Thunderstorm Dynamics

Houze sections 7.4, 8.3, 8.5, Refer back to equations in Section 2.3 when necessary.

Bluestein Vol. II section 3.4.6.

Review article "Dynamics of Tornadic Thunderstorms" by Klemp – handout.

A. Equations of Motion

Boussinesq approximated equations (neglecting friction and Coriolis force)

$$\frac{du}{dt} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x} \qquad (1a)$$

$$\frac{dv}{dt} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial y} \qquad (1b)$$

$$\frac{dw}{dt} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial z} + B \qquad (1c)$$

where buoyancy B includes effects of air density and water loading. The prime is with respect to a horizontally homogeneous base state.

Written in a vector form:

$$\frac{d\vec{V}}{dt} = -\frac{1}{\rho_0}\nabla p' + B\hat{k} \tag{3}$$

or

$$\frac{\partial\vec{V}}{\partial t} = -\vec{V}\cdot\nabla\vec{V} - \frac{1}{\rho_0}\nabla p' + B\hat{k}\,. \tag{4}$$

Verify it for yourself that

$$\vec{V}\cdot\nabla\vec{V} = \nabla\left(\frac{\vec{V}\cdot\vec{V}}{2}\right) - \vec{V}\times\left(\nabla\times\vec{V}\right) \tag{4a}$$

Therefore

$$\frac{\partial\vec{V}}{\partial t} = -\nabla\left(\frac{\vec{V}\cdot\vec{V}}{2} + \frac{p'}{\rho_0}\right) + \vec{V}\times\vec{\omega} + B\hat{k} \tag{5}$$

where $\vec{\omega} = \nabla\times\vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$ is the 3D vorticity vector .

B. Vorticity Equation

$$\vec{\omega} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k} .$$

Derive 3-D vorticity equation by taking $\nabla \times (5)$ and recall $\nabla \times \nabla(\ ) = 0$ →

$$\frac{\partial \vec{\omega}}{\partial t} = \nabla \times (\vec{V} \times \vec{\omega}) + \nabla \times (B\hat{k}) \qquad (6)$$

1. Development of rotation (vertical vorticity)

To investigate the development of rotation in thunderstorms, look at the vertical component of vorticity $\varsigma = \hat{k} \cdot \vec{\omega}$ →

$$\frac{\partial \varsigma}{\partial t} = \hat{k} \cdot \nabla \times (\vec{V} \times \vec{\omega}) + \hat{k} \cdot \nabla \times (B\hat{k}) = \hat{k} \cdot \nabla \times (\vec{V} \times \vec{\omega})$$

Note that $\hat{k} \cdot \nabla \times (B\hat{k}) = 0$ (verify yourself) therefore buoyancy does not directly generate vertical rotation in thunderstorms! Buoyancy only generates horizontal vorticity which can be tilted into the vertical direction.

Let ξ, η and ς be the *x*, *y* and *z* component of vorticity, respectively.

$$\vec{V}\times\vec{\omega}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ u & v & w\\ \xi & \eta & \varsigma\end{vmatrix}=\left(v\varsigma-w\eta\right)\vec{i}+\left(w\xi-u\varsigma\right)\vec{j}+\left(u\eta-v\xi\right)\vec{k}$$

and

$$\hat{k}\cdot\nabla\times(\vec{V}\times\vec{\omega})=\frac{\partial}{\partial x}\left(w\xi-u\varsigma\right)-\frac{\partial}{\partial y}\left(v\varsigma-w\eta\right)$$

$$\frac{\partial\varsigma}{\partial t}=-u\frac{\partial\varsigma}{\partial x}-v\frac{\partial\varsigma}{\partial y}-w\frac{\partial\varsigma}{\partial z}-\varsigma\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\xi\frac{\partial w}{\partial x}+\eta\frac{\partial w}{\partial y}\quad \text{(we have used } \nabla\cdot\vec{\omega}\equiv\frac{\partial\xi}{\partial x}+\frac{\partial\eta}{\partial y}+\frac{\partial\varsigma}{\partial z}=0\text{)}\quad (7)$$

or

$$\frac{\partial\varsigma}{\partial t}=-\vec{V}\cdot\nabla\varsigma-\varsigma\nabla_H\cdot\vec{V}+\vec{\omega}_H\cdot\nabla_H w \qquad (8)$$

The last term is called 'tilting' term. It turns horizontal vorticity into the vertical component through differential vertical motion.

Making use of Boussinesq mass continuity equation

$$\nabla\cdot\vec{V}=\nabla_H\cdot\vec{V}+\frac{\partial w}{\partial z}=0$$

Eq.(8) can be rewritten as

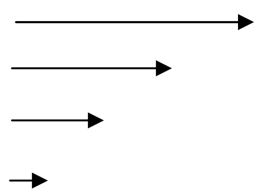
$$\frac{\partial \varsigma}{\partial t} = -\vec{V} \cdot \nabla \varsigma + \varsigma \frac{\partial w}{\partial z} + \vec{\omega}_H \cdot \nabla_H w \qquad (9)$$

Local change of vertical	Advection term	Stretching term	Tilting term

Stretching term - if $\varsigma > 0$ and w increases with height (stretching of air column), the term is positive → increases in ς. Vertical stretching corresponds to horizontal convergence – angular momentum conservation principle!

Tilting term.

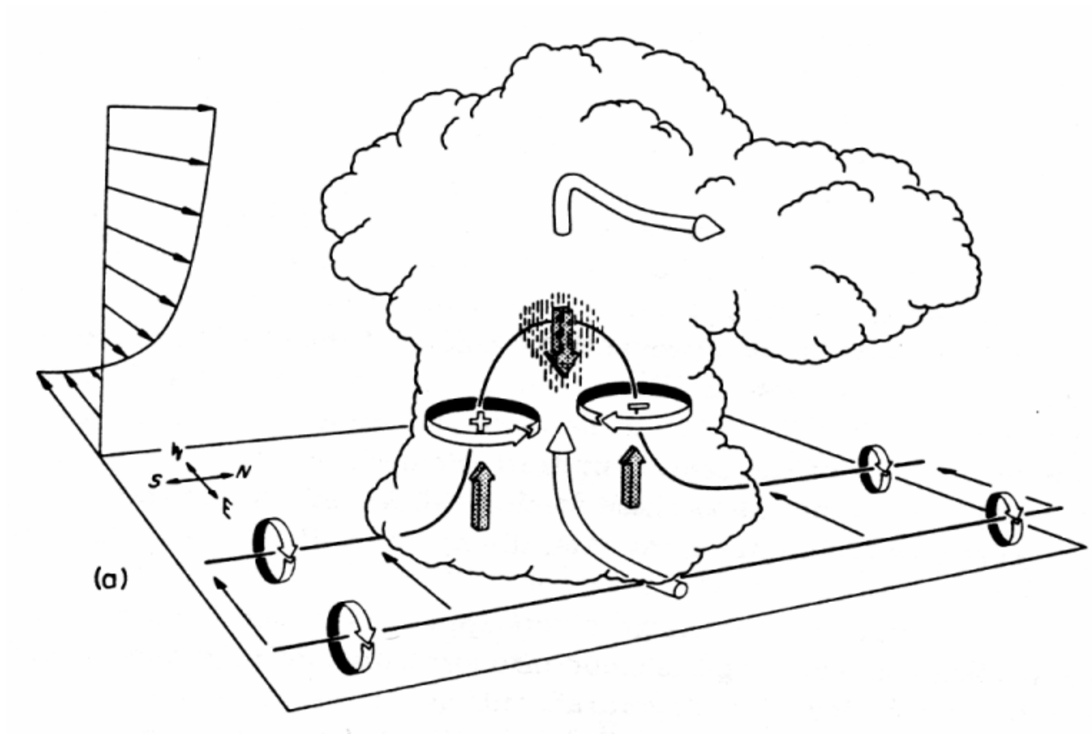
If shear is in u only,



$\frac{\partial u}{\partial z} > 0$, the horizontal vorticity vector points in positive y direction.

On the south side of updraft, $\frac{\partial w}{\partial y} > 0$ → $\frac{\partial \varsigma}{\partial t} = \eta \frac{\partial w}{\partial y} = \frac{\partial u}{\partial z}\frac{\partial w}{\partial y} > 0$ → the tilting creates positively vertical vorticity.

Similarly, one the north side, the tilting due to updraft motion creates negative vertical vorticity.



2. Generation of Horizontal Vorticity (rotation about horizontal axis)

Where the environment has vertical shear, the environment contains horizontal vorticity – the vertical shear is a reservoir of horizontal vorticity. We saw earlier that through tilting of horizontal vortex tubes, horizontal vorticity can be transformed into vertical one – contributing to the thunderstorm rotation.

We pointed earlier that buoyancy does not produce vertical vorticity, what about horizontal vorticity? Yes – it does. Remember horizontal vorticity generation at the gust front where there is strong horizontal buoyancy gradient?

To investigate generation of vorticity about the horizontal, we need the equation for horizontal vorticity. Let's consider the y component of vorticity first:

$$\eta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}.$$

Take $\frac{\partial}{\partial z}(Eq.1a) - \frac{\partial}{\partial x}(Eq.1c)$, we obtain (show it yourself)

$$\boxed{\frac{d\eta}{dt} = -\frac{\partial B}{\partial x},} \qquad (10)$$

therefore (apart from friction), horizontal gradient of buoyancy provides the only source of horizontal vorticity processed by an air parcel.

When the low-level air parcel trajectory has a significant parallel component to the gust front, the vorticity generation by the horizontal gradient can be significant, and the tilting of it into the vertical is believed to contribute significantly to the thunderstorm rotation too.

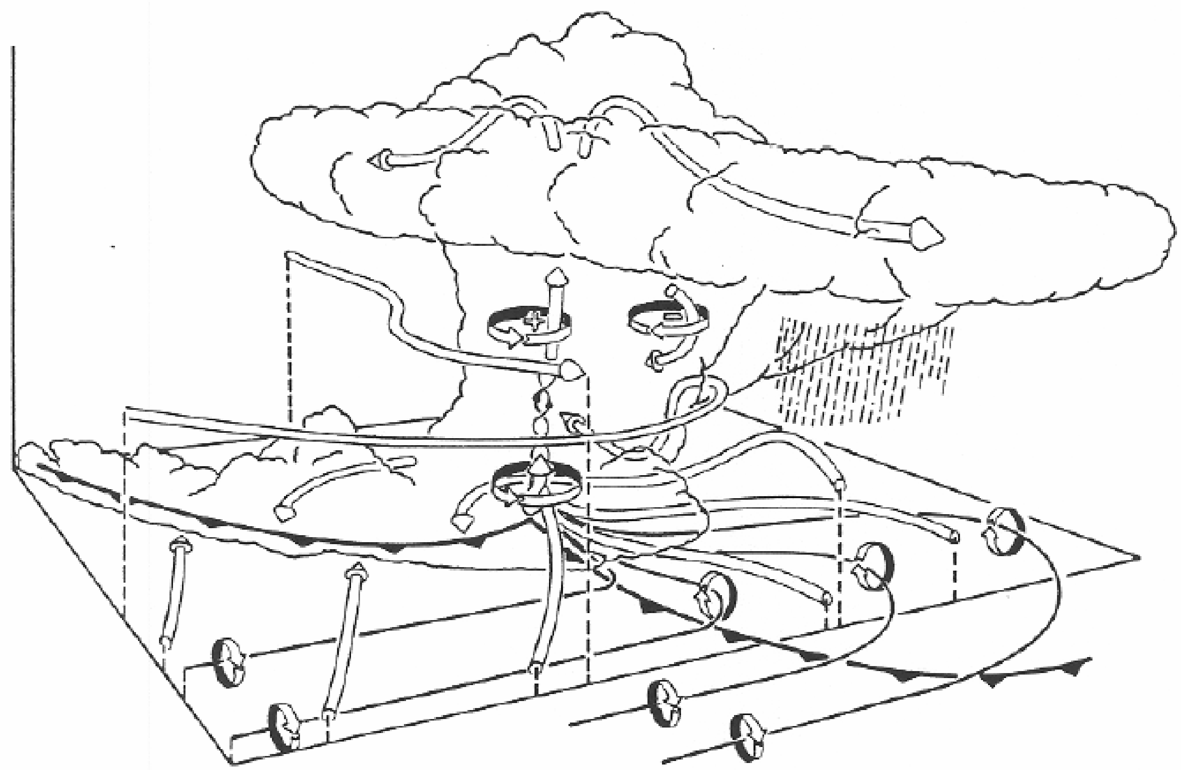


Figure 12 Three-dimensional schematic view of a numerically simulated supercell thunderstorm at a stage when the low-level rotation is intensifying. The storm is evolving in westerly environmental wind shear and is viewed from the southeast. The cylindrical arrows depict the flow in and around the storm. The thin lines show the low-level vortex lines, with the sense of rotation indicated by the circular-ribbon arrows. The heavy barbed line marks the boundary of the cold air beneath the storm.

C. Pressure Perturbation Equation

The rotational dynamics with supercell storms has a lot to do with the pressure perturbations created by the air flow. It is this effect that makes supercells special.

Take $\nabla \cdot$(equation of motion), i.e., $\frac{\partial}{\partial x}(1a)+\frac{\partial}{\partial y}(1c)+\frac{\partial}{\partial z}(1d)$:

1sr term: $$\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=-\frac{1}{\rho_0}\frac{\partial^2 p'}{\partial x^2},$$

2nd term: $$\frac{\partial}{\partial y}\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right]=-\frac{1}{\rho_0}\frac{\partial^2 p'}{\partial y^2},$$

3rd term: $$\frac{\partial}{\partial z}\left[\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right]=-\frac{1}{\rho_0}\frac{\partial^2 p'}{\partial z^2}+\frac{\partial B}{\partial z}.$$

Taking derivatives, combining terms and remembering $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$, we have

$$\begin{aligned}\frac{1}{\rho_0}\nabla^2 p' = &-\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right] \\ &-2\left[\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+\frac{\partial u}{\partial z}\frac{\partial w}{\partial x}+\frac{\partial v}{\partial z}\frac{\partial w}{\partial y}\right]+\frac{\partial B}{\partial z}\end{aligned} \qquad (11)$$

Now let's partition winds between environmental wind and thunderstorm induced perturbation winds:

$$u(x,y,z,t) = \overline{u}(z) + u'(x,y,z,t)$$
$$v(x,y,z,t) = \overline{v}(z) + v'(x,y,z,t)$$
$$w(x,y,z,t) = 0 + w'(x,y,z,t)$$

Substituting them into (11), we have

$$\nabla^2 p' = -\rho_0\left[\left(\frac{\partial u'}{\partial x}\right)^2 + \left(\frac{\partial v'}{\partial y}\right)^2 + \left(\frac{\partial w'}{\partial z}\right)^2 + 2\frac{\partial u'}{\partial y}\frac{\partial v'}{\partial x} + 2\frac{\partial u'}{\partial z}\frac{\partial w'}{\partial x} + 2\frac{\partial v'}{\partial z}\frac{\partial w'}{\partial y}\right]$$
$$-\rho_0\left[2\frac{\partial \overline{u}}{\partial z}\frac{\partial w'}{\partial x} + 2\frac{\partial \overline{v}}{\partial z}\frac{\partial w'}{\partial y}\right] + \rho_0\frac{\partial B}{\partial z} \qquad (12)$$

It's an elliptic diagnostic equation for pressure p'.

Dividing the total perturbation pressure into

$$p' = p'_{dyn} + p'_B$$
$$= p'_L \qquad + \qquad p'_{NL} \qquad + \qquad p'_B$$
$$2nd\ term \qquad \text{"}fluid\ extension\ term\text{"} + shear\ term \qquad last\ term$$

Each part is attributed to certain terms on the right hand side of Eq.(12).

Applications:

1. Updraft enhancement in rotating thunderstorms and cell splitting.

We saw earlier that a strong updraft in an environment of significant vertical shear produces a pair of counter-rotating vortices. Consider p'_{NL} term only. It can be verified that the 3 shear terms can be written in the following form:

$$
\begin{aligned}
& 2\frac{\partial u'}{\partial y}\frac{\partial v'}{\partial x} + 2\frac{\partial u'}{\partial z}\frac{\partial w'}{\partial x} + 2\frac{\partial v'}{\partial z}\frac{\partial w'}{\partial y} \\
&\quad = \frac{1}{2}\left[\left(\frac{\partial v'}{\partial x} + \frac{\partial u'}{\partial y} \right)^2 + \left(\frac{\partial v'}{\partial z} + \frac{\partial w'}{\partial y} \right)^2 + \left(\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} \right)^2 \right. \\
&\qquad \left. - \left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y} \right)^2 - \left(\frac{\partial v'}{\partial z} - \frac{\partial w'}{\partial y} \right)^2 - \left(\frac{\partial u'}{\partial z} - \frac{\partial w'}{\partial x} \right)^2 \right]
\end{aligned}
$$

If we assume pure rotation (no div, deformation) and ignore extension terms (i.e., look at the effect of vertical rotation only), then

$$
\nabla^2 p'_{NL} = \frac{\rho_0}{2}\left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y} \right)^2 = \frac{\rho_0}{2}\varsigma'^2 .
$$

Since the left hand side is a Laplacian operator, $\nabla^2 p'_{NL} \propto -p'_{NL}$, therefore

$$p'_{NL} \propto -\varsigma'^2. \quad (13)$$

→ **Both cyclonic and anticyclonic rotation produces <u>negative</u> pressure perturbation**. The low pressure center is actually required to give a PGF that balances the centrifugal force!

Negative p'_{NL} is largest where rotation is the strongest, which is usually at the mid-levels of thunderstorms where the mesocyclone is found - the p'_{NL} "low" there is up to 2-4 mb.

Earlier figure shows that because of tilting, **vertical rotation is strongest at the flanks of updraft, and negative p' at the mid-levels creates an upward pressure gradient force there that promotes new updrafts there – a dynamic cause for cell splitting**.

Now, let's consider the fluid extension terms,

$$\nabla^2 p' = -\rho_0 \left[\left(\frac{\partial u'}{\partial x} \right)^2 + \left(\frac{\partial v'}{\partial y} \right)^2 + \left(\frac{\partial w'}{\partial z} \right)^2 + \ldots \right] < 0$$

$$p' \propto \rho_0 \left[\left(\frac{\partial u'}{\partial x} \right)^2 + \left(\frac{\partial v'}{\partial y} \right)^2 + \left(\frac{\partial w'}{\partial z} \right)^2 + \ldots \right] > 0$$

So **positive p' is largest where stretching is largest**. This occurs at the low and high levels.

So we have

$p'_{NL} > 0$ H

$p'_{NL} < 0$ L

$\Uparrow$ vertical PGF

$p'_{NL} > 0$ H

Therefore the **nonlinear pressure perturbation due to shear and stretching creates additional upward lifting (pressure gradient) force at the lower atmosphere that enhances the updraft beyond that based on buoyancy! Therefore, supercell storms tend to be stronger than regular storms, given the same amount of CAPE.**

Rule of thumb: 1mb VPGF over 1 km ~ same forcing as 3°C of buoyancy.

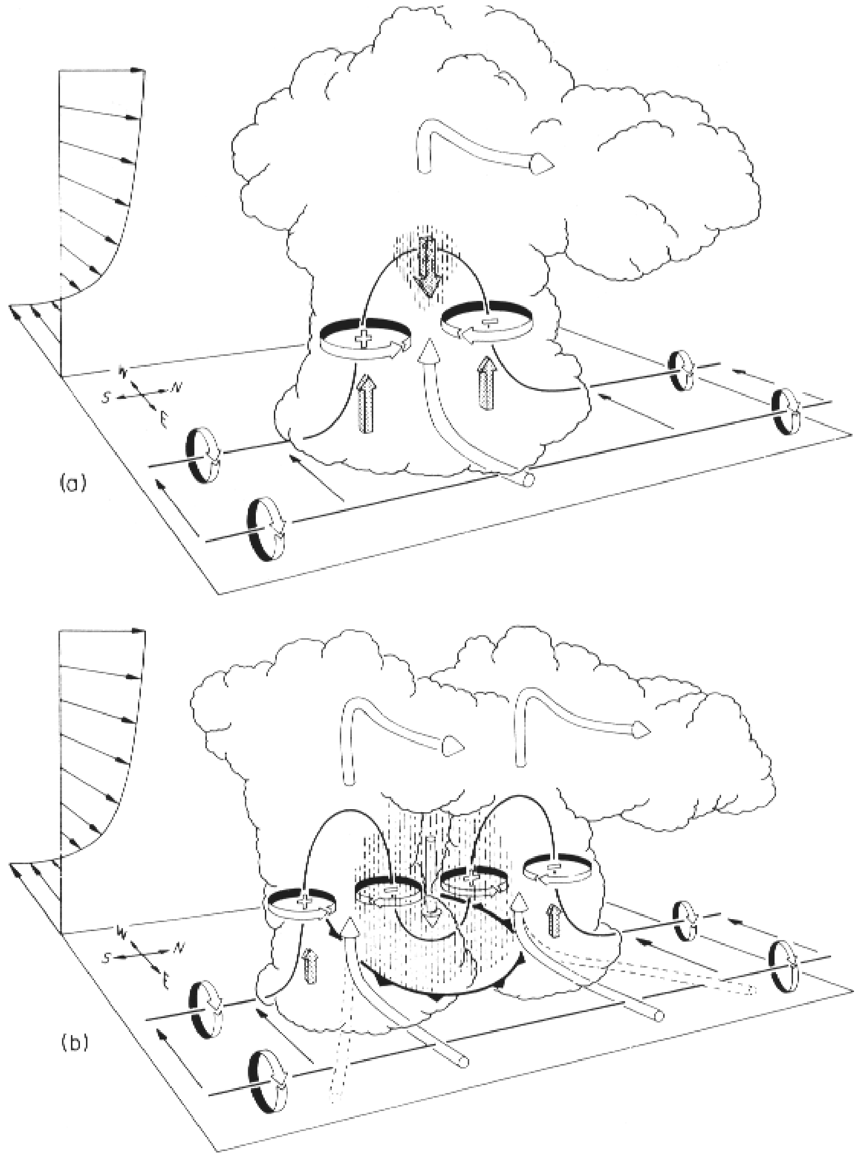


Figure 3.23 Schematic diagram depicting how a typical vortex line (streamline of three-dimensional vorticity vector) contained within (westerly) environmental shear is deformed as it interacts with a convective cell (viewed from the southeast). Direction of cloud-relative airflow (cylindrical arrows); vortex lines (solid lines), with the sense of rotation indicated by circular arrows; the forcing influences that promote new updraft and downdraft growth (shaded arrows); regions of precipitation (vertical dashed lines). (a) Initial stage: Vortex line loops into the vertical as it is swept into the updraft. (b) Splitting stage: Downdraft forming between the splitting updraft cells tilts vortex line downward, producing two vortex pairs. Boundary of the cold air spreading out beneath the storm (cold-front symbol at the

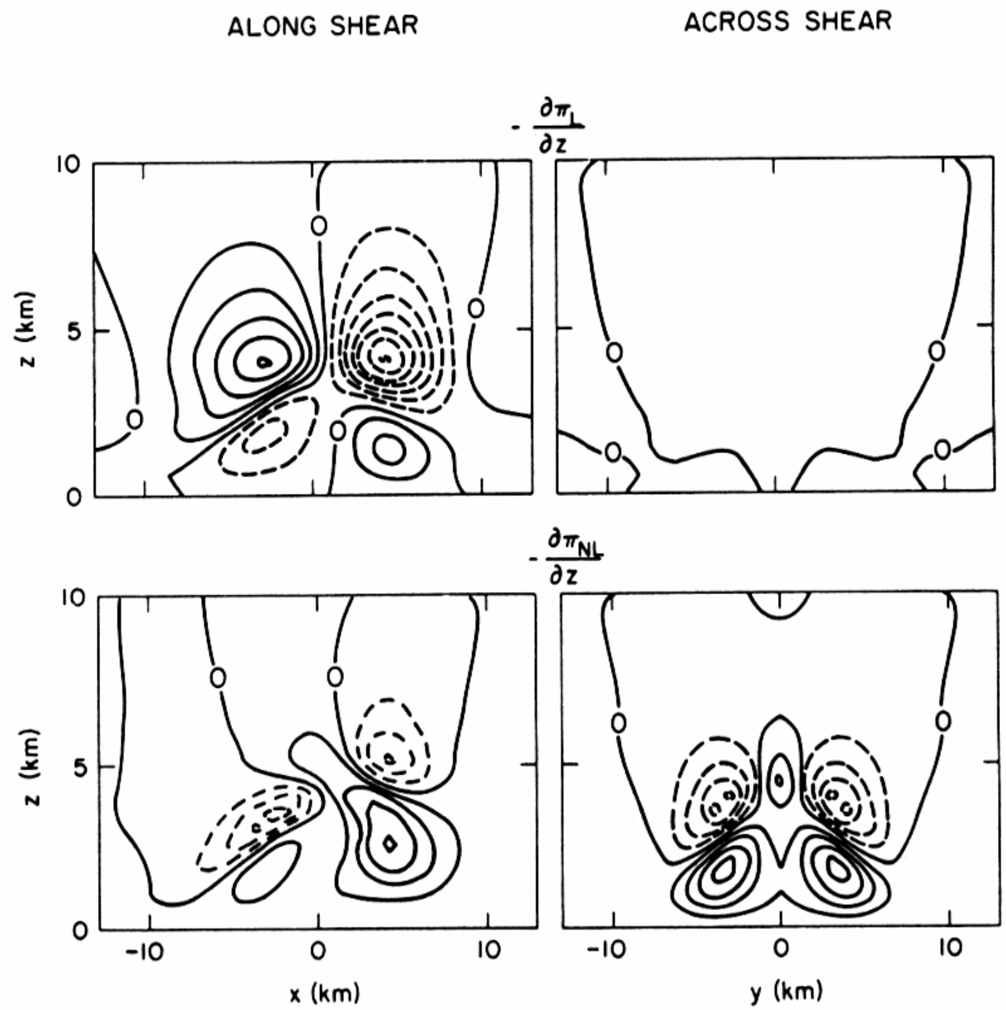


Figure 3.22 Vertical cross section of acceleration induced by vertical perturbation-pressure force for the linear part of the wind field ($-\partial\pi_L/\partial z$ (top) and the nonlinear part of the wind field ($-\partial\pi_{NL}/\partial z$) (bottom) in a numerical simulation 10 min after storm initiation. The environmental wind profile has a straight-line hodograph. Contours plotted every 0.004 m s^{-2} (from Rottuno and Klemp, 1982). (Courtesy of the American Meteorological Society)

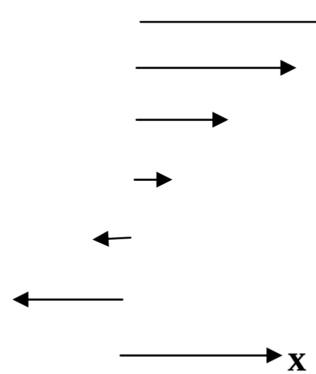
2. Preferred enhancement of right-moving or left-moving storm

Consider the linear p'_{dyn} term:

$$\nabla^2 p' = -\rho_0 \left[2\frac{\partial \overline{u}}{\partial z}\frac{\partial w'}{\partial x} + \frac{\partial \overline{v}}{\partial z}\frac{\partial w'}{\partial y} \right] = -2\rho_0 \frac{\partial \vec{\overline{V}}}{\partial z} \cdot \nabla w'$$

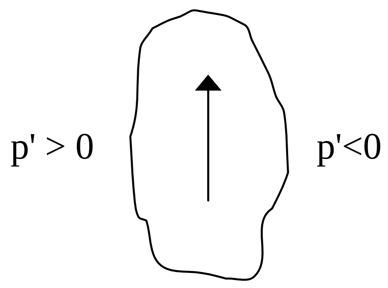
so $$p' \propto \frac{\partial \vec{\overline{V}}}{\partial z} \cdot \nabla w'. \qquad (14)$$

For <u>unidirectional shear</u>



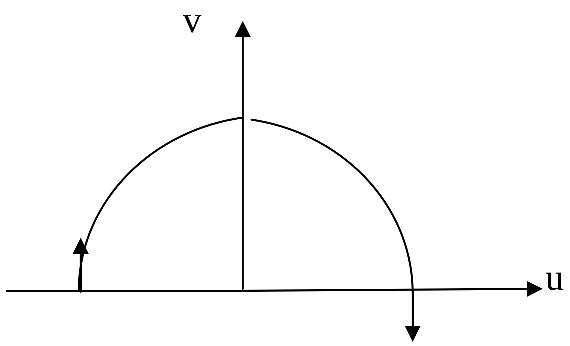
$p' \propto \frac{\partial \overline{u}}{\partial z}\frac{\partial w'}{\partial x} > 0$ on the west/upshear flank of updraft

$p' \propto \frac{\partial \overline{u}}{\partial z}\frac{\partial w'}{\partial x} < 0$ on the east/downshear flank of updraft and p' largest at the mid-levels



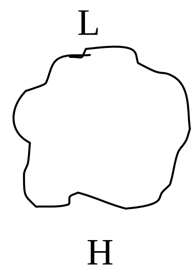
→ new cell growth on the downshear flank

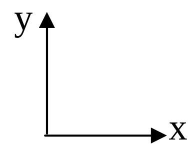
If the hodograph is clockwide curved,



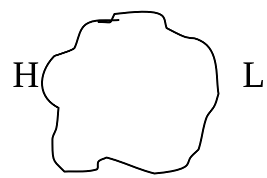
then we also have to consider $\frac{\partial \overline{v}}{\partial z}\frac{\partial w'}{\partial y}$ term in (14).

At low levels, $\frac{\partial \overline{v}}{\partial z}\frac{\partial w'}{\partial y}$, produces

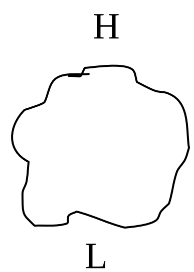




At the mid-levels, $\frac{\partial \overline{u}}{\partial z}\frac{\partial w'}{\partial x}$ produces



At the upper levels $\frac{\partial \overline{v}}{\partial z}\frac{\partial w'}{\partial y}$ produces



→ there is a upward vertical PGF on the right flank of the storm (downward PGF on the left flank) → new cell growth is enhanced to the right, rotating updraft becomes a 'right mover'.

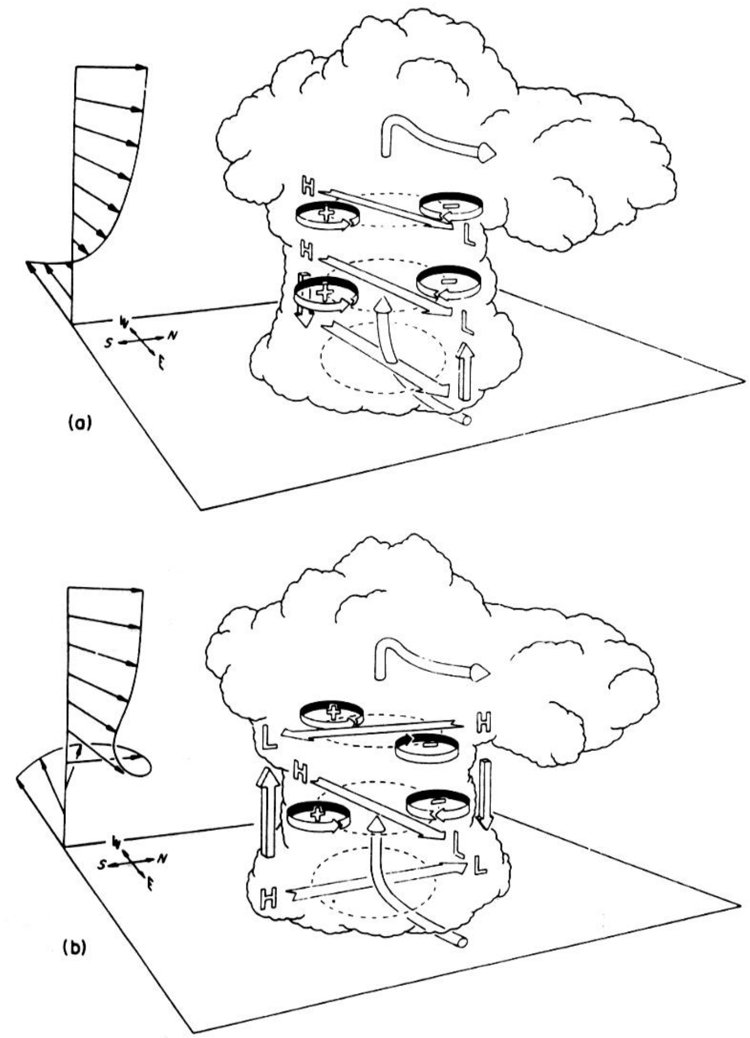


Figure 3.21 Schematic diagram illustrating the pressure and vertical vorticity perturbations arising as an updraft interacts with an environmental vertical wind shear vector that (a) does not change direction with height and (b) turns clockwise with height. The high (H) to low (L) horizontal pressure-gradient forces parallel to the shear vectors (flat arrows) are labeled along with the preferred location of cyclonic (+) and anticyclonic (−) vorticity. Shaded arrows depict orientation of resulting vertical pressure-gradient forces (from Klemp, 1987; adapted from Rotunno and Klemp, 1982). (Courtesy of the American Meteorological Society)

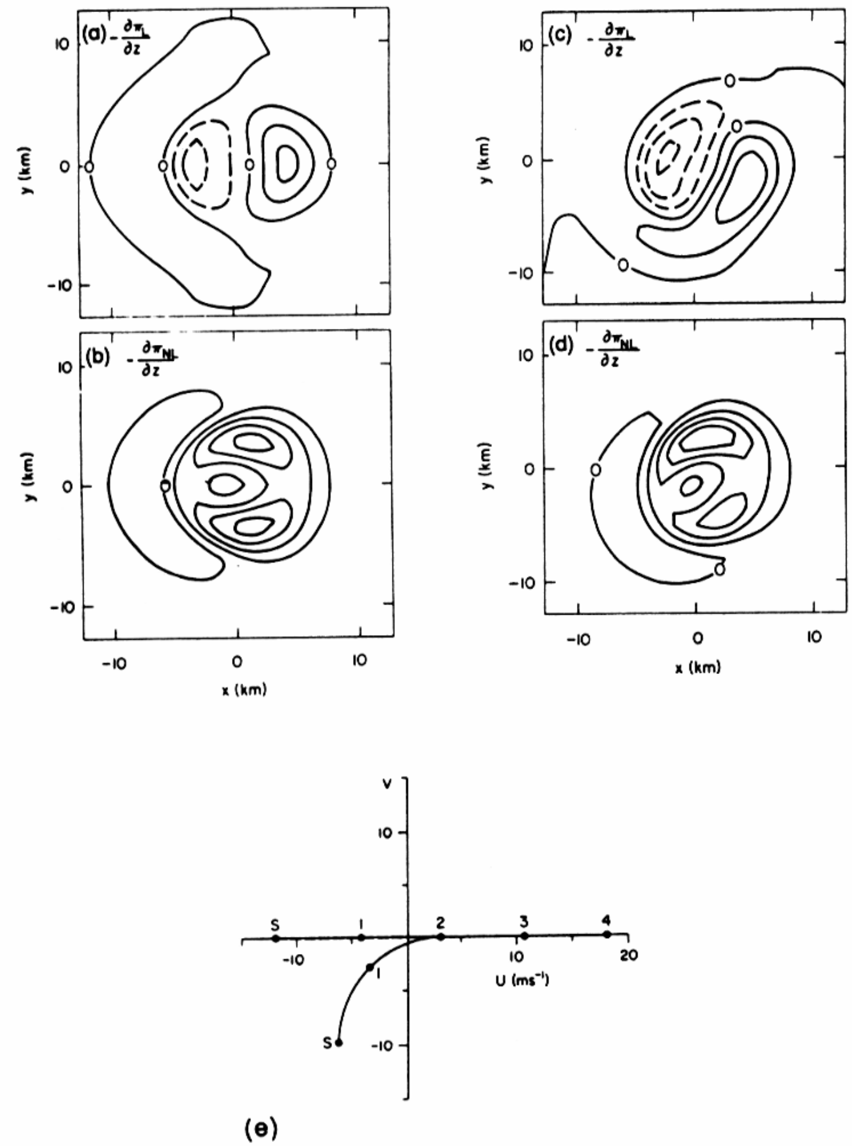


Figure 3.24 Acceleration induced by vertical perturbation-pressure gradient force at 1.5 km in a numerical simulation 10 min after storm initiation for (a) the linear part of the wind field for a straight-line hodograph; (b) the nonlinear part of the wind field for a straight-line hodograph; (c) the linear part of the wind field for a clockwise-turning hodograph; (d) the nonlinear part of the wind field for a clockwise-turning hodograph. Contours plotted every 0.004 m s^{-2}; (e) hodographs used in the simulations; clockwise-turning hodograph indicated by solid line; [illegible] AGL (from Rotunno and Klemp, 1982). (Courtesy of the American

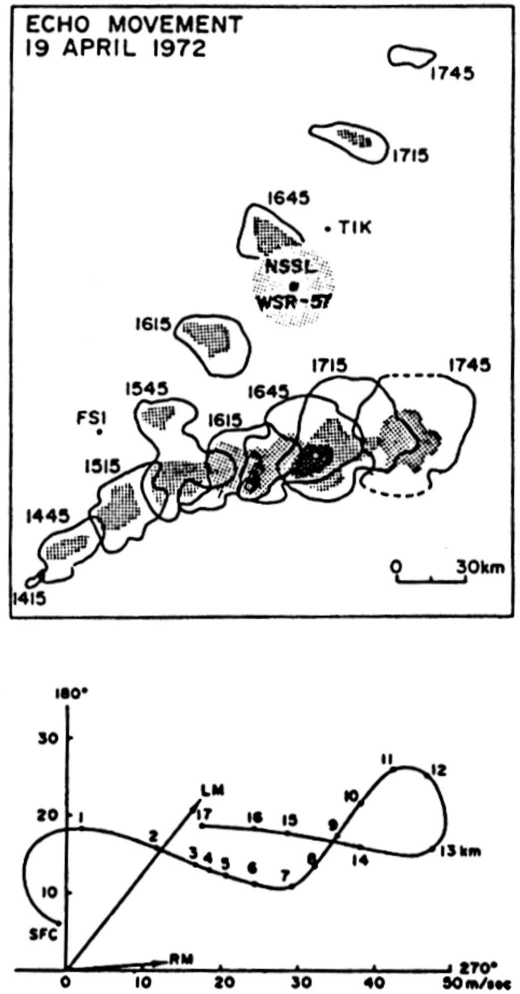


Figure 3.25 (Top) Radar-echo history of a splitting storm observed in south-central Oklahoma. Radar reflectivity of 10 dBZ (solid lines); radar reflectivity in excess of 40 dBZ (stippled regions). Times adjacent to each outline are CST. (Bottom) Hodograph representative of the storm's environment. Heights in km AGL. Motion of the right-moving (RM) and left-moving (LM) cells (from Weisman and Klemp, 1986; adapted from Burgess, 1974). (Courtesy of the American Meteorological Society)

Summary:

The nonlinear-shear effect that promotes new or continued cell growth on the flanks of (alongside) the old cell;

The linear effect of tilting biases the cell movement toward the right (left) if the environmental hodograph is curved in a clockwise (counterclockwise) manner;

Unidirectional shear promotes storms that split, with each member of the split pair having components of motion normal to the shear vector and opposite to each other.

New buoyant updrafts form off the axis of the shear because upward-directed perturbation pressure gradients induce upward accelerations there and lift air to its LFC.

Owing to the low-level convergence associated with upward-moving air, vorticity increases through the stretching of the existing vorticity and is advected upward by the updraft:

Right movers tend to develop cyclonic rotation, while left movers tend to develop anti-cyclonic rotation. The cyclonic vorticity produced in storms that grow in an environment of clockwise turning shear is not due to the Earth's rotation.