Mesoscale Meteorology

1   Introduction

In this class, we will try to understand many of the mesoscale phenomena that we encounter in real life.

Such phenomena include, but are not limited to:

- mountain waves
- density currents
- gravity waves
- land/sea breezes
- heat island circulations
- clear air turbulence
- low-level jets
- fronts
- mesoscale convective complexes
- squall lines
- supercells
- tornadoes
- hurricanes

We will focus on the physical understanding of these phenomena, and use dynamic equations to explain their development and evolution.

First, we must define ***mesoscale***.

1.1   Definition of Mesoscale

We tend to classify weather systems according to their *intrinsic or characteristic time and space scales*.

Often, *theoretical considerations* can determine the definition. There are two commonly used approaches for defining the scales: dynamical and scale-analysis.

1.1.1 Dynamical

The dynamical approach asks questions such as the following:

- What controls the time and space scales of certain atmospheric motion?
- Why are thunderstorms a particular size?
- Why is the planetary boundary layer (PBL) not 10 km deep?
- Why are raindrops not the size of baseball?
- Why most cyclones have diameters of a few thousand kilometers not a few hundred of km?
- Tornado and hurricanes are both rotating vortices, what determine their vastly different sizes?

There are theoretical reasons for them! There are many different scales in the atmospheric motion. Let's look at a couple of examples:
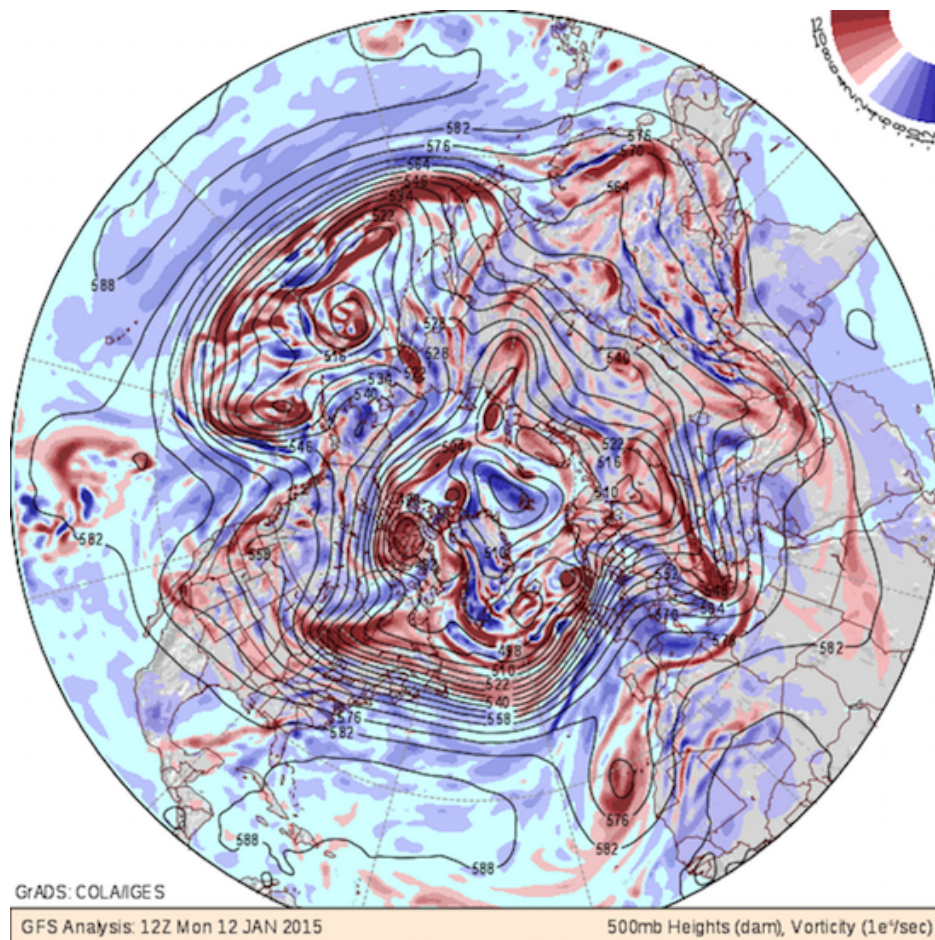
Figure 1: Hemispherical plot of 500mb height (contour) and vorticity (color) showing planetary scale waves.
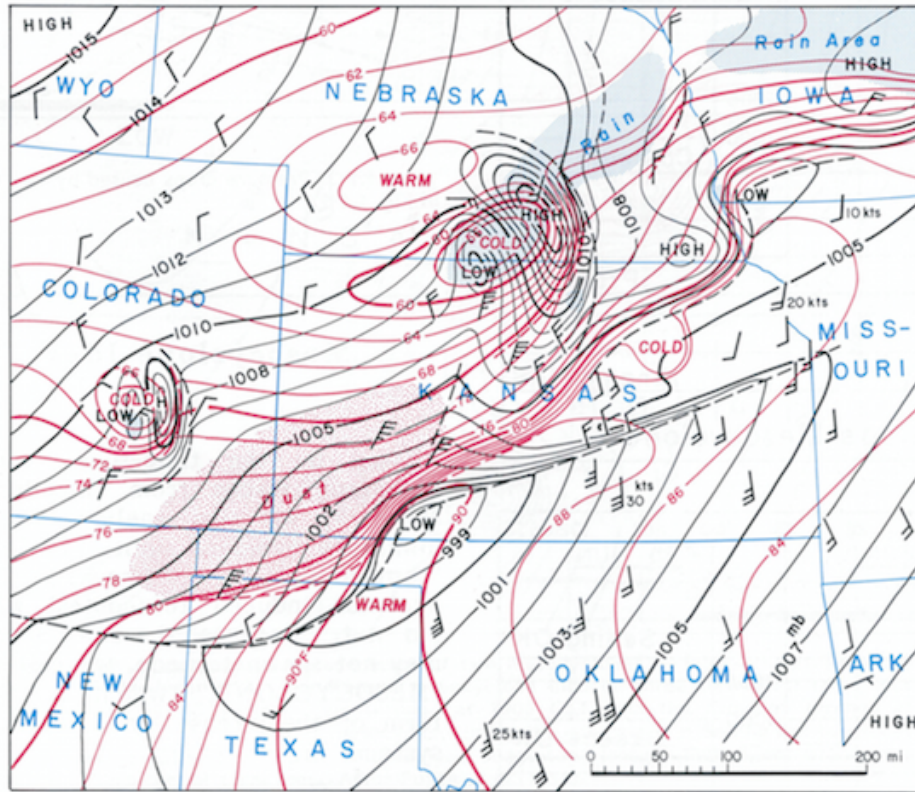
Figure 2: Sea level pressure (black contours) and temperature (red contours) analysis at 0200 CST 25 June 1953. A squall line was in progress at the time in northern Kansas, eastern Nebraska, and Iowa. [From Markowski and Richardson 2010]

1.1.2 Atmospheric Energy Spectrum

Atmospheric motions exist continually across space and time scales. Spatial scales range from $\sim 0.1\ \mu$m (mean free path of molecules) to $\sim 40,000$ km (circumference of Earth), while temporal scales range from sub-second (small-scale turbulence) to multi-week (planetary-scale Rossby waves). Meteorological features with short (long) time scales are generally associated with small (large) space scales. The ratio of horizontal space and time scales is approximately the same order of magnitude (~ 10 ms^{-1}) for these features.

There are a few dominant time scales in the atmosphere when looking at the kinetic energy spectrum plotted as a function of time (see Fig. 3). The figure also shows that the *energy spectrum* of the atmospheric motion is actually *continuous*!

There is a local peak around one day (associated with the diurnal cycle of solar heating), and a large peak near one year (associated with the annual cycle due to the change in the earth's rotation axis relative to the sun). These time scales are mainly determined by forcing external to the atmosphere.

There is also a peak in the a few days up to about one month range. These scales are associated with synoptic scale cyclones up to planetary-scale waves. There isn't really any external forcing that is dominant at a
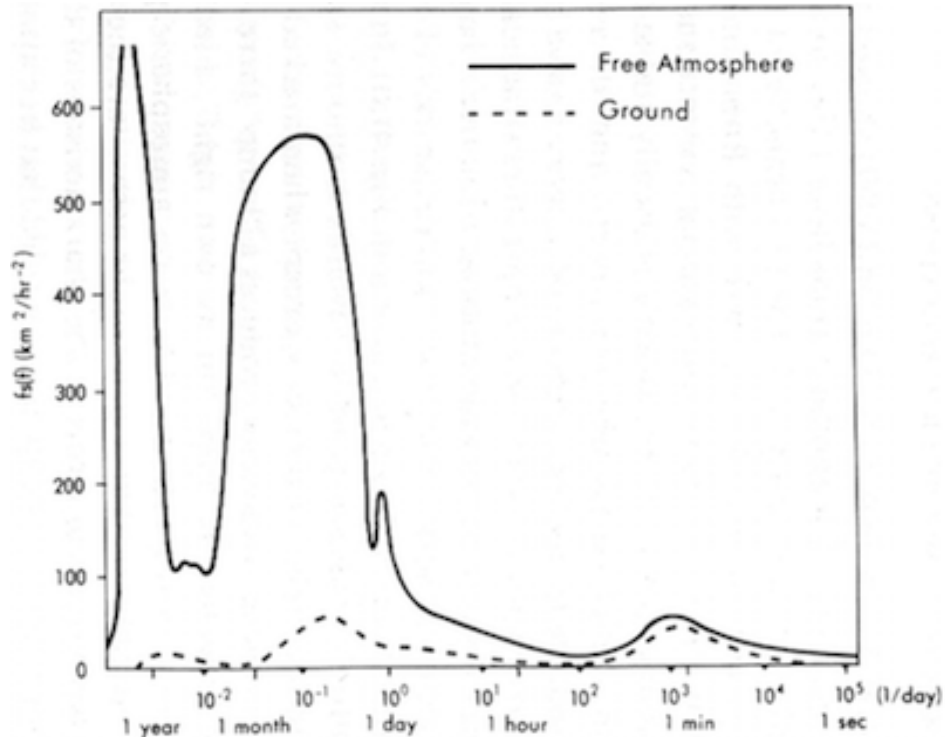
Figure 3: Average kinetic energy of west-east wind component in the free atmosphere (solid line) and near the ground (dashed line). (After Vinnichenko 1970; see also Atkinson 1981)

period of a few days. Thus, this peak must be related to the something internal to the atmosphere. It is actually the scale of the most unstable atmospheric motion, such as those associated with baroclinic and barotropic instability.

There is another peak around one minute. This appears to be associated with small-scale turbulent motions, including those found in convective storms and the PBL.

There appears to be a 'gap' between several hours to $\sim$ 30 minutes (there remain disputes about the interpretation of this 'gap'). This 'gap' actually corresponds to the *mesoscale*, the subject of our class.

We know that many weather phenomena occur on the *mesoscale*, although they tend to be intermittent in both time and space. The intermittency (unlike the ever present large-scale waves and cyclones) may be the reason for the 'gap'.

Mesoscale is believed to play an important role in transferring energy from large scales down to the small scales.

Quoting from Dr. A. A. White, of the British Met Office: *At any one time there is not much water in the outflow pipe from a bath, but it is inconvenient if it gets blocked.*

It's like the mid-latitude convection – it does not occur every day, but we can not do without it. Otherwise, the heat and moisture will accumulate near the ground and we will not be able live at the surface of the earth!

1.1.3 Energy Cascade

As scales decrease, we see finer and finer structures. Many of these structures are due to certain types of instabilities that inherently limit the size and duration of the phenomena. Also, there exists the exchange of energy, heat, moisture, and momentum among all scales.

Most of the energy transfer in the atmosphere is downscale – starting from differential heating with latitude and land-sea contrast on the planetary scales. Energy in the atmosphere can also transfer upscale, however. We call the energy transfer among scales the *energy cascade*.

Example: A thunderstorm feeds off convective instabilities (as measured by CAPE) created by *e.g.*, synoptic-scale cyclones. A thunderstorm can also derive part of its kinetic energy from the mean flow. The thunderstorm in turn can produce tornadoes by concentrating vorticity into small regions. Strong winds in the tornado create turbulent eddies which then dissipate and eventually turn the kinetic energy into heat. Convective activities can also feed back into the large scale and enhancing synoptic scale cyclones (up-scale transfer).

What does this have to do with the *mesoscale*?

It turns out that the definition of the mesoscale is not easy. Historically, the mesoscale was first introduced by Ligda (1951) in an article reviewing the use of weather radar. It was described as the scale between the visually observable convective storm scale (a few kilometers or less) and the limit of resolvability of a synoptic observation network – *i.e.*, it was a scale that could not be observed. The mesoscale was, in early 1950's, anticipated to be observed by weather radars.
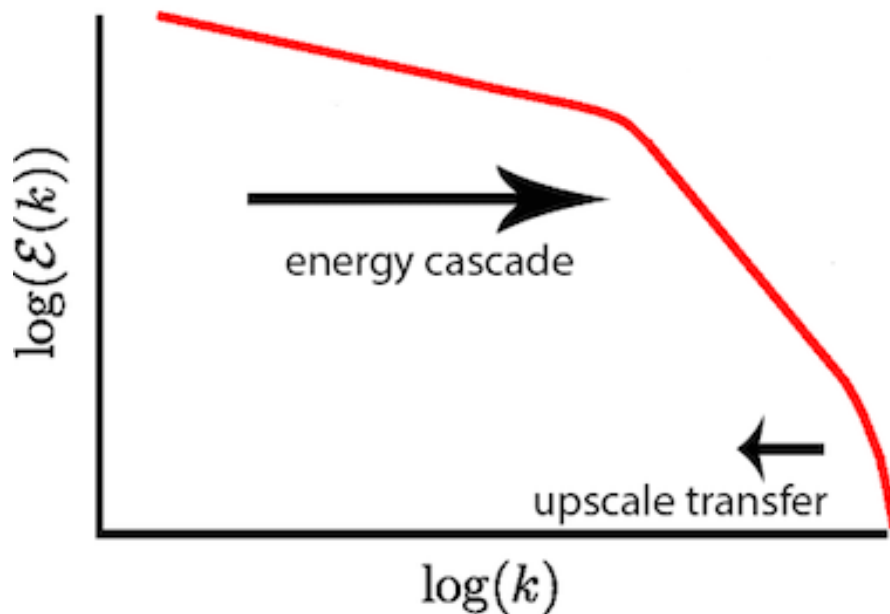

Figure 4: Depiction of the energy cascade. Here, E is energy and k is wavenumber.

1.1.4 A Rough Definition

The mesoscale may be considered the scale for which both ageostrophic advection and Coriolis parameter f are important, which is smaller than the *Rossby radius of deformation* ($L = NH/f \sim 1000$ km). In other words, it is the scale of atmospheric motions that are driven by a variety of mechanisms rather than by one dominant instability.

For now, let us use a more qualitative definition and try to relate the mesoscale to something more concrete. Roughly consider that the word *mesoscale* defines meteorological events having spatial dimensions of the order of one state. Thus, individual thunderstorms or cumulus clouds are excluded since their scale is on the order of a few kilometers. Similarly, synoptic-scale cyclones are excluded since their scale is on the order of several thousands of kilometers.

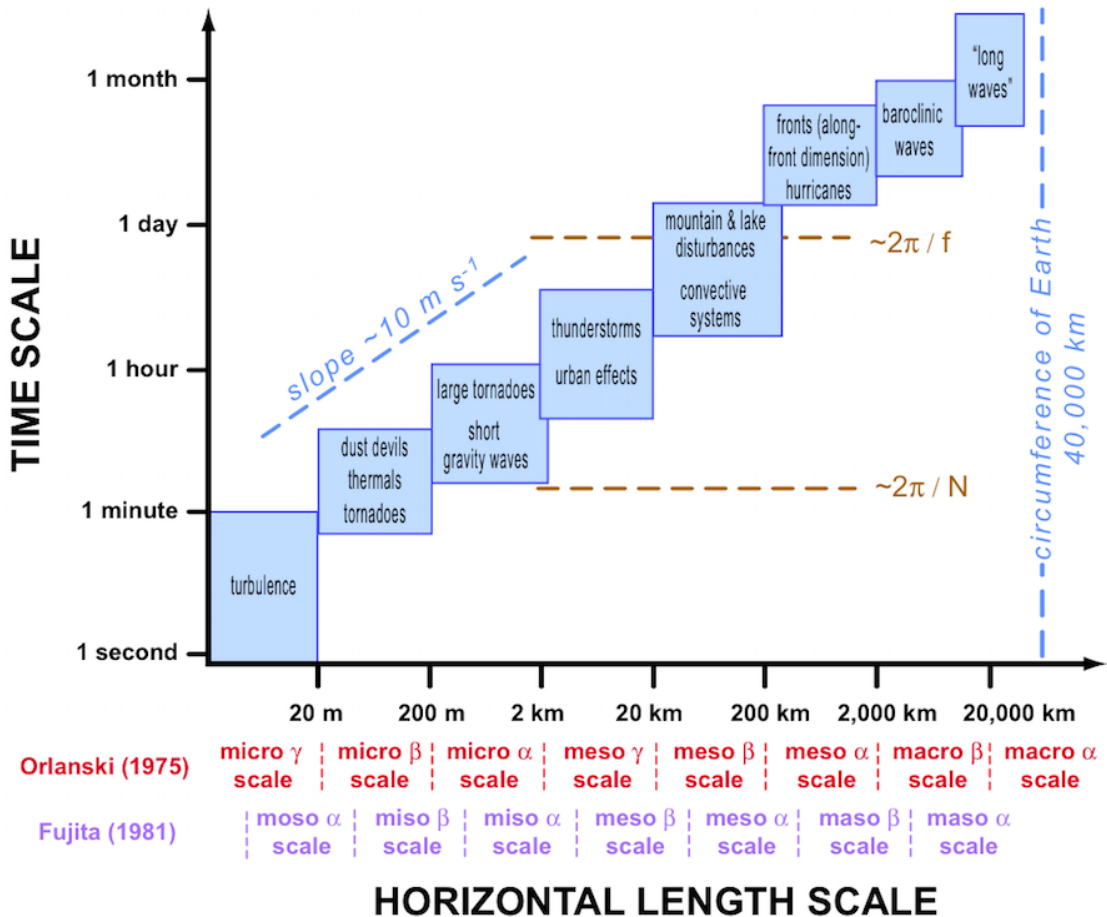### 1.1.5 Classification of Scales

Figure 5: Scale definitions and the characteristic time and horizontal length scales of a variety of atmospheric phenomena. Classification schemes from Orlanski (1975) and Fujita (1981) also are indicated. [From Markowski and Richardson 2010]

1.1.6   Scale Analysis

Some temporal and spatial scales in the atmosphere are obvious.

Time scales:

- diurnal cycle
- annual cycle
- inertial oscillation period – due to Earth's rotation, the Coriolis parameter f
- advective time scale – time taken to advect over certain distance

Space scales:

- global – related to earth's radius
- scale height of the atmosphere – related to the total mass of the atmosphere and gravity
- scale of fixed geographical features – mountain height, width, width of continents, oceans, lakes

Scale analysis (you should have learned this tool in Dynamics) is a very useful method for establishing the importance of various processes in the atmosphere and terms in the governing equations. Based on the relative importance of these processes/terms, we can deduce much of the behavior of motion at such scales.

Consider this simple example:

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv$$

What is this equation? Can you identify the terms in it?

With scale analysis, we try to assign the characteristic values for each of the variables in the equation in order to estimate the magnitude of each term and determine their relative importance. For example,

$$\frac{du}{dt} \sim \frac{\Delta u}{\Delta t} \sim \frac{V}{T},$$

where V is the velocity scale (typical magnitude or amplitude if described as a wave component), and T the time scale (typical length of time for velocity to change by Δu, or the period for oscillations). It should be emphasized here that it is the typical magnitude of change that defines the scale, which is not always the same as the magnitude of the quantity itself. The absolute temperature is a good example, as is surface pressure.

For different scales, the terms in the equation of motion have different importance. This leads to different behavior of the motion, meaning there is a dynamic significance to the scale!

1.1.7 Synoptic Scale Motion

Let's do the scale analysis for the *synoptic scale motion* using the horizontal equation of motion

- $V \sim 10 \ \text{ms}^{-1}$ (the typical variation or change in horizontal velocity over the typical distance)

- $W \sim 0.1 \ \text{ms}^{-1}$ (typical magnitude or range of variation of vertical velocity)

- $L \sim 1000 \ \text{km} = 10^6 \ \text{m}$ (about the radius of a typical cyclone)

- $H \sim 10 \ \text{km} = 10^4 \ \text{m}$ (depth of the troposphere)

- $T \sim L/V \sim 10^5 \ \text{s}$ (the time for an air parcel to travel for 1000 km)

- $f \sim 10^{-4} \ \text{s}^{-1}$ (for the mid-latitude)

- $\rho \sim 1 \ \text{kg} \, \text{m}^{-3}$

- Δp in horizontal $\sim 10 \ \text{mb} = 1000 \ \text{Pa}$ (about the variation of pressure from the center to the edge of a cyclone. Note that it is the typical variation that determines that typical scale, not the value itself, as in this example. Using the scale of 1000 mb will give you the wrong result)

$$\frac{\partial u}{\partial t} \qquad +u\frac{\partial u}{\partial x} \qquad +w\frac{\partial u}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad +fv$$

$$\frac{V}{T} \qquad\qquad \frac{VV}{L} \qquad\qquad \frac{WV}{H} \qquad\qquad \frac{\Delta p}{\rho L} \qquad\qquad fV$$

$$\frac{10}{10^5} \qquad\qquad \frac{10^2}{10^6} \qquad\qquad \frac{0.1 \times 10}{10^4} \qquad\qquad \frac{10^3}{1 \times 10^6} \qquad\qquad 10^{-4} \times 10$$

$$10^{-4} \qquad\qquad 10^{-4} \qquad\qquad 10^{-4} \qquad\qquad 10^{-3} \qquad\qquad 10^{-3}$$

It turns out that the time tendency and advection terms are one order of magnitude smaller for synoptic scale flows.

The pressure gradient force and Coriolis force are in rough balance. What kind of flow do you get in this case? The *quasi-geostrophic* flow.

Similar scale analysis can be performed on the vertical equation of motion.

- Δp over vertical length scale $H \sim 1000 \text{ mb} = 10^5 \text{ Pa}$

$\dfrac{\partial w}{\partial t}$	$+u\dfrac{\partial w}{\partial x}$	$+w\dfrac{\partial w}{\partial z}$	$= -\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}$	$-g$
$\dfrac{W}{T}$	$\dfrac{VW}{L}$	$\dfrac{WW}{H}$	$\dfrac{\Delta p}{\rho H}$	g
$\dfrac{0.1}{10^5}$	$\dfrac{10 \times 0.1}{10^6}$	$\dfrac{0.1 \times 0.1}{10^4}$	$\dfrac{10^5}{1 \times 10^4}$	10
10^{-6}	10^{-6}	10^{-6}	10	10

Clearly, the vertical acceleration is much smaller than the vertical pressure gradient term and the gravitational term, which are of the same order of magnitude.

The balance between these two terms gives the hydrostatic balance, and this balance is a very good approximation for synoptic scale flows.

Also, the vertical motion is much smaller than the horizontal motion. We can deduce the latter from the mass continuity equation ($\nabla \cdot \vec{V} \approx 0$).

Therefore, we obtain, based on the scale analysis along, the following basic properties of flows at the synoptic-scale flows: such flows are quasi-two-dimensional, quasi-geostrophic and hydrostatic.

We saw a good example of horizontal scale determining the dynamics of motion!

In summary, synoptic (and up) scale flows are

- quasi-two-dimensional (because $w << u$)

- hydrostatic (we can see it by performing scale analysis for vertical equation of motion)

- Coriolis force is a dominant term in the equation of motion and it is in rough balance with the PGF

- the flow is quasi-geostrophic

1.1.8 Mesoscale Motion

What about the mesoscale? We said earlier we define the mesoscale to be on the order of hundreds of kilometers in space and hours in time. Repeat the scale analysis done earlier:

$$\frac{\partial u}{\partial t} \qquad +u\frac{\partial u}{\partial x} \qquad +w\frac{\partial u}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad +fv$$

$$\frac{V}{T} \qquad \frac{VV}{L} \qquad \frac{WV}{H} \qquad \frac{\Delta p}{\rho L} \qquad fV$$

$$\frac{10}{10^4} \qquad \frac{10^2}{10^5} \qquad \frac{1\times 10}{10^4} \qquad \frac{10^2}{1\times 10^5} \qquad 10^{-4}\times 10$$

$$10^{-3} \qquad 10^{-3} \qquad 10^{-3} \qquad 10^{-3} \qquad 10^{-3}$$

we see that the all terms in the equation are of the same magnitude – none of them can be neglected – we no longer have geostrophy! For the vertical direction, the hydrostatic approximation is still reasonable for the mesoscale.

In the vertical direction:

$$\frac{\partial w}{\partial t} \qquad +u\frac{\partial w}{\partial x} \qquad +w\frac{\partial w}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial z} \qquad -g$$

$$\frac{W}{T} \qquad \frac{VW}{L} \qquad \frac{WW}{H} \qquad \frac{\Delta p}{\rho H} \qquad g$$

$$\frac{1}{10^4} \qquad \frac{10\times 1}{10^5} \qquad \frac{1\times 1}{10^4} \qquad \frac{10^5}{1\times 10^4} \qquad 10$$

$$10^{-4} \qquad 10^{-4} \qquad 10^{-4} \qquad 10 \qquad 10$$

Therefore, mesoscale motion is not geostrophic (i.e., the ageostrophic component is significant – see earlier definition), the Coriolis force remains important, and hydrostatic balance is roughly satisfied. The motion is quasi-two-dimensional ($w << u$ or v). This can be a *dynamic definition of the mesoscale*, or in Orlanski's definition, the *meso-β* scale.

In summary, meso-β scale flows are

- quasi-two-dimensional

- nearly hydrostatic

- Coriolis force is non-negligible

We can go one step further down the scale, looking at cumulus convection or even supercell storms: $L \sim 10$ km, $V \sim 10$ ms^{-1}, $T \sim 1000$ s.

$$\frac{\partial u}{\partial t} \qquad +u\frac{\partial u}{\partial x} \qquad +w\frac{\partial u}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad +fv$$

$$\frac{V}{T} \qquad\qquad \frac{VV}{L} \qquad\qquad \frac{WV}{H} \qquad\qquad \frac{\Delta p}{\rho L} \qquad\qquad fV$$

$$\frac{10}{10^3} \qquad\qquad \frac{10^2}{10^4} \qquad\qquad \frac{10^2}{10^4} \qquad\qquad \frac{10^2}{10^4} \qquad\qquad 10^{-4}\times 10$$

$$10^{-2} \qquad\qquad 10^{-2} \qquad\qquad 10^{-2} \qquad\qquad 10^{-2} \qquad\qquad 10^{-3}$$

Now we see that the Coriolis force is an order of magnitude smaller, it can therefore be neglected when studying cumulus convection that lasts for an hour or so. Again, the acceleration term is as important as the PGF.

The scale analysis of the vertical equation of motion, based on the Boussinesq equations of motion (see e.g., page 354 of Bluestein 1993) is as follows:

$$\frac{\partial w}{\partial t} \qquad +u\frac{\partial w}{\partial x} \qquad +w\frac{\partial w}{\partial z} \qquad = -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial z} \qquad +\frac{\theta'}{\bar{\theta}}g$$

$$\frac{W}{T} \qquad\qquad \frac{VW}{L} \qquad\qquad \frac{WW}{H} \qquad\qquad \frac{\Delta p}{\bar{\rho}H} \qquad\qquad \frac{\Delta\theta}{\theta_0}g$$

$$\frac{10}{10^3} \qquad\qquad \frac{10^2}{10^4} \qquad\qquad \frac{10^2}{10^4} \qquad\qquad \frac{10^2}{0.5\times 10^4} \qquad\qquad \frac{1\times 10}{300}$$

$$10^{-2} \qquad\qquad 10^{-2} \qquad\qquad 10^{-2} \qquad\qquad 2\times 10^{-2} \qquad\qquad 3\times 10^{-2}$$

Here we are using the vertical mean density as the density scale in PGF term. The Boussinesq form of equation is used because it is the residual between the PGF and buoyancy force terms that drives the vertical motion, therefore we want to estimate the terms in terms of the deviations/perturbations from the hydrostatically balanced base state.

Clearly, the vertical acceleration term is now important. Therefore, the hydrostatic approximation is no longer valid. According to the definition of Orlanski (1975), this falls into the *meso-γ* range, sometimes it's referred to as small scale or convective scale. At this scale, the flow will be ageostrophic, nonhydrostatic, and three dimensional ($w \sim u$ and v).

In summary, meso-γ scale flows are

- three-dimensional
- nonhydrostatic
- ageostrophic and the Coriolis force is negligible

As one goes further down to the micro-scales, the basic dynamics becomes similar to the small scale flows - the flow is

- three-dimensional
- nonhydrostatic
- ageostrophic and the Coriolis force is negligible

1.1.9 Summary

There are more than one way to define the scales of weather systems. The definition can be based on the time or space scale (or both) of the system. It can also be based on certain physically meaningful non-dimensional parameters (for example, Rossby number based on the Lagrangian time scale as advocated by Emanuel 1986).

The most important thing to know is the key characteristics associated with weather systems/disturbances at each of these scales, as revealed by the scale analysis. The scale analysis can lead to non-dimensional parameters in non-dimensionalized governing equations. Physically, the last approach (that based on non-dimensional parameters) makes most sense.

In the course, we will focus on the meso-β (or classical mesoscale) and meso-γ (or small or convective scale), with some coverage on meso-α and micro-α scale (e.g., tornadoes).

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