QG Height Tendency

Arranged by Andrew Moore
Material from Thomas Galarneau

QG Height Tendency

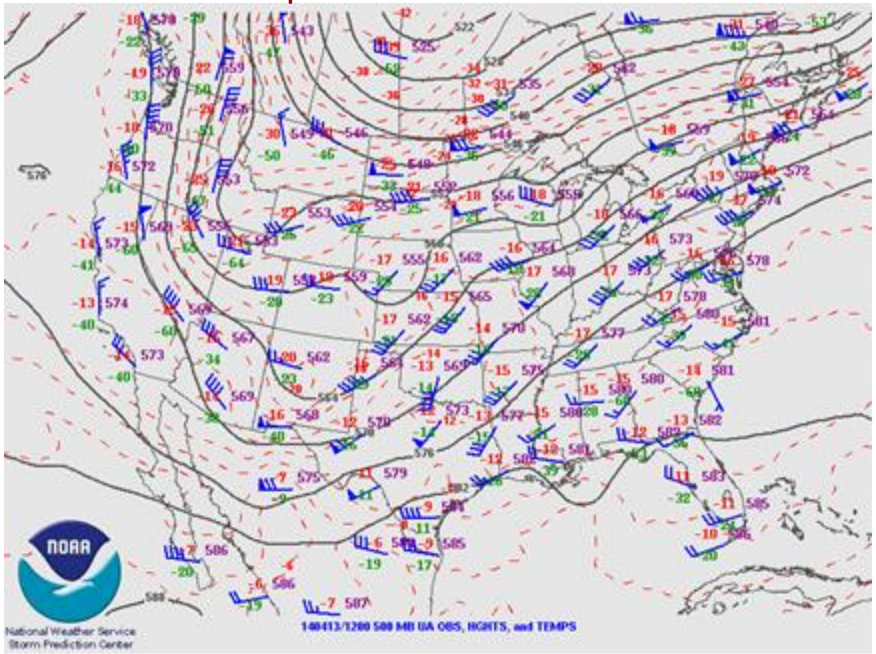
Why do we care?

The QG height tendency equation allows us to anticipate:

- The evolution of upper air and surface patterns
- The evolution of certain severe weather parameters (e.g. shear, lift, etc…)

It is also relatively easy to use!

What will this pattern look like in 12-24 hours?



QG Height Tendency

A quick note:

There are alternatives to QG theory (for example, IPV theory), that will work just as well.

We will focus on QG theory in this class for two reasons:

1) It's easy to interpret from basic weather charts
2) Most U.S. weather entities use QG theory (not the case elsewhere…)

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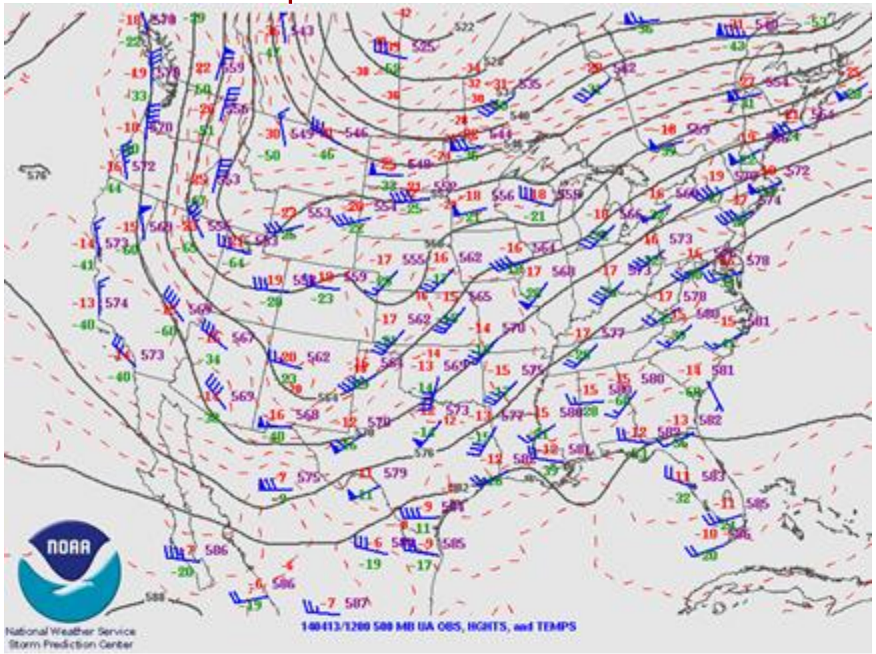


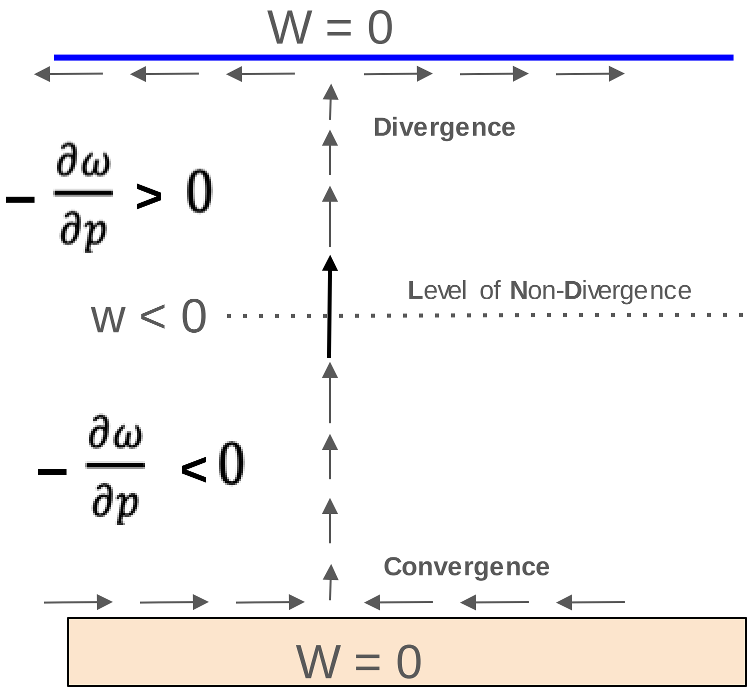
Some Background Concepts

Mass Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

We're going to assume:

- On synoptic scales, the troposphere is incompressible.
- Hydrostatic approximation applies
- Vertical velocity is zero at the surface and at the tropopause.
- **Because of mass continuity, any vertical motion is associated with horizontal convergence and divergence**



Some Background Concepts

1.4 thermal wind balance

(1) $u_g = -\frac{g}{f}\frac{\partial Z}{\partial y}$ geostrophic wind

(2) $\frac{\partial Z}{\partial p} = -\frac{RT}{gp}$ hypsometric eqn

plug (2) into (1)

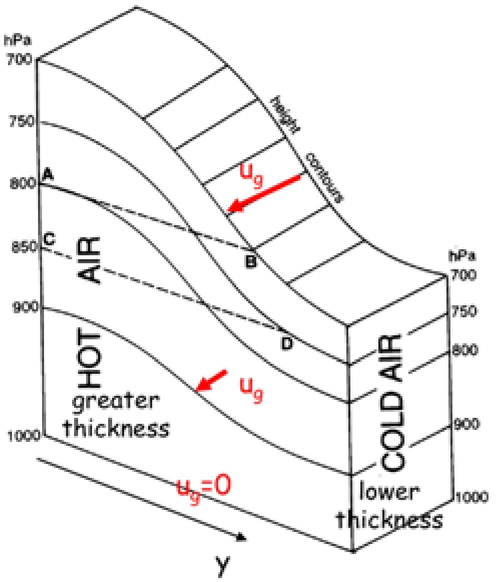
$$\frac{\partial u_g}{\partial p} = \frac{g}{f}\frac{\partial\left(RT/gp\right)}{\partial y}$$

$$= \frac{R}{fp}\frac{\partial T}{\partial y}$$

finite difference expression:

$$\Delta u_g = \frac{R}{f}\frac{\Delta p}{p}\frac{\Delta \overline{T}}{\Delta y}$$

this is the thermal wind: an increase in wind with height due to a temperature gradient

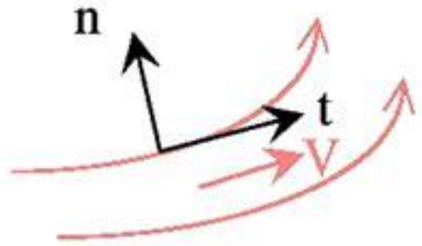


The thermal wind blows ccw around cold pools in the same way as the geostrophic wind blows ccw around lows. The thermal wind is proportional to the T gradient, while the geostrophic wind is proportional to the pressure (or height) gradient.

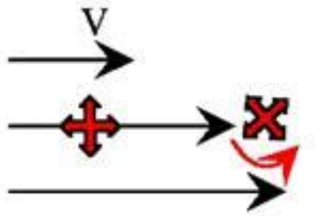
Some Background Concepts

Vorticity:

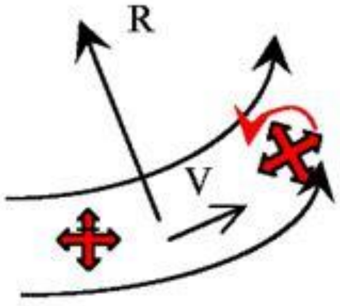
- Vorticity is the curl of the wind field
- Exists in all 3 dimensions, but for today we'll only consider the X/Y dimensions
- Vorticity can be generated by curvature in the flow and/or speed shear in the flow
- Vorticity is directly related to vertical motions and convergence/divergence due to the conservation of angular momentum and conservation of mass



natural coordinate system



shear vorticity



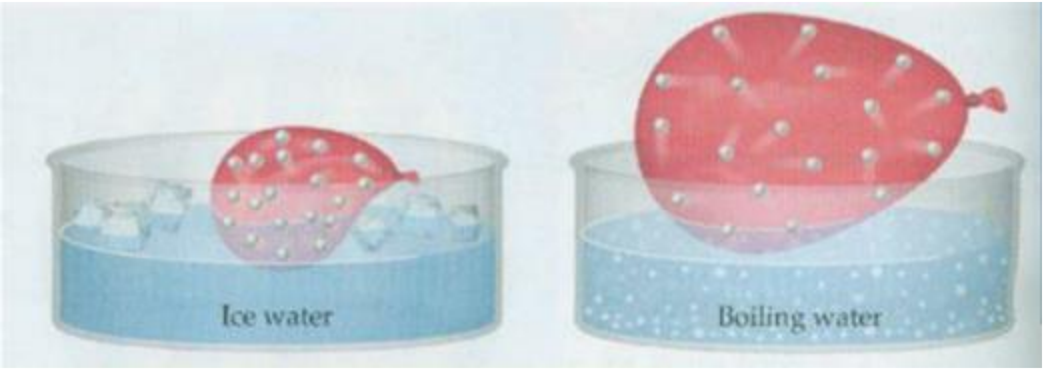
curvature vorticity

Physical Intuition

Jacques Charles
(1746-1823).

Charles's Law

- The volume of a gas is directly proportional to the temperature of the gas at a constant pressure.
- If the gas heats up -> it expands!
- If the gas cools down -> it contracts!

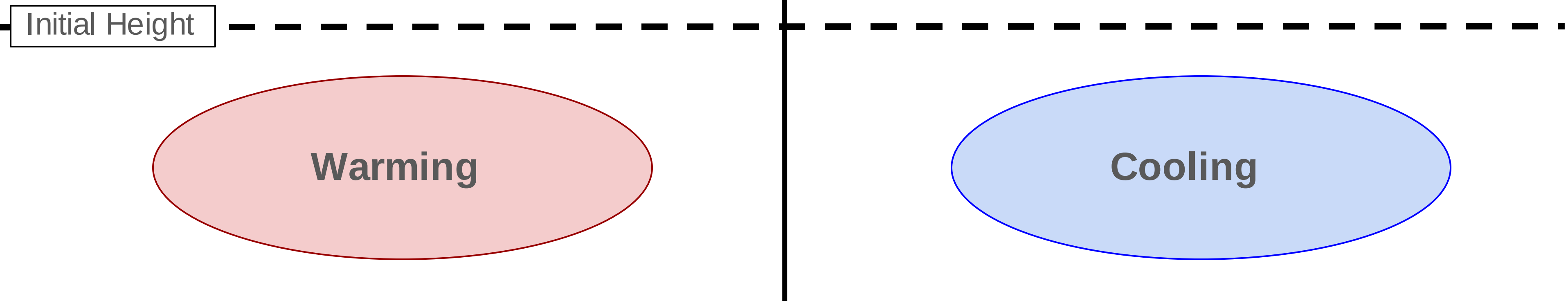


Physical Intuition

Consider a layer of the atmosphere in contact with the surface:

Warming the layer

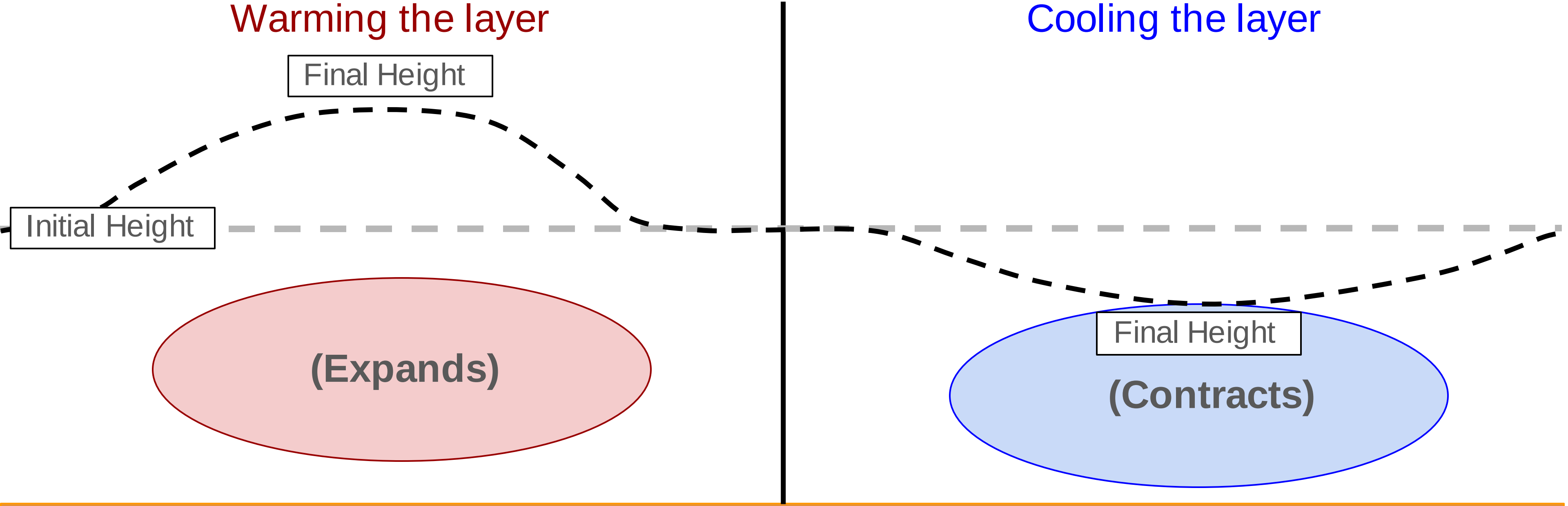
Cooling the layer



The ground can't move - so the top of the layer must move!

Physical Intuition

Consider a layer of the atmosphere in contact with the surface:



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Physical Intuition

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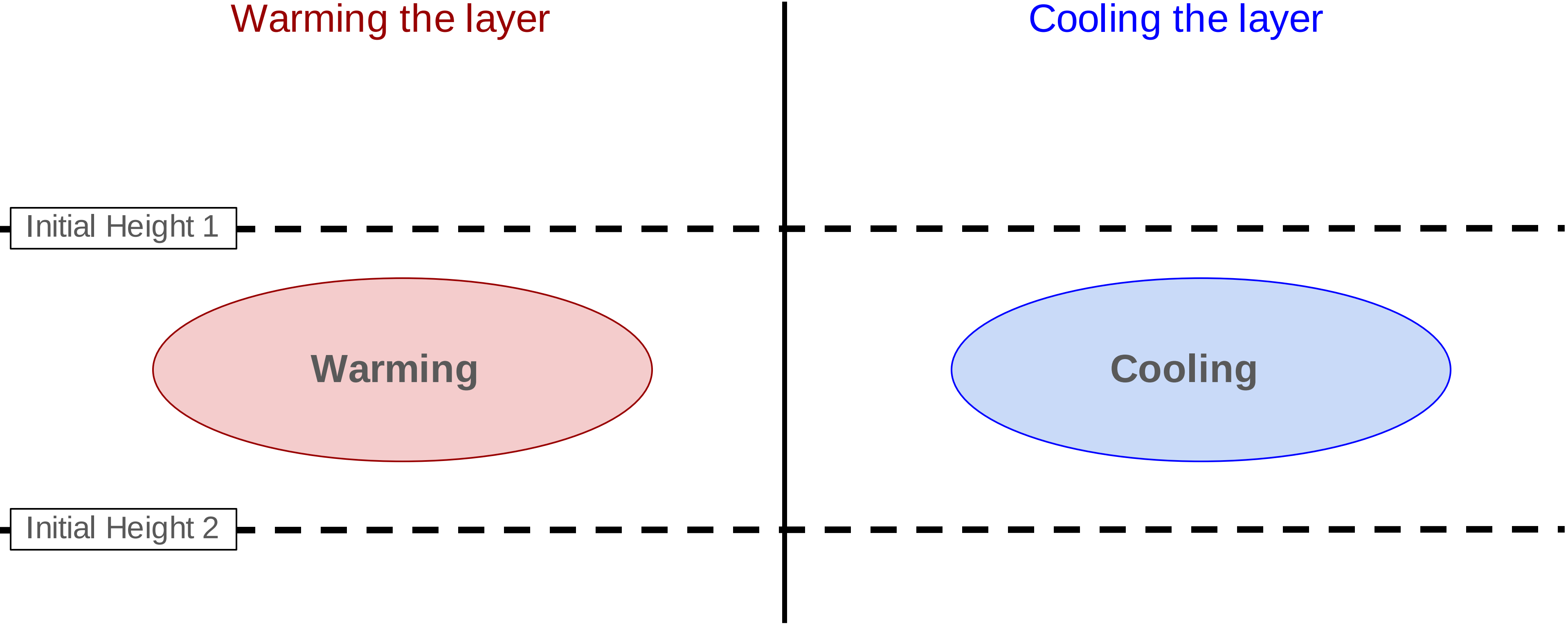
Cooling the layer



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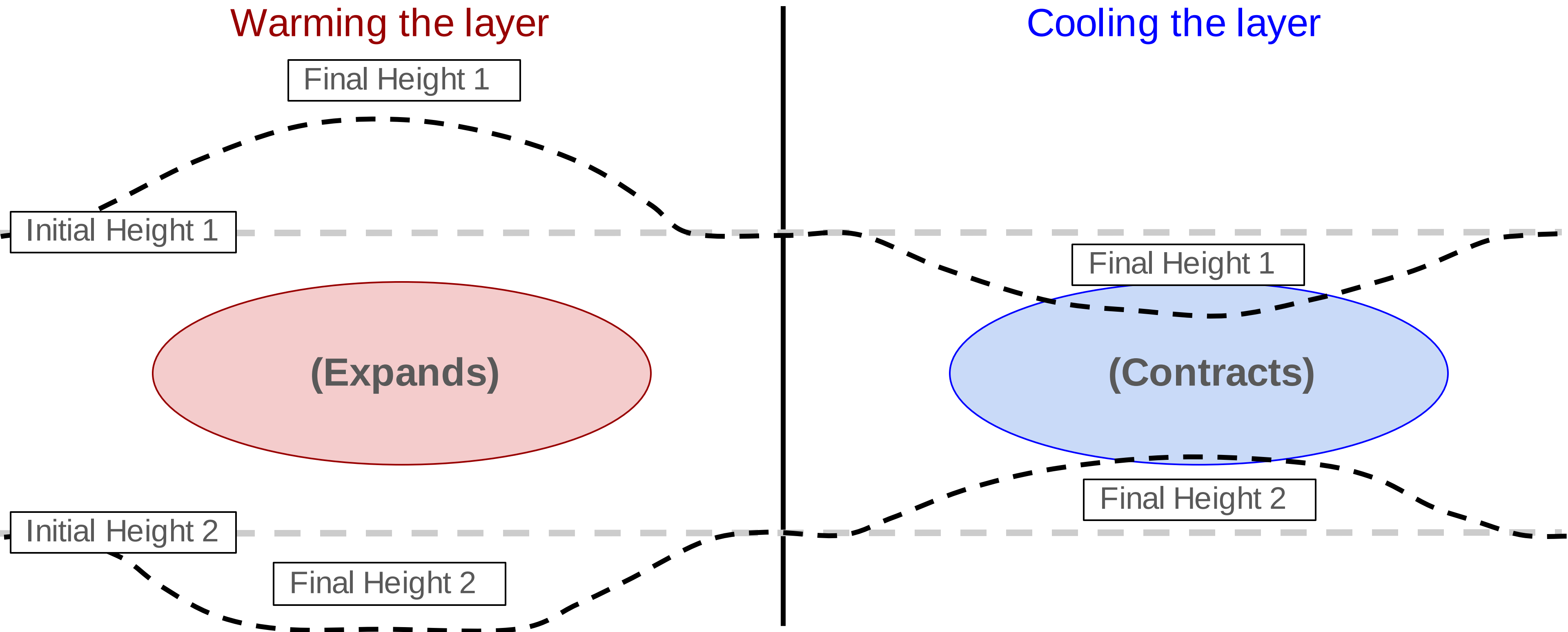


Physical Intuition

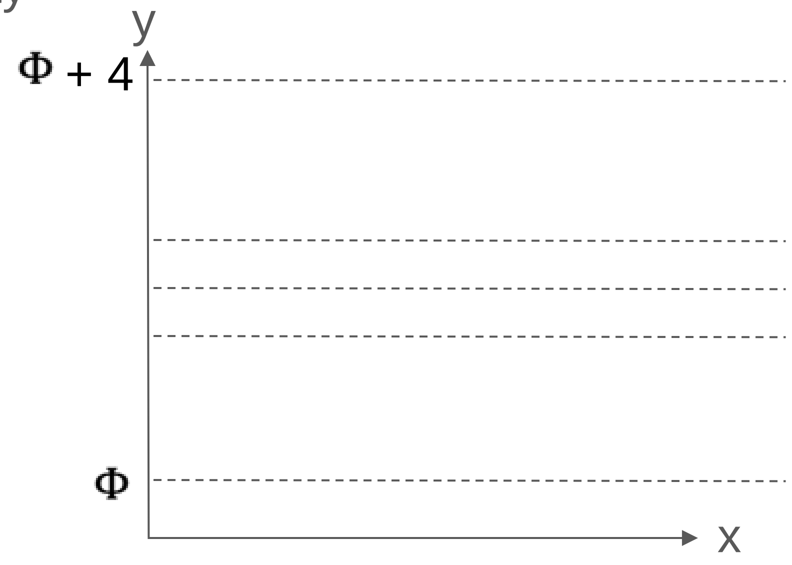
Geostrophic Vorticity:

Plug in geostrophic wind balance into the vorticity equation.

You end up with a form of geostrophic vorticity that relates to the Laplacian of the geopotential height field.

Thus, if you locally change the vorticity at a location, you must also change the geopotential height (i.e. thickness) field!

Consider this geopotential height field.
What would vorticity look like?



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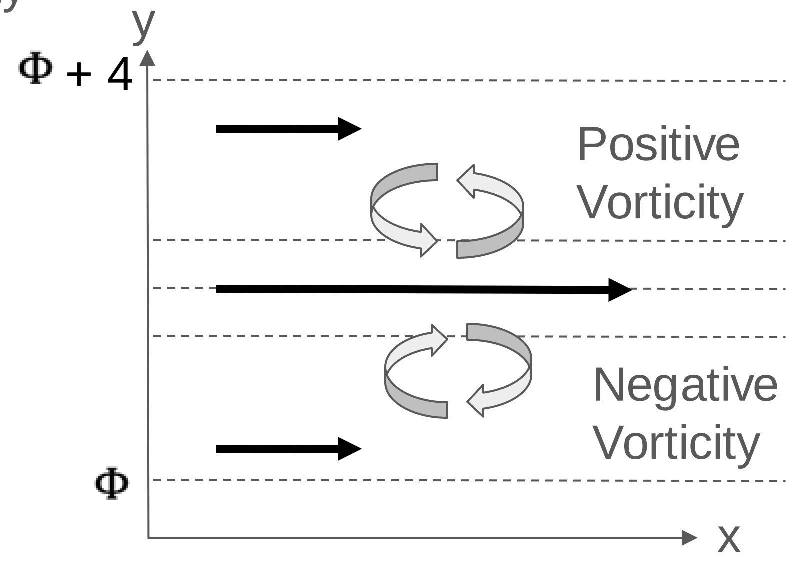

Use the thermal wind relation to confirm!

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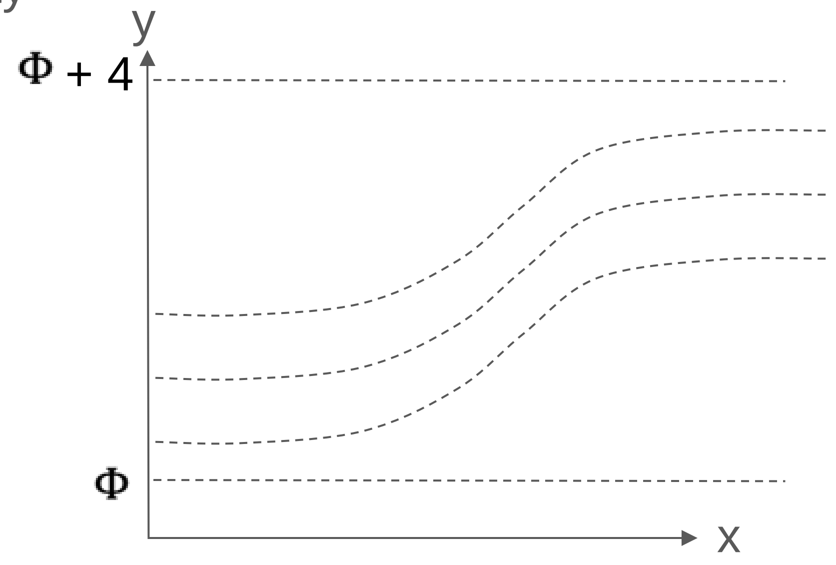


If we change the geopotential height field, how will the vorticity field change (and vise versa)?

QG Vorticity and Thermo Equations

QG vorticity equation

$$\frac{d(\zeta_g + f)}{dt} = f_0 \frac{\partial \omega}{\partial p}$$

Rate of change of absolute vorticity

Vertical motion (or convergence/divergence)

See Bluestein Vol. 1 page 329 for details

QG thermodynamic equation

$$\frac{dT}{dt} = \frac{p}{R_d} \sigma \omega$$

Rate of change of temperature

Vertical motion (or expansion/contraction from vertical motion)

QG Height Tendency Equation

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi = -f_0 \boldsymbol{V}_g \cdot \boldsymbol{\nabla}_p \left(\frac{1}{f_0}\nabla_p^2 \Phi + f\right) - \frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[\boldsymbol{V}_g \cdot \boldsymbol{\nabla}_p \left(-\frac{\partial \Phi}{\partial p}\right)\right] - \frac{\partial H}{\partial p}$$

2nd derivative operator

Absolute vorticity advection

Differential thermal advection

Diabatic heating

QG X Breakdown

2nd Derivative Operator:

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi$$

On synoptic scales, we can roughly assume that the height field (and thus the height tendency field) is sinusoidal.

Take the second derivative of a sine function:

d(d(sin(x)))

= d(cos(x))

= -sin(x)

This assumption turns the 2nd derivative operation into a simple minus sign!

QG X Breakdown

$$-f_0 \boldsymbol{V_g} \cdot \boldsymbol{\nabla_p} \left(\frac{1}{f_0} \nabla_p^2 \Phi + f \right)$$

OR

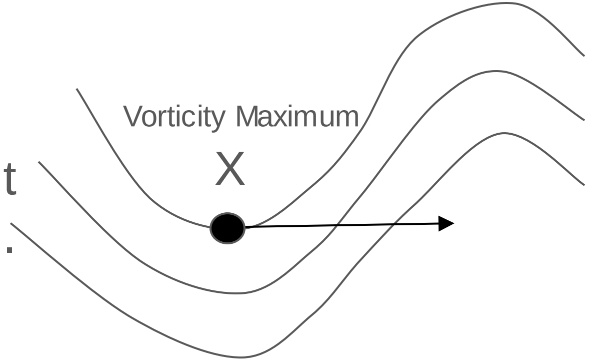
$$-f_0 \boldsymbol{V_g} \cdot \boldsymbol{\nabla_p} (\zeta_g + f)$$

Advection of absolute vorticity:

- This considers both relative vorticity (related to the height field) and planetary vorticity (related to the Coriolis force).

- Here we see how moving the height field (more specifically, the Laplacian of the height field) can change the height field.

Consider this case:



Vg = 0
Ug > 0

QG X Breakdown

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OR

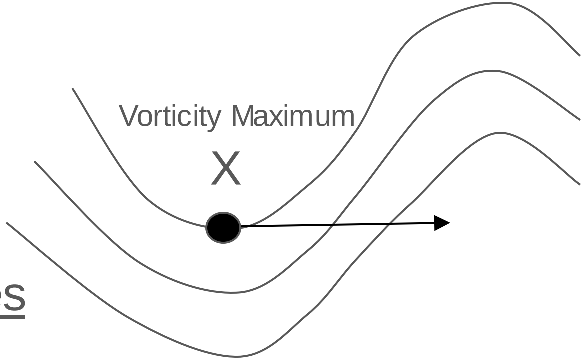
$$-f_0 \boldsymbol{V_g} \cdot \boldsymbol{\nabla_p} (\zeta_g + f)$$

Advection of absolute vorticity:

- Advecting cyclonic vorticity (CVA) leads to <u>height falls</u>

- Advecting anticyclonic vorticity (AVA) leads to <u>height rises</u>

Consider this case:



Vg = 0
Ug > 0

QG X Breakdown

$$-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[\boldsymbol{V_g}\cdot\boldsymbol{\nabla_p}\left(-\frac{\partial\Phi}{\partial p}\right)\right]$$

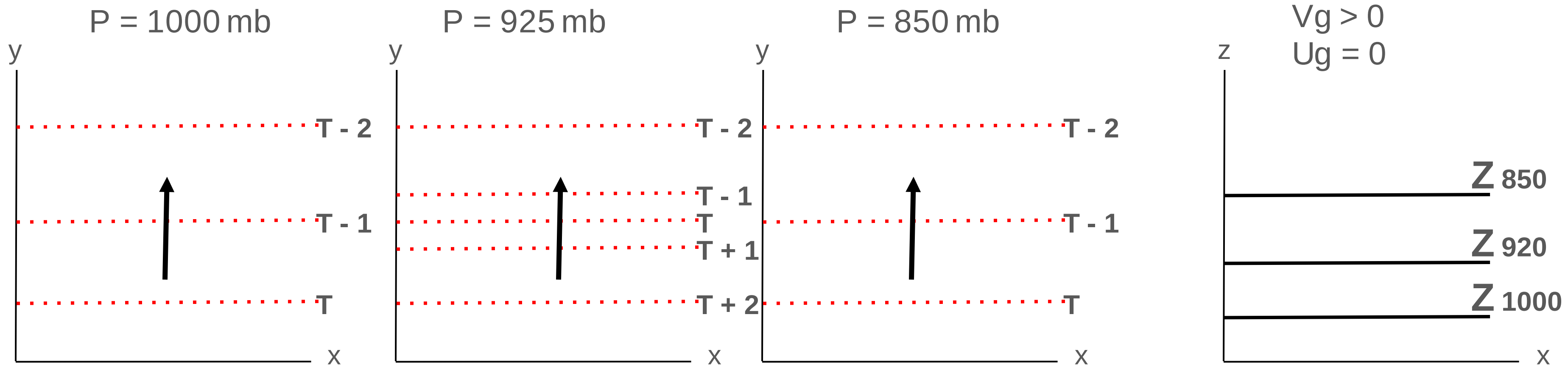
OR

$$-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[-\frac{R}{p}\boldsymbol{V_g}\cdot\boldsymbol{\nabla_p}T\right]$$

Differential Thermal Advection

Assume:

Vg > 0
Ug = 0



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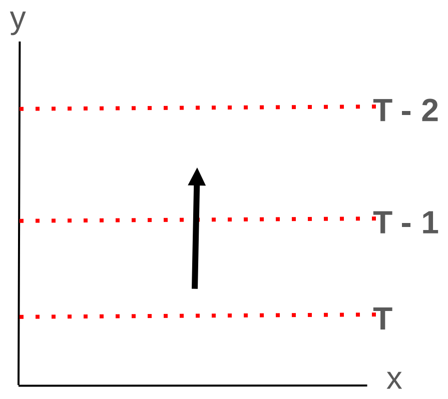
Differential Thermal Advection

Assume:

Vg > 0
Ug = 0

P = 1000 mb



Consider only y component of thermal advection:

Vg > 0

dT/dy < 0

-Vg(dT/dy) > 0

x

QG X Breakdown

$$-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[\boldsymbol{V}_g \cdot \boldsymbol{\nabla}_p\left(-\frac{\partial \Phi}{\partial p}\right)\right]$$

OR

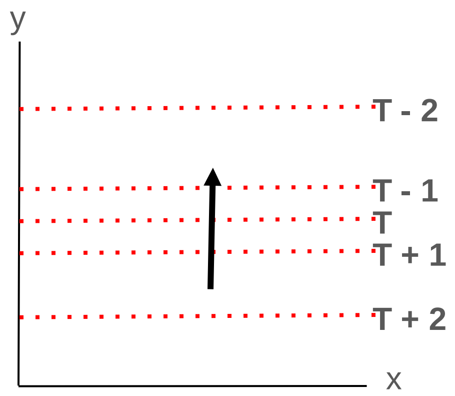
$$-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[-\frac{R}{p}\boldsymbol{V}_g \cdot \boldsymbol{\nabla}_p T\right]$$

Differential Thermal Advection

Assume:

Vg > 0
Ug = 0

P = 925 mb



Consider only y component of thermal advection:

Vg > 0

dT/dy << 0

-Vg(dT/dy) >> 0

QG X Breakdown

$$-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[\boldsymbol{V}_g \cdot \boldsymbol{\nabla}_p\left(-\frac{\partial \Phi}{\partial p}\right)\right]$$

OR

$$-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[-\frac{R}{p}\boldsymbol{V}_g \cdot \boldsymbol{\nabla}_p T\right]$$

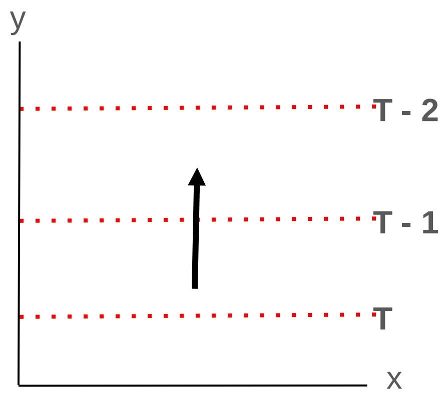
Differential Thermal Advection

Assume:

Vg > 0
Ug = 0

Consider only y component of thermal advection:

P = 850 mb



Vg > 0

dT/dy < 0

-Vg(dT/dy) > 0

QG X Breakdown

$$-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[\boldsymbol{V}_g \cdot \boldsymbol{\nabla}_p\left(-\frac{\partial \Phi}{\partial p}\right)\right]$$

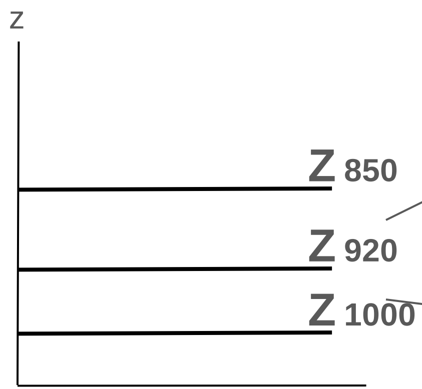
OR

$$-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[-\frac{R}{p}\boldsymbol{V}_g \cdot \boldsymbol{\nabla}_p T\right]$$

Can’t forget this -1!

Differential Thermal Advection

Now consider differential portion between the pressure levels:



Between 850 - 925:

d(Tadv)/dP > 0

$-\chi \propto$ -1 * (positive value) = positive value

Between 925 - 1000:

d(Tadv)/dP < 0

$-\chi \propto$ -1 * (negative value) = negative value

QG X Breakdown

$$-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[\boldsymbol{V}_g \cdot \boldsymbol{\nabla}_p\left(-\frac{\partial \Phi}{\partial p}\right)\right]$$

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Differential Thermal Advection

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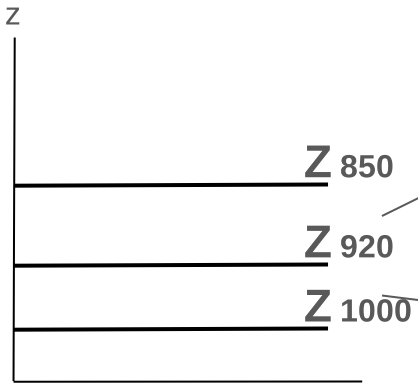
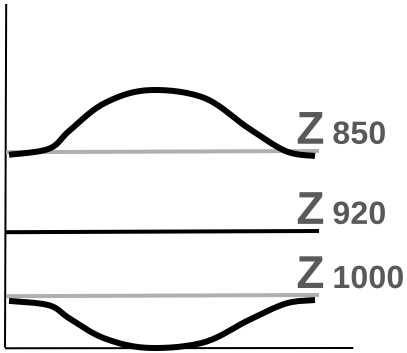


Between 850 - 925:

Positive Value = Height Rises!

Between 925 - 1000:

Negative Value = Height Falls!



QG X Breakdown

$$-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[\boldsymbol{V}_g \cdot \boldsymbol{\nabla}_p\left(-\frac{\partial \Phi}{\partial p}\right)\right]$$

OR

$$-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[-\frac{R}{p}\boldsymbol{V}_g \cdot \boldsymbol{\nabla}_p T\right]$$

Differential Thermal Advection:

- Warm air advection at a given pressure level induces height rises above that level, and height falls below that level

- Cold air advection does the opposite: it cases height falls above the given pressure level, and height rises below.

- This ties back directly to Charles's Law!

QG X Breakdown

$$-\frac{\partial H}{\partial p}$$

Diabatic Heating:

- Similar in concept to differential thermal advection:
  Diabatic heating in a layer causes the layer to expand;
  Diabatic cooling in layer causes the layer to contract.

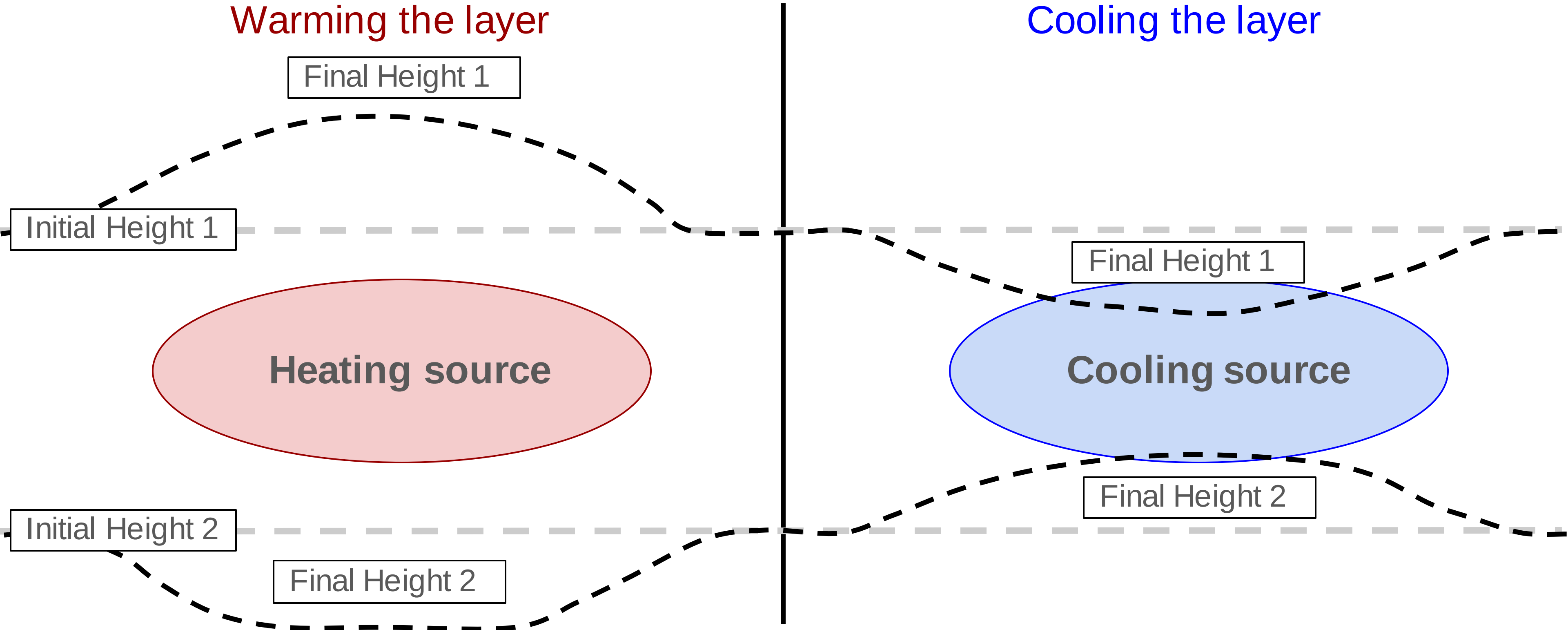
- Causes height rises (falls) above (below) the source of heating.

- Causes height rises (falls) below (above) the source of cooling

- Examples:
  - Latent heat release from large systems (hurricanes, large cyclones, etc…)
  - Mesohighs in the wake of strong MCSs

QG X Breakdown

$$-\frac{\partial H}{\partial p}$$



QG X Application

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi = -f_0 \boldsymbol{V_g} \cdot \boldsymbol{\nabla_p}\left(\frac{1}{f_0}\nabla_p^2\Phi + f\right) - \frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left[\boldsymbol{V_g} \cdot \boldsymbol{\nabla_p}\left(-\frac{\partial\Phi}{\partial p}\right)\right] - \frac{\partial H}{\partial p}$$

How do we use this?

- Use the vorticity advection term to help anticipate where a vorticity maximum or minimum (i.e. a trough or a ridge) will go.

- Use the differential thermal advection term to anticipate whether or not a trough or ridge will amplify.

- The diabatic heating term is similar to the thermal advection term, but is typically not as consequential.

QG X Application

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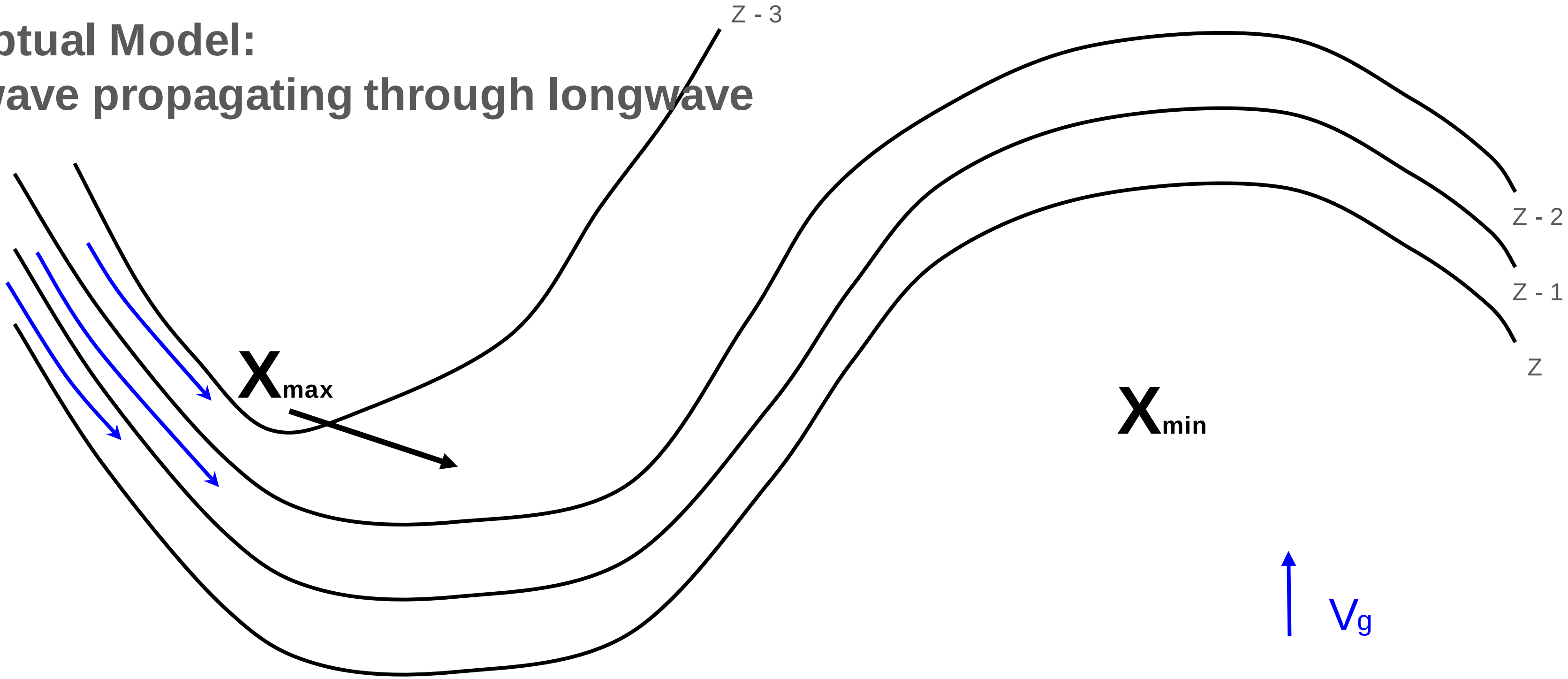
Conceptual Model:
Shortwave propagating through longwave



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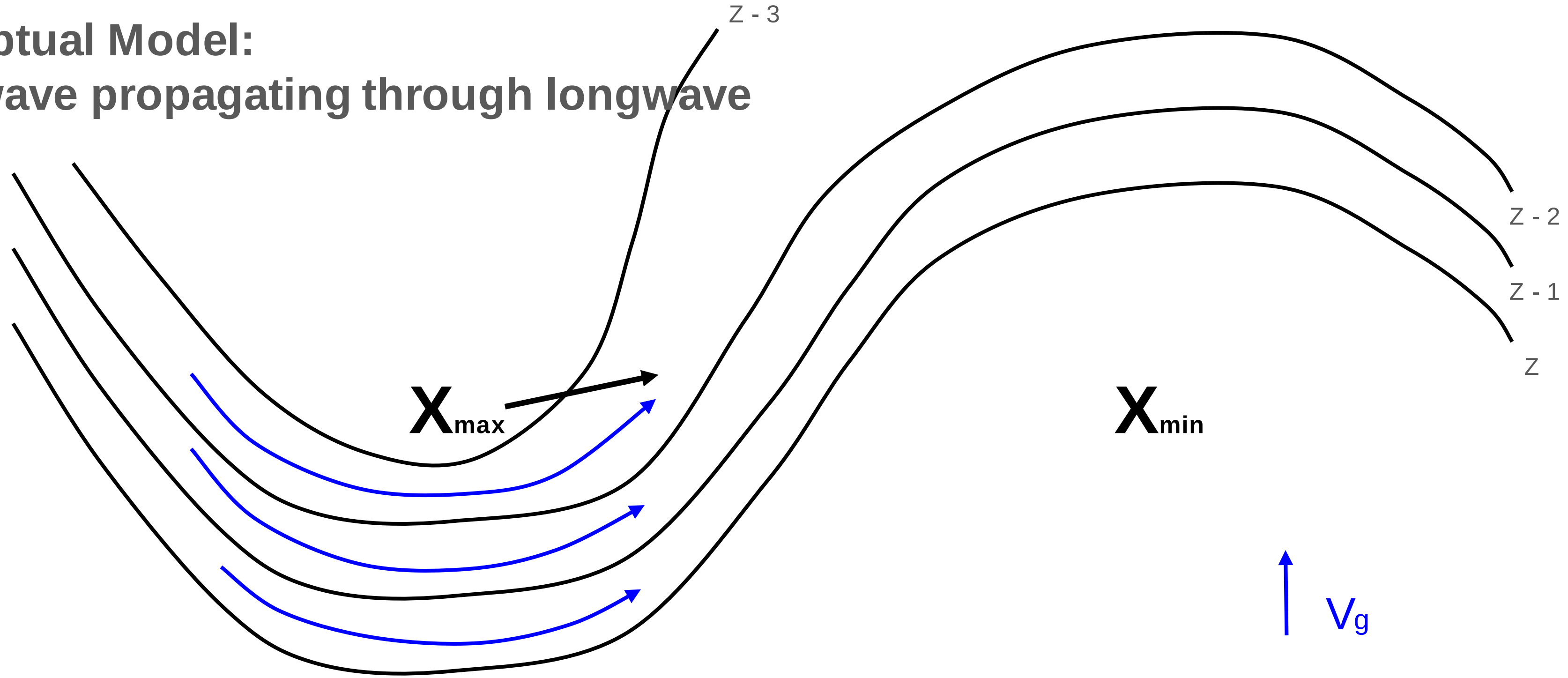

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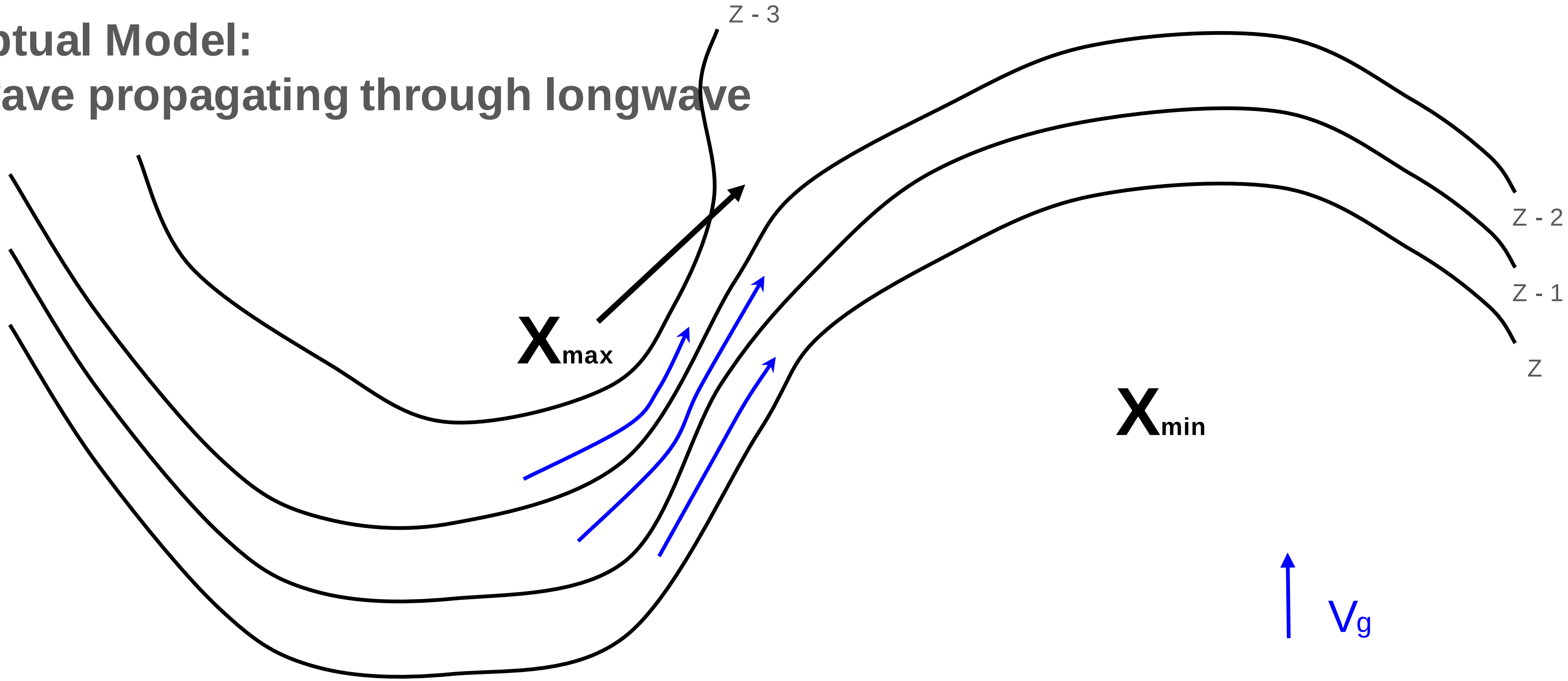

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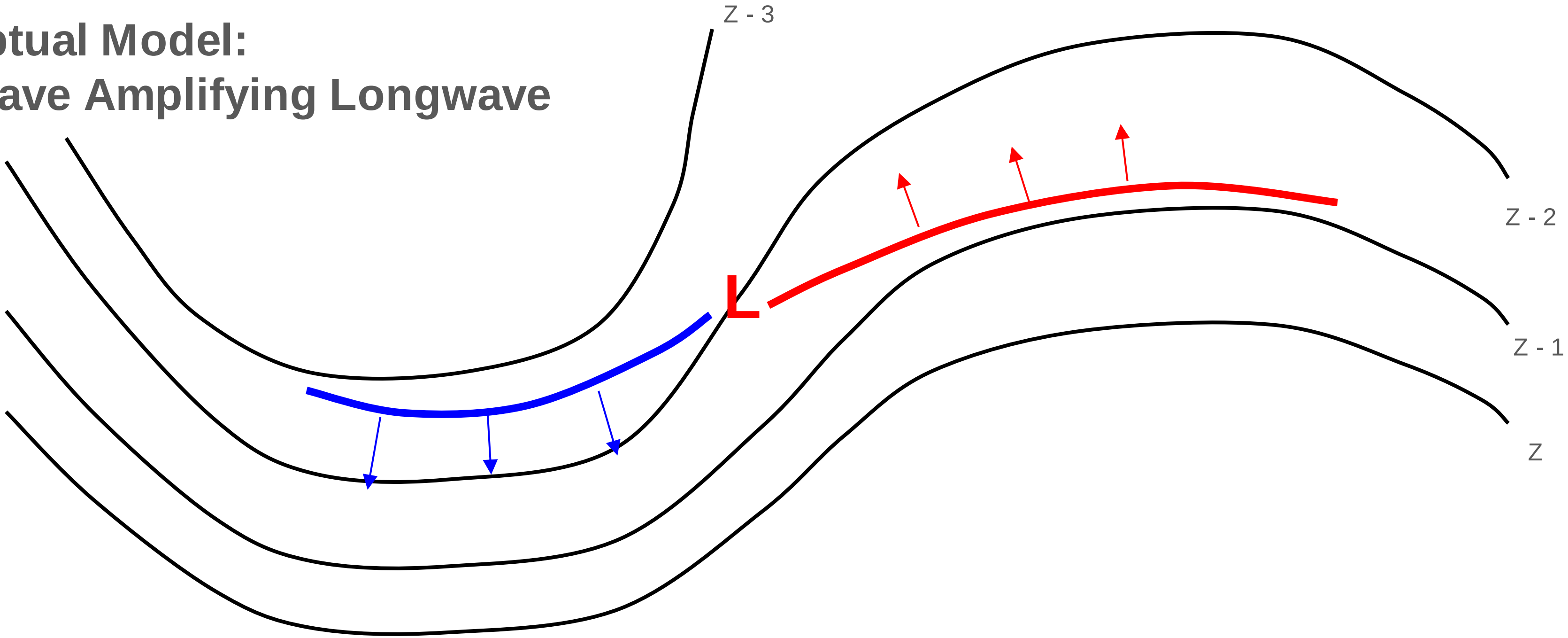

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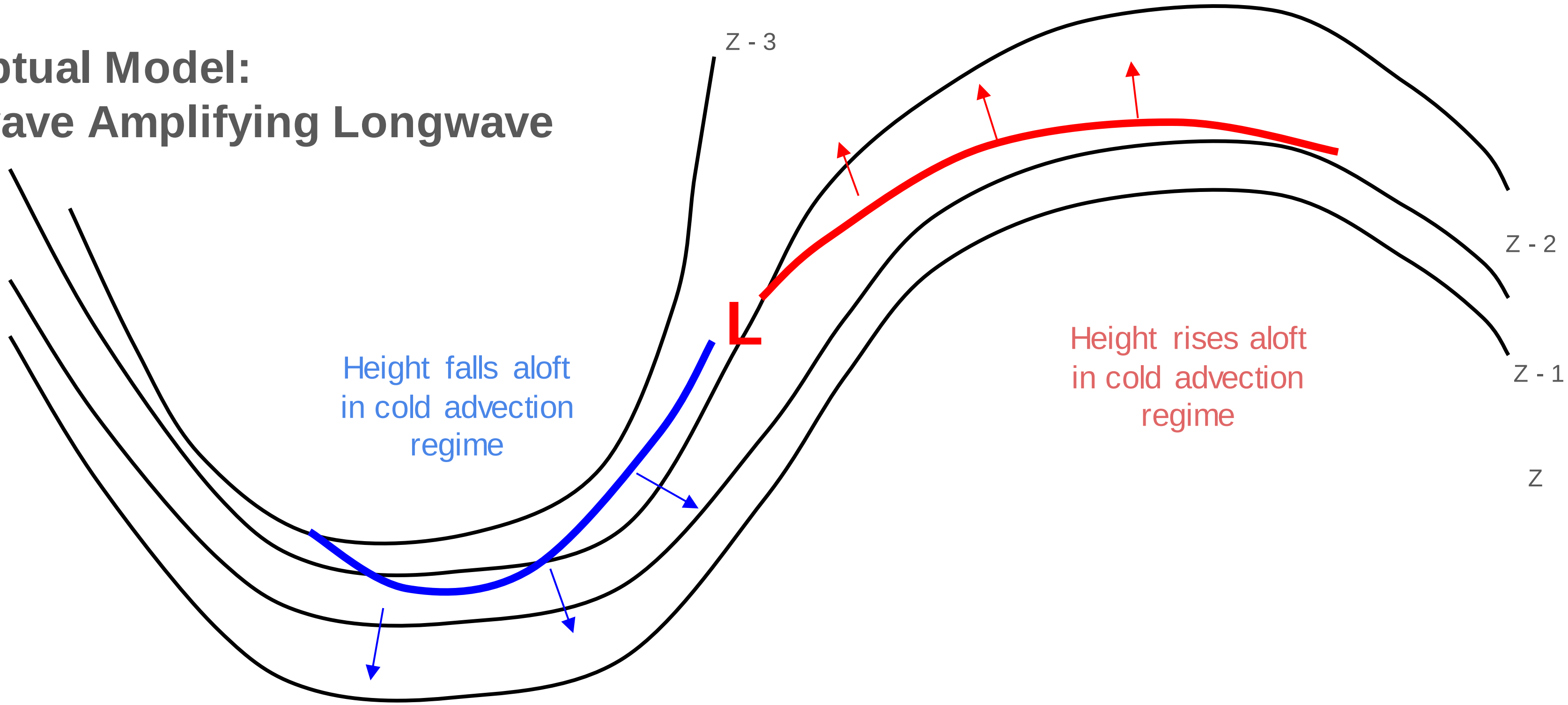
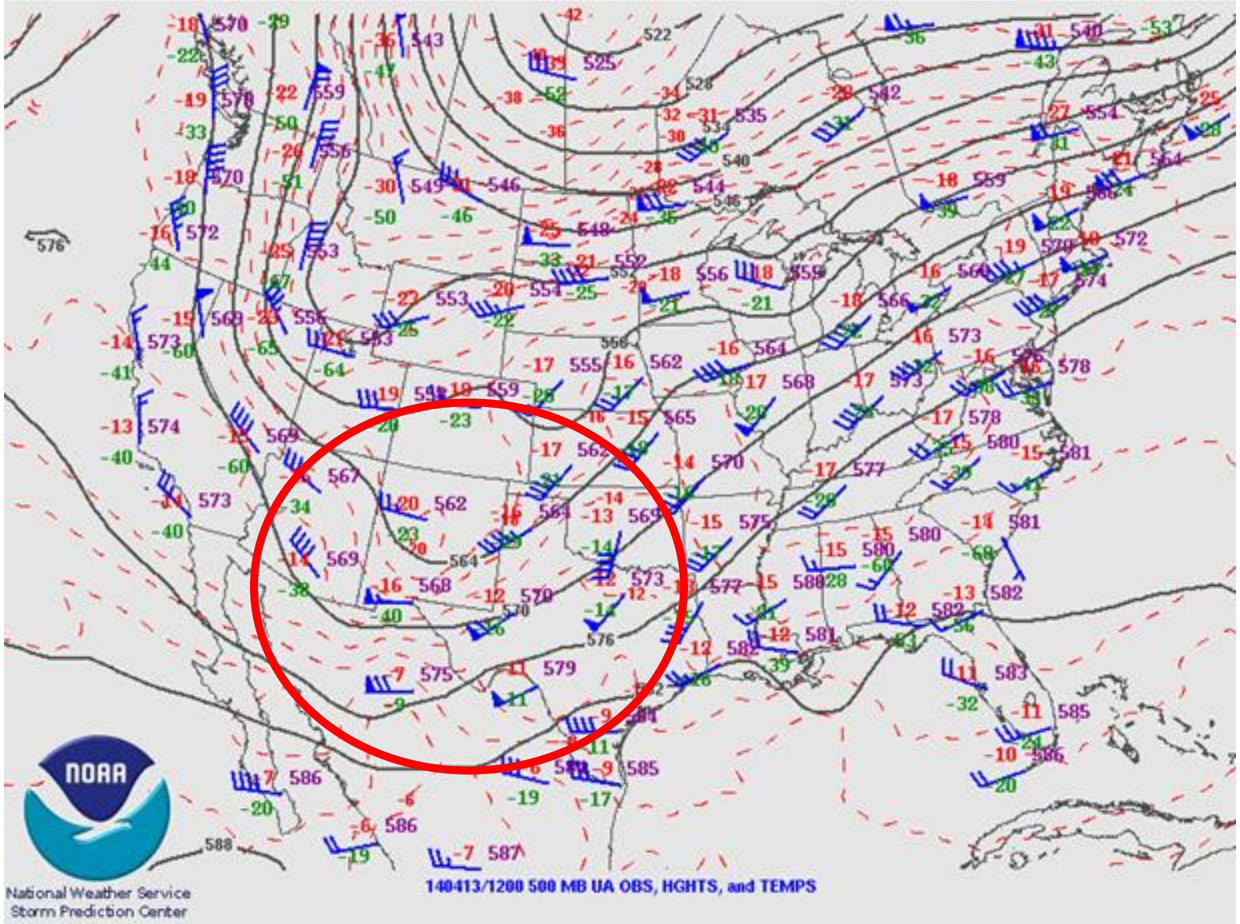


QG X Examples

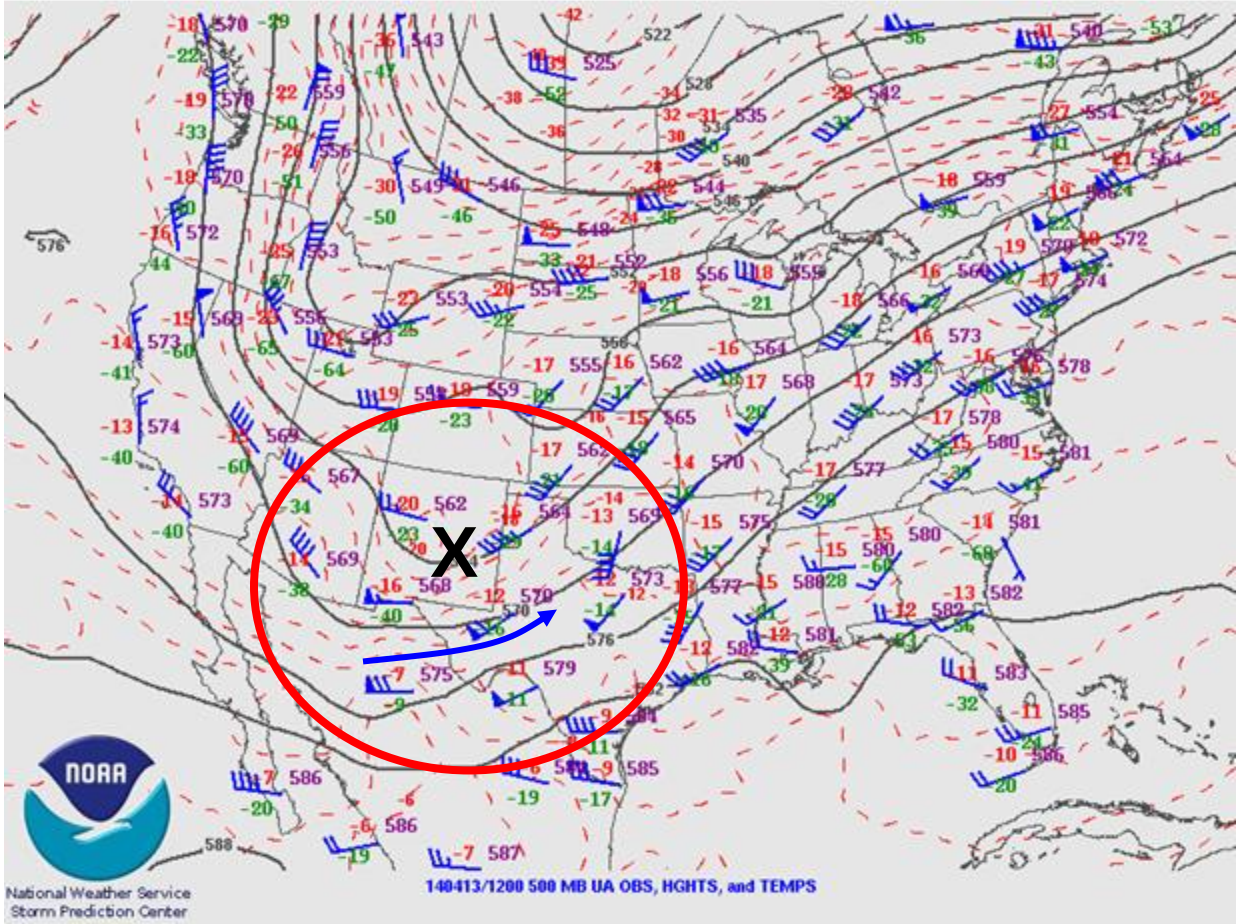


Focus on the shortwave trough in New Mexico.

Where is the vorticity maximum at?

Where will the 500 mb winds advect this vort. max?

QG X Examples



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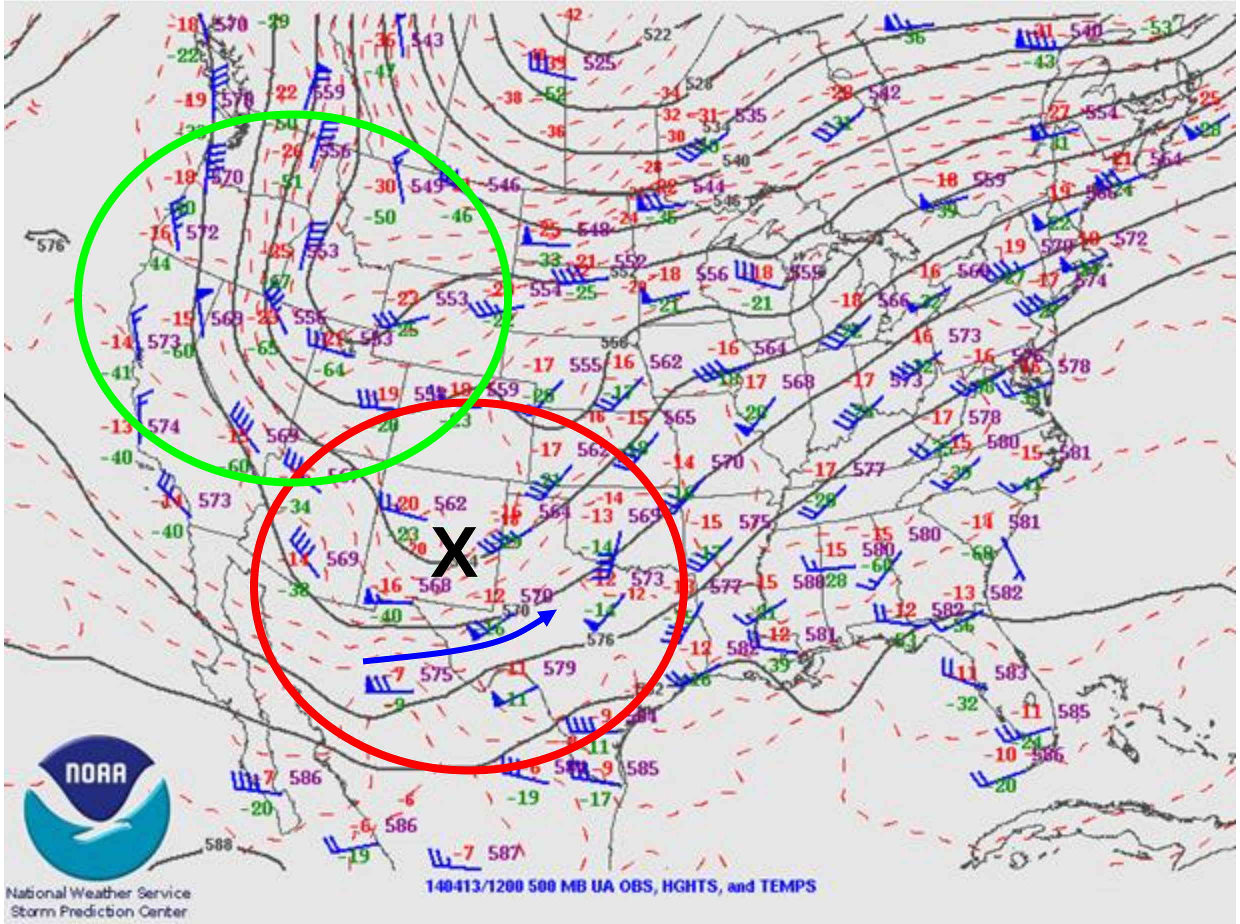

How about this shortwave over the northern Great Basin?

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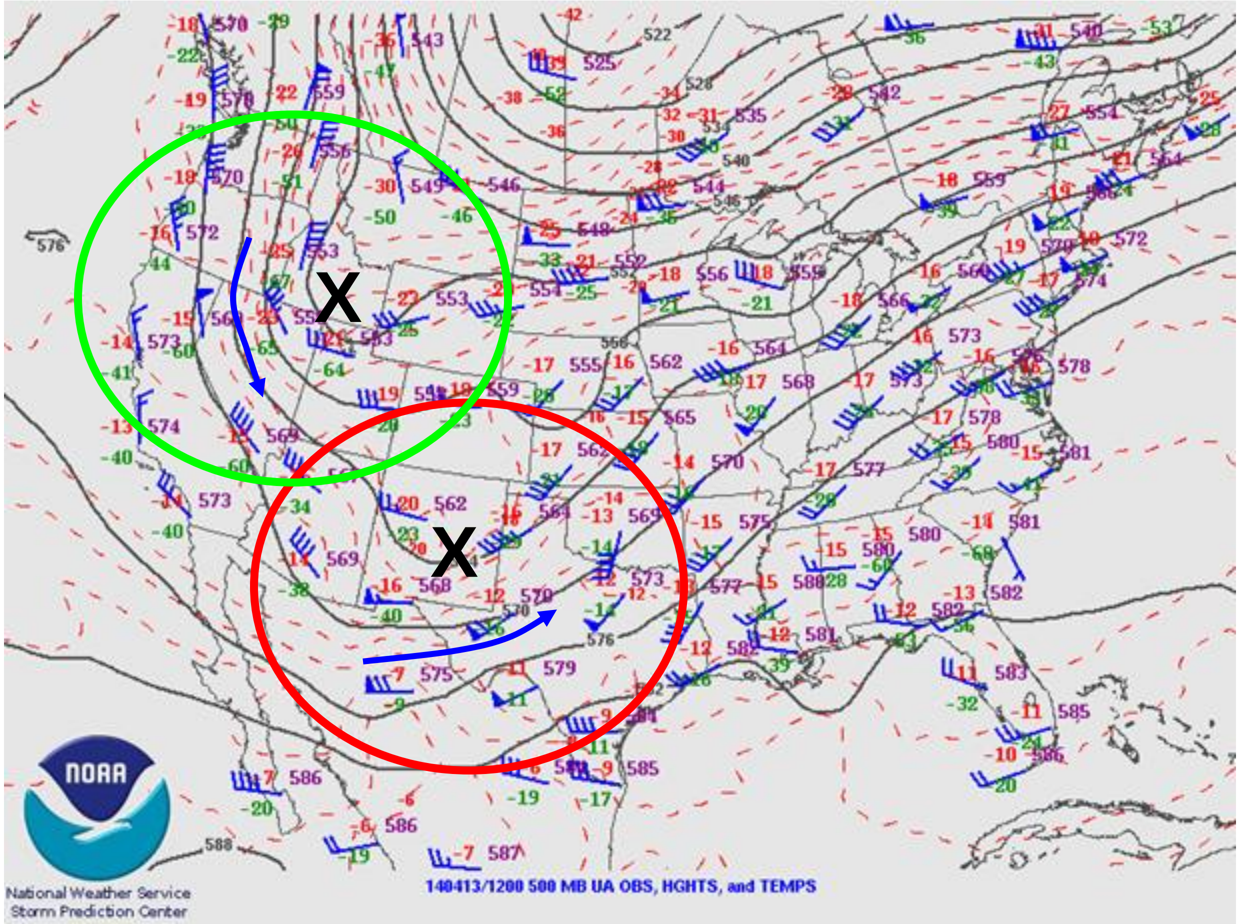
QG X Examples



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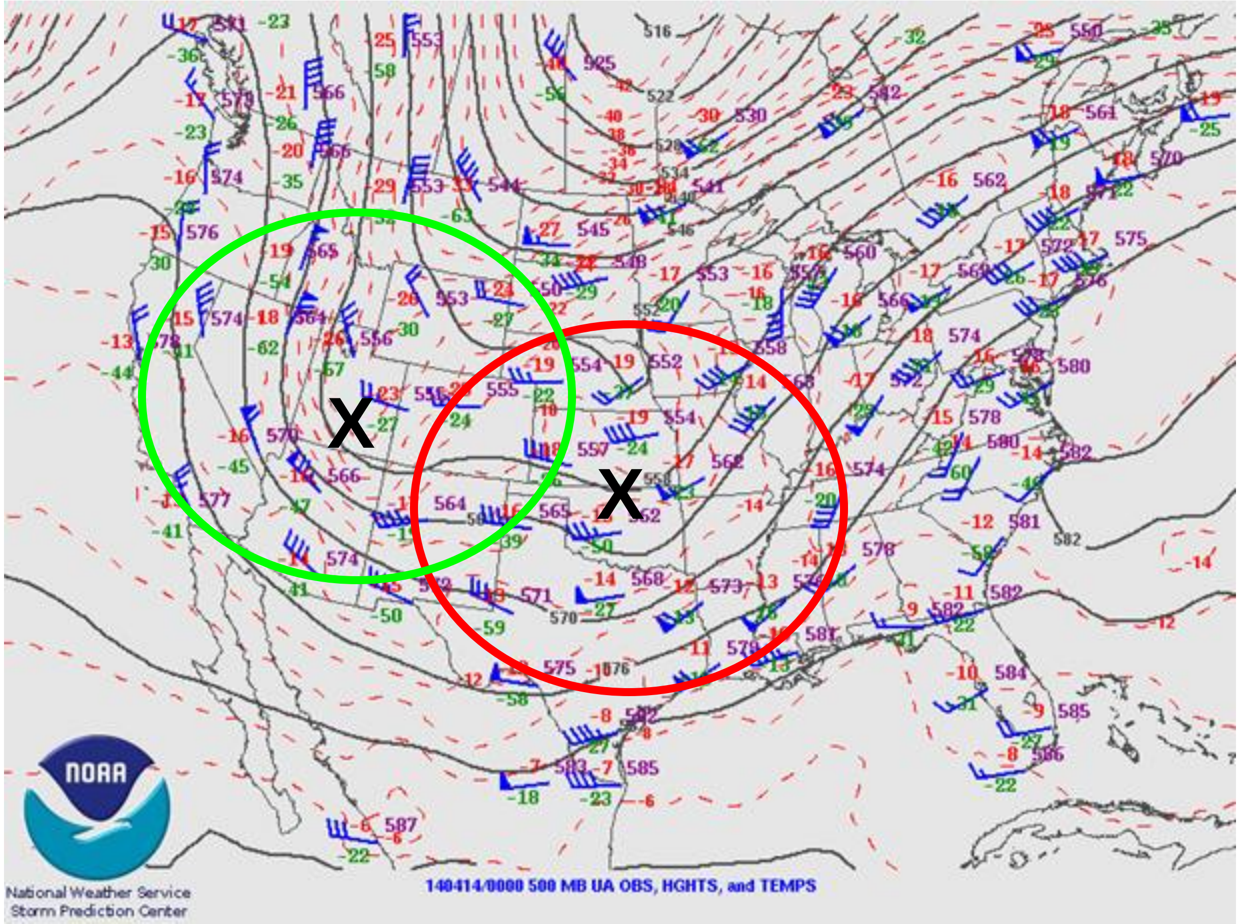
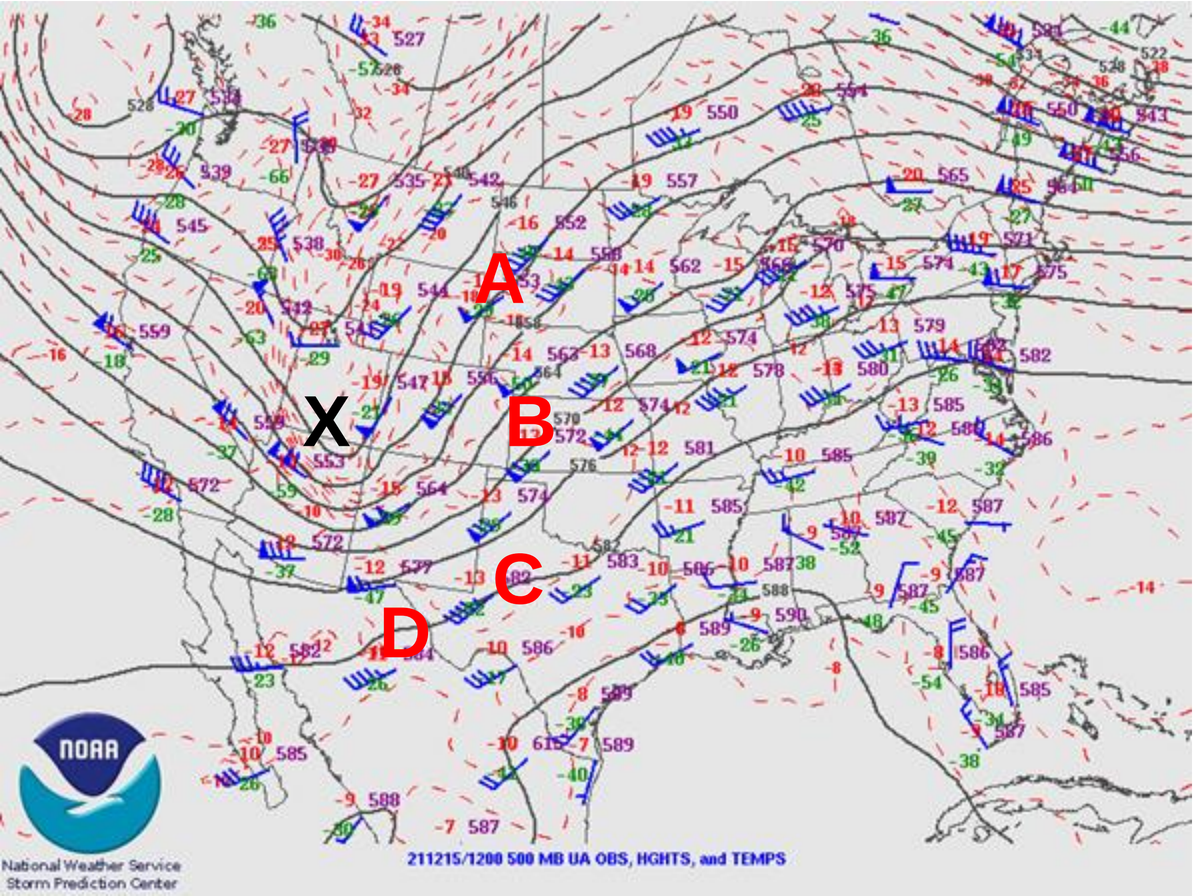
Where will the geostropshic winds advect the vorticity?

QG X Examples



Did this meet your expectations?

QG X Examples



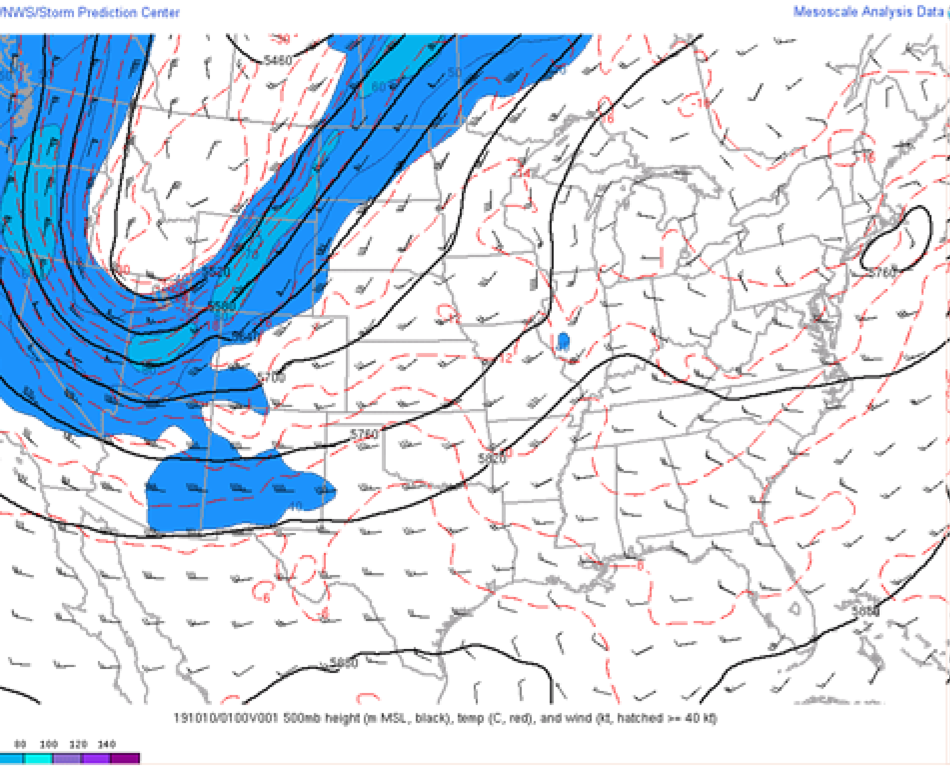
How about this case?

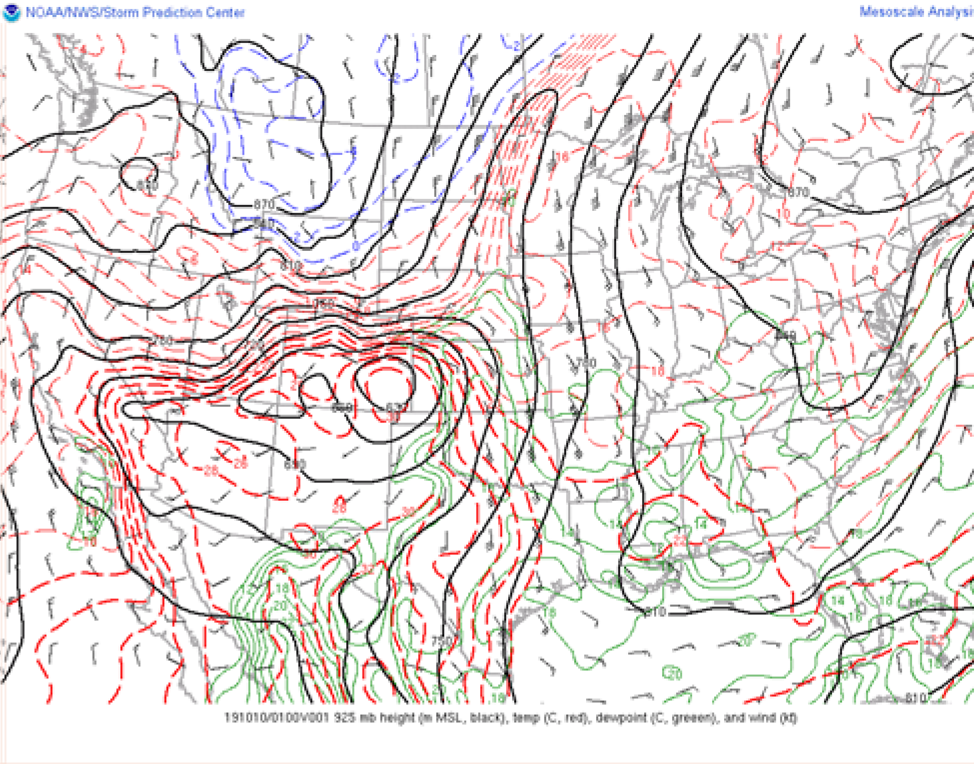
Where do you think this trough will go in the next 12 hours?

(Choose A, B, C, or D)

QG X Examples

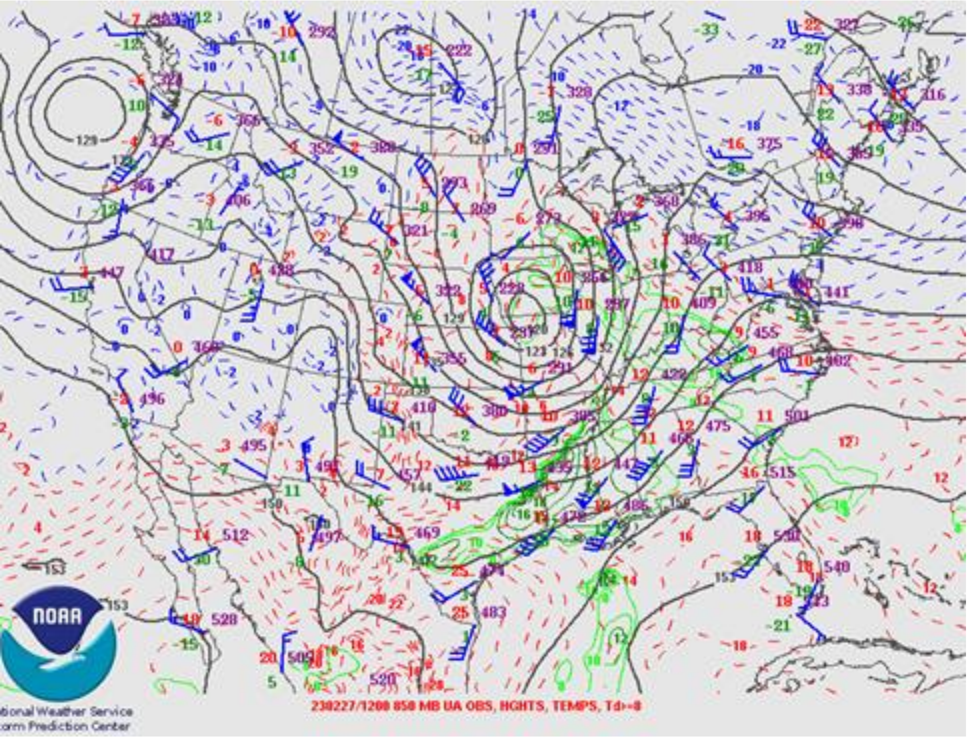
Watch how the 500 mb trough deepens as the 925 mb cold front surges south into the Plains.
This is an example of differential thermal advection.

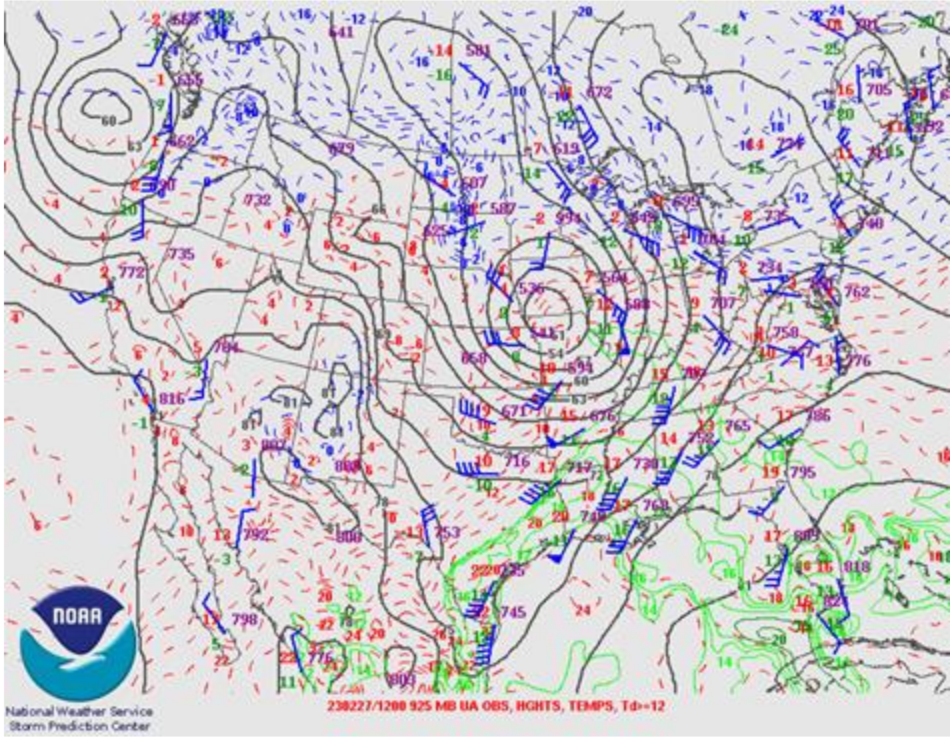




QG X Examples

Where is the strongest warm air advection at 850 mb?
This will tell you where the 925 mb low should go!





QG X Examples

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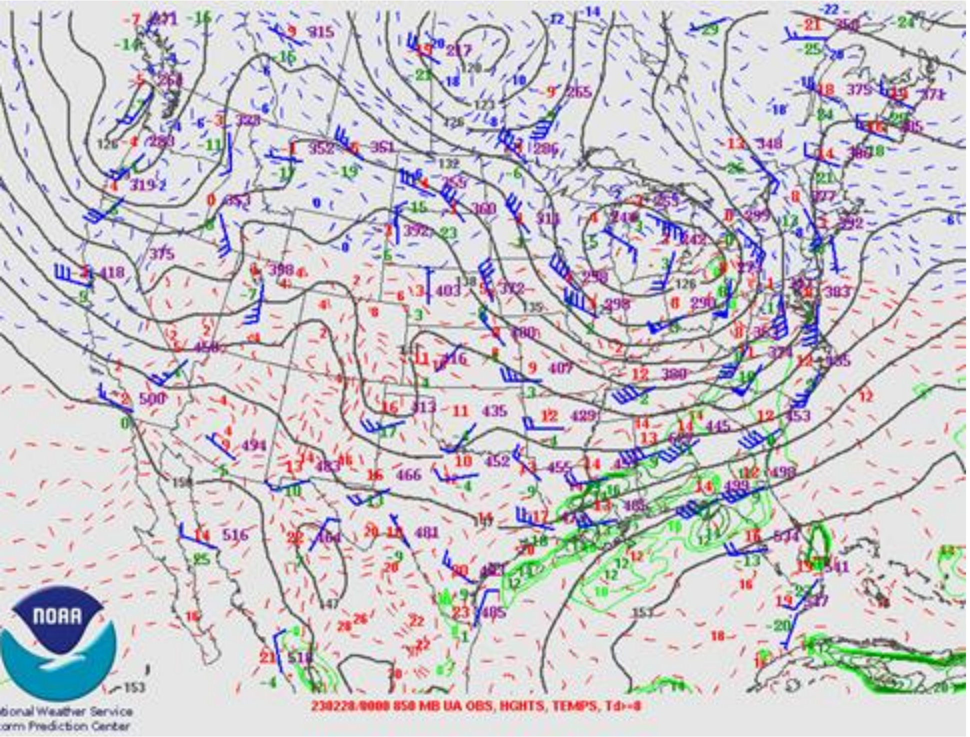
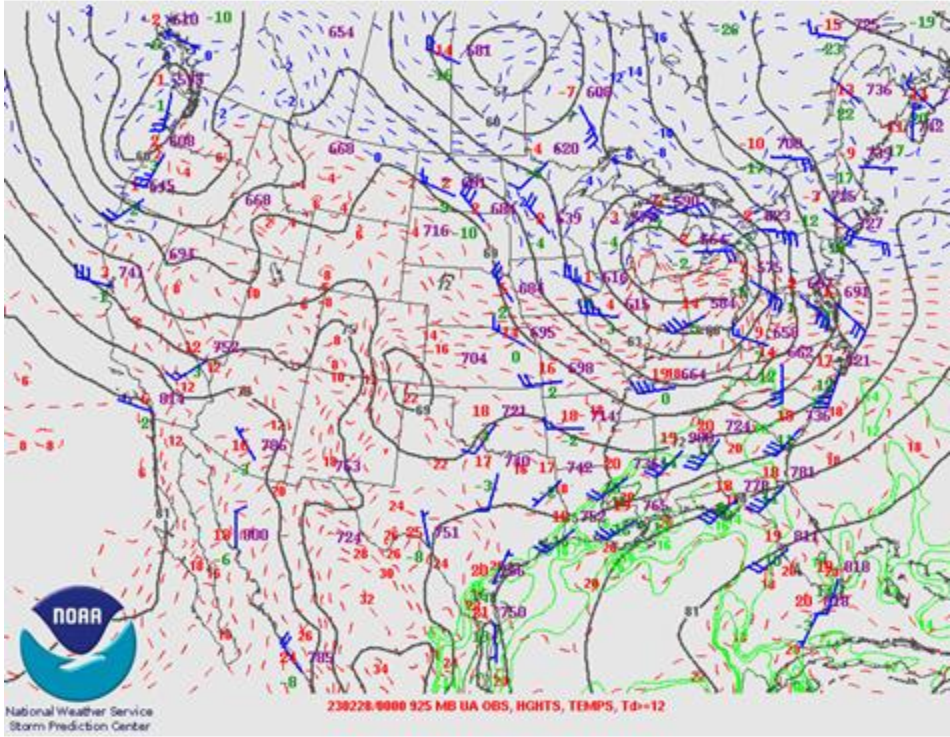
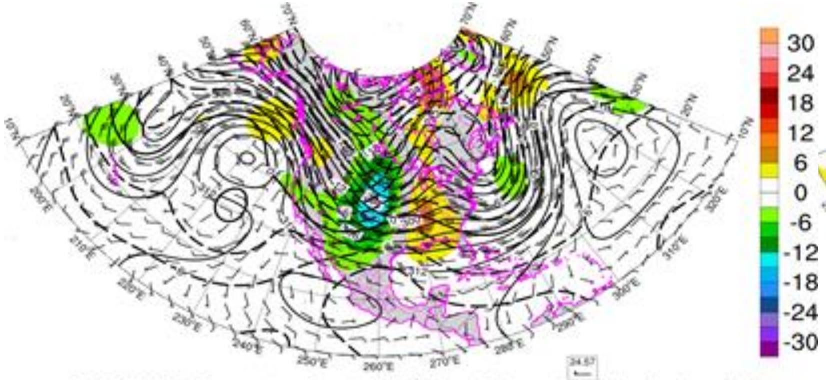


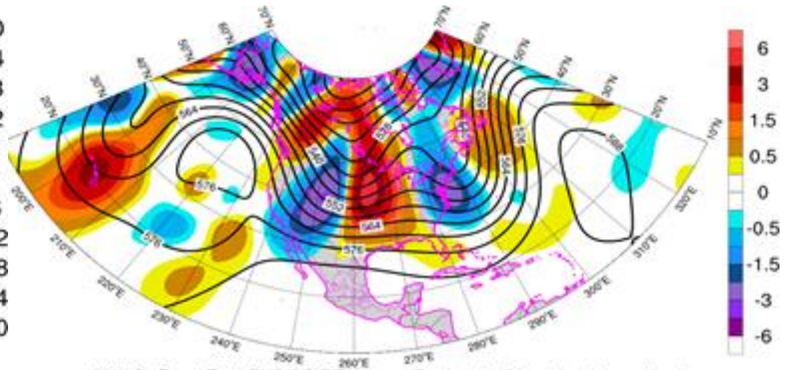


QG Resources

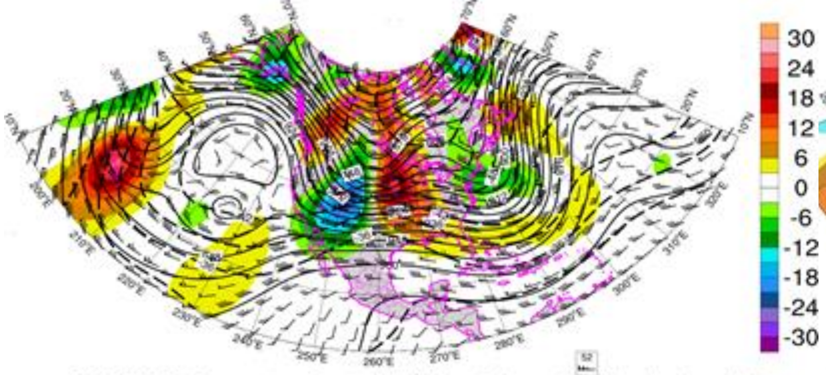
https://inside.nssl.noaa.gov/tgalarneau/real-time-qg-diagnostics/



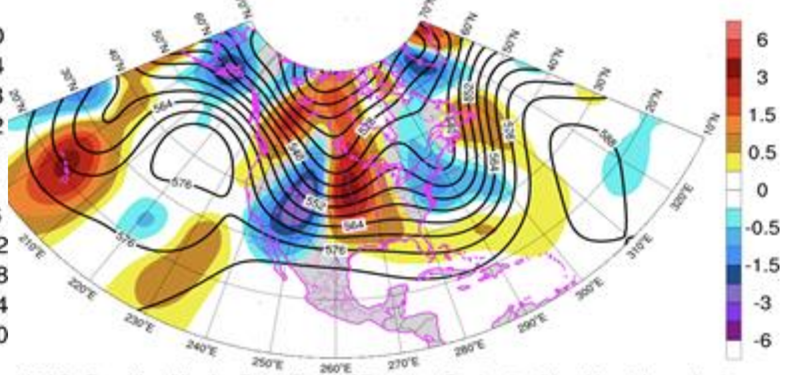
700 hPa Height, Temperature, Geostrophic Wind, and Temperature Advection (smoothed)
0-h CMC Global forecast at 2021020412



500 hPa Z and Total RHS QG Z Tendency Forcing (traditional form) (smoothed)
0-h CMC Global forecast at 2021020412



300 hPa Height, Temperature, Geostrophic Wind, and Temperature Advection (smoothed)
0-h CMC Global forecast at 2021020412



500 hPa Z and Amplification Term (B) QG Z Tendency Forcing (traditional form) (smoothed)
0-h CMC Global forecast at 2021020412