.

7. EXAMPLES

Several experiments were made with the data for 0000 GMT, August 20, 1959. For convenience in representation, the analyses were confined to the area shown in figure 4. In order to make the examples more interesting, a 24-hour forecast was used as the first guess instead of the usual 12-hour forecast. Analyses were conducted with the following variations:

(a) A complete 500-mb. analysis.

(b) An analysis using the reported heights only and disregarding the reported winds.

(c) An analysis using the reported winds only and disregarding the reported heights.

(d) An analysis using the balance equation as the wind law.

Verifications of the analyses were made by computing the r.m.s. difference between the reported heights and the heights interpolated from the analyses to the locations of each observation. The interpolation was made in the plane fit by least-squares to the four heights at the corners of the grid square containing the observations. The r.m.s. vector wind difference between reported winds and geostrophic winds from the analyses was also computed for each map. The interpolation in the analysis was made linearly in each grid square. In the case of the non-geostrophic analyses, the wind relation of equation (7) was used. The results are given in table 4. Figure 4 shows the height analysis obtained by item (2) in table 4, together with the data used.

One of the most interesting results shown in table 4 is the excellent quality of the analysis obtained when using only the reported winds. In an analysis for winds only, the mean height over the area is not changed. If the area analyzed is large enough that the mean algebraic height error of the first guess is very small, the heights of the resulting analysis will be accurate. So far as this example is representative, one could conclude that most of the information regarding the 500-mb. contours in a dense data area is contained in the winds, the heights being relatively redundant information.

Even in an area of dense data the results of this analysis scheme are not completely independent of the first guess. The slightly lower height error obtained with the 24-hour forecast as first guess as compared with the 12-hour forecast is probably attributable to the apparent diurnal variations of the 500-mb. surface.

Table 4.—*Comparison of analyzed values and the data for experiments on August 20, 1959, 0000* gmt.

	r.m.s. height error (meters)	r.m.s. wind error (meters/sec.)
(1) 24-hr. forecast	35.3	7.0
(2) height and winds used: 24-hr. forecast as 1st guess	14.0	5.2
(3) heights only: 24-hr. forecast as 1st guess	18.2	6.0
(4) winds only: 24-hr. forecast as 1st guess	14.6	5.2
(5) non-geostrophic analysis	17.8	5.2
(6) heights and winds used: 12-hr. forecast as 1st guess	16.2	4.9

8. DIRECT WIND ANALYSIS

An analysis of the u and v components of the wind can be made by the above method by obtaining a first guess of the components of the wind from a stream function map. Then by using the correction equation (1) and by going through a series of scans, as described earlier, a separate analysis of each wind component can be obtained. This adaptation of the analysis program was made by Mr. A. Kneer and Capt. J. Neilon, USAF, of the JNWP Unit, who made several such analyses. The divergence field computed from one of these analyses is shown in figure 5.

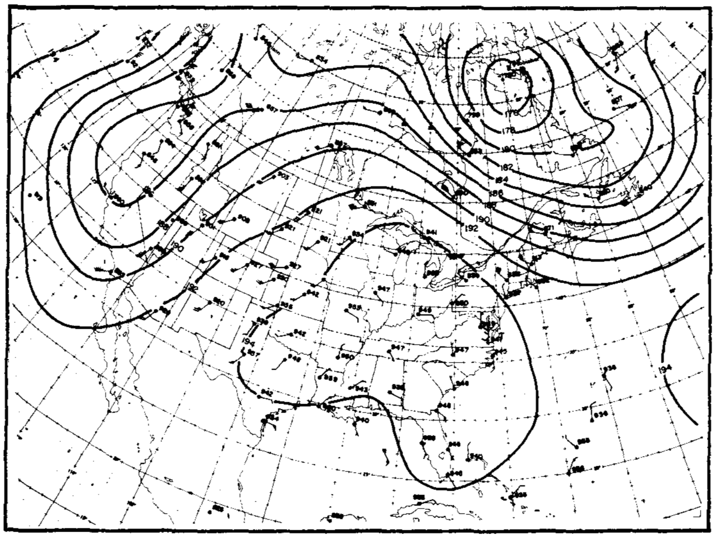

Figure 4.—Objective analysis for a North American section of the map for 0000 gmt, August 20, 1959. The contours were traced from the band-contour output of the IBM 717 printer.

Referring to figure 4, we can see that the divergence field of figure 5 is relatively smooth and has believable magnitudes in areas of dense wind coverage. In other areas, the magnitudes seem suspiciously large for 500 mb. in a summer situation. For example, the wind at Churchill, Manitoba, seems to be considerably subgeostrophic on figure 4, and is largely responsible for the relatively strong convergence and divergence patterns indicated to the west and east of that station. The reality of this situation is difficult to estimate without additional supporting data.

Variations in the analysis procedure were tried, but did not effect changes in the divergence of more than 20 percent. It is therefore concluded that the divergence pattern of figure 5, over the United States, is indicated by

Figure 5.—Horizontal divergence computed from a direct wind field analysis. Values are in units of 10^{-6} sec.$^{-1}$. Map for 500 mb., 0000 gmt, August 20, 1959.

the observed winds, and does not depend very much on the details of the analysis procedure. Further studies will be made of divergence patterns using this method of analysis.

9. THE PROBLEM OF INTERNAL CONSISTENCY

The internal consistency required of an analysis system depends on the use to be made of the analysis. Since direct wind-field analysis over a large part of the hemisphere does not seem feasible with today's data, forecast models of the next few years will probably have to use the height field and its derivatives as input data. A procedure suitable for currently-used multi-level models has been to use the first two surfaces analyzed in an extrapolation to the other levels (see Brown [3]). This extrapolation is used as the first guess, and is modified only by reported data. However, this method still does not insure that the implied static stability will everywhere remain positive if the surfaces analyzed are close together in the vertical. For this problem it is planned to use a special program which makes the necessary small adjustments in the heights of the pressure surfaces to insure positive stabilities everywhere.

10. CONCLUSIONS

The analysis system described above has proven to be a flexible, inexpensive, and satisfactory system for the present requirements of numerical prediction, and has been in continuous operational use for over a year.

For more advanced prediction models, particularly those using the "primitive" equations of motion, the demands for internal consistency may exceed the capabilities of this system. Suggestions for more elegant methods of achieving internal consistency have been made by Sasaki [10].

The problem of automatic recognition and correction or removal of erroneous data is a very urgent one. The methods described in this paper have worked fairly well in operational use. However, a much more satisfactory solution would involve improved methods of automatic detection and correction of the communication errors directly on the circuits, since these account for the majority of the "erroneous" reports.

ACKNOWLEDGMENTS

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REFERENCES

1. H. A. Bedient and G. P. Cressman, "An Experiment in Automatic Data Processing," *Monthly Weather Review*, vol. 85, No. 10, Oct. 1957, pp. 333–340.
2. P. Bergthórssen and B. Döös, "Numerical Weather Map Analysis," *Tellus*, vol. 7, No. 3, Aug. 1955, pp. 329–340.
3. J. A. Brown, "Multiple Linear Regression Equations Expressing Heights of Certain Pressure Surfaces," Technical Memorandum No. 15, Joint Numerical Weather Prediction Unit, April 1959, 5 pp.
4. G. P. Cressman, "An Objective Analysis Study," Technical Memorandum No. 12, Joint Numerical Weather Prediction Unit, June 1957, 12 pp.
5. B. Gilchrist and G. P. Cressman, "An Experiment in Objective Analysis," *Tellus*, vol. 6, No. 4, Nov. 1954, pp. 309–318.
6. O. Haug, "A Method for Numerical Weather Map Analysis," *Scientific Report*, No. 5, Det Norske Meteorologiske Institutt, 1959, 10 pp.
7. D. H. Johnson, "Preliminary Research in Objective Analysis," *Tellus*, vol. 9, No. 3, Aug. 1957, pp. 316–322.
8. M. Neiburger, T. Sherman, W. W. Kellogg, and A. F. Gustafson, "On the Computation of Wind From Pressure Data," *Journal of Meteorology*, vol. 5, No. 3, June 1948, pp. 87–92.
9. H. A. Panofsky, "Objective Weather Map Analysis," *Journal of Meteorology*, vol. 6, No. 6, Dec. 1949, pp. 386–392.
10. Y. Sasaki, "An Objective Analysis Based on the Variational Method," *Journal of Meteorological Society of Japan*, Series II, vol. 36, No. 3, June 1958, pp. 77–88.
11. F. G. Shuman, "Numerical Methods in Weather Prediction: I. The Balance Equation," *Monthly Weather Review*, vol. 85, No. 10, Oct. 1957, pp. 329–332.
12. F. G. Shuman, "Numerical Methods in Weather Prediction: II. Smoothing and Filtering," *Monthly Weather Review*, vol. 85, No. 11, Nov. 1957, pp. 357–361.