

**METR 4433 – Mesoscale Meteorology  
Spring 2007**

**Problem Set #1  
Mountain Waves**

Distributed Tuesday February 13, 2007  
Due Tuesday February 27, 2007

1. (40%) Linear mountain wave solutions over 2D sinusoidal mountain
  - (a) Verify that solutions for  $w'$  given in Eqs. (18) and (30) of Chapter 2 Lecture Notes Part I are the solutions to Eq. (5) under the forcing of a sinusoidal mountain whose profile is given by Eq. (8).
  - (b) Derive solutions for  $u'$ ,  $p'$  and  $\theta'$  given in Eqs. (21) – (23) and Eqs. (32) – (34).
  - (c) Discuss the similarity and differences of the above two sets of solutions. Use schematics if helpful.
  
2. (60%) In the case of  $l^2 \gg k^2$  (or  $Na \gg U$ ), the linear steady-state hydrostatic mountain wave solution for a constant ( $U = \text{constant}$ ) Boussinesq flow (density is assumed constant with height) with constant static stability ( $N = \text{constant}$ ), forced by a bell shaped mountain [ $h(x) = \frac{h_m a^2}{x^2 + a^2}$ ] is, given as Eq. (46) in the lecture notes:

$$\eta(x, z) = \frac{h_m a (a \cos lz - x \sin lz)}{x^2 + a^2}$$

where  $\eta$  is the vertical displacement of streamlines from their far upstream height level.

- (a) Find the solutions of variables  $w'$ ,  $u'$ ,  $p'$ , and  $\theta'$  from the above solution. Note that  $w' = U \frac{\partial \eta}{\partial x}$ .
  
- (b) Numerically calculate the height of streamlines that start at the 1, 2, 3, ... km height levels at the far upstream for a domain of 100 km wide and 13 km deep, with the bell shaped mountain located at the center of the domain, for  $a = 10$  km,  $N = 0.01 \text{ s}^{-1}$ ,  $U = 10 \text{ m s}^{-1}$ , and for two difference cases of  $h_m = 500$  m and 1.5 km. Note that the actual height of the streamlines will be  $z = z_0 + \eta$  where  $z_0 = 1, 2, 3, \dots$  km for the individual streamlines. Since the above equations assumes  $x = 0$  at the mountain peak, therefore your  $x$ -coordinate for the domain should be from  $-50$  km to  $+50$  km. Evaluate  $\eta$  at 1 km horizontal grid intervals. Draw/plot curves for these streamlines in a single graph for each of the two mountain heights. A Fortran program, called *hw1.f90* located in /home/mxue/mm2007 of Fractal.ou.edu is provided for you to create the plots. You just need fill arrays zeta, and (w\_prt and theta\_prt for the next part) using the solutions/formula that you obtained.
  
- (c) Evaluate the solutions of  $w'$  and  $\theta'$  numerically, on the same grid used earlier (100 km wide and 13 km deep, and using 1 km horizontal and 250 m vertical grid intervals) for  $a = 10$  km,  $N = 0.01 \text{ s}^{-1}$ ,  $U = 10 \text{ m s}^{-1}$ , and for  $h_m = 500$  m and 1.5 km cases, and plot the contour maps of these two fields for the two cases and discuss the plots. What is the vertical wave length of these waves and how does the mountain height affect the solutions of  $w'$ ,  $\theta'$  and  $\eta$ ? Again, assume the mountain is centered at  $x = 50$  km. Use  $\theta_0 = 300\text{K}$ ,  $g = 10 \text{ m s}^{-2}$ . Again use *hw1.f90* provided.

## Commands to run the program:

Login onto fractal.ou.edu using

```
ssh -X fractal.ou.edu -l your_user_name
```

Copy files from /home/mxue:

```
cp -rp /home/mxue/mm2007 .
```

Go into directory *mm2007*

```
cd mm2007
```

Edit *hw1.f90* program using your favorite text editor (e.g., *vi hw1.f90*). If you do not know how to edit under Linux, send the file to a PC and edit it and save it as text file, and send back.

Take a look at script *do\_run*, then run the script which compiles *hw1.f90* and runs its executable if the compilation is successful.

```
./do_run
```

If your program compiled and ran correctly, it will generate a Postscript graphics file called *hw1.ps*.

You can view the graphics on a terminal supporting X-windows, using

```
ggv hw1.ps
```

If you log onto Fractal from a PC using, e.g., SSH Secure Shell client, without X-window support, you can convert the PS file to PDF format first and copy the PDF file to your personal web space provided by SOM. Then you will be able to view the PDF file easily within a Web Browser. The steps are as follows:

```
ps2pdf hw1.ps  
cp hw1.pdf /webhost/your_use_name/http/hw1.hdf
```

Then open a web browser window and enter address *http://weather.ou.edu/~your\_user\_name/hw1.pdf*. The above does assume that directory */webhost/your\_use\_name/http* already exists, otherwise the copying will fail. In that case, you need to create the directory first, by entering:  
*mkdir /webhost/your\_use\_name/http*. Replace *your\_user\_name* with your user name on Fractal.

When you are satisfied with the graphics output, you can print it on the printer in Room 5402 from Fractal (make sure you pick up your own output) by entering

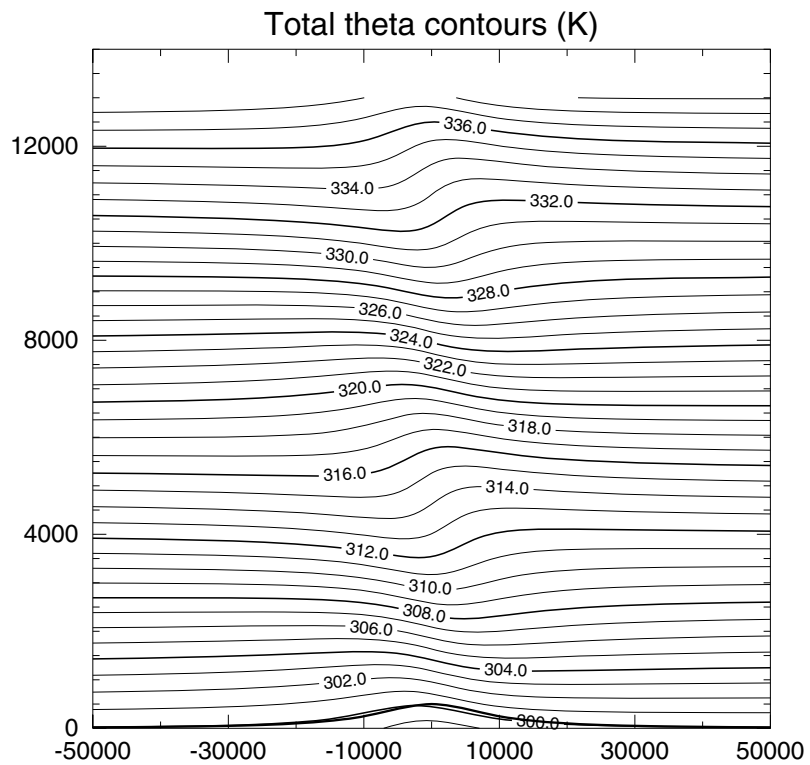
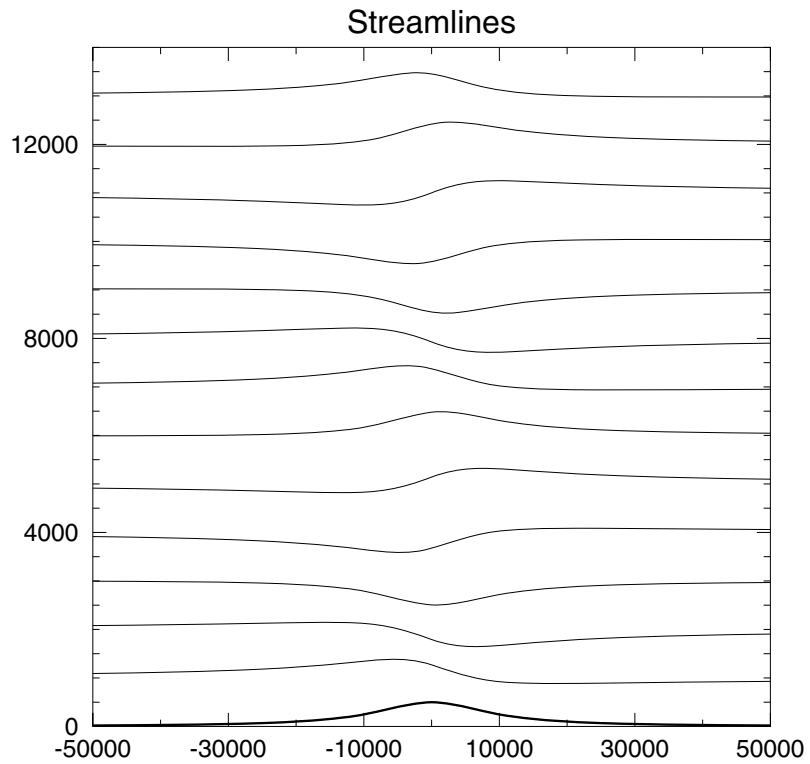
```
lpr hw1.ps
```

or print the PDF file from your PC using the web-server-based approach.

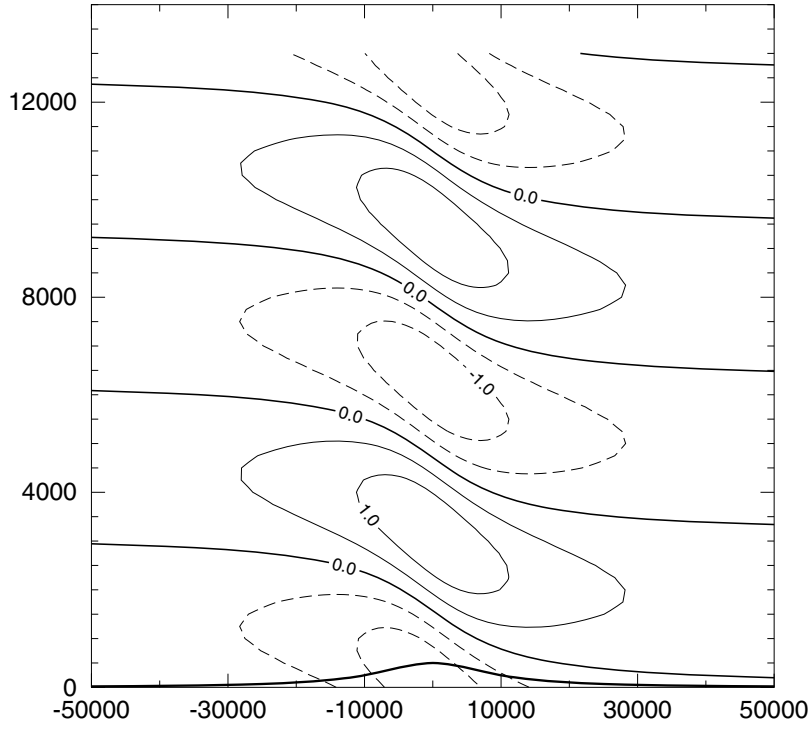
Include the plots with your homework report. Discuss the plots of course. Remember you need to run the program for two different mountain heights.

<http://www-solar.mcs.st-and.ac.uk/~steveb/course/notes/set2.pdf> is a brief introduction on Fortran.

For  $h_m = 500$  m, you should get the following:



theta prt contours (K)



w prt contours (m/s)

