

Hodographs and Storm-Relative Helicity etc.

Bluestein Vol II. Pages 471-476.

Dowsett, 1991: A review for forecasters on the application of hodographs to forecasting severe thunderstorms. *National Weather Digest*, **16** (No. 1), 2-16.

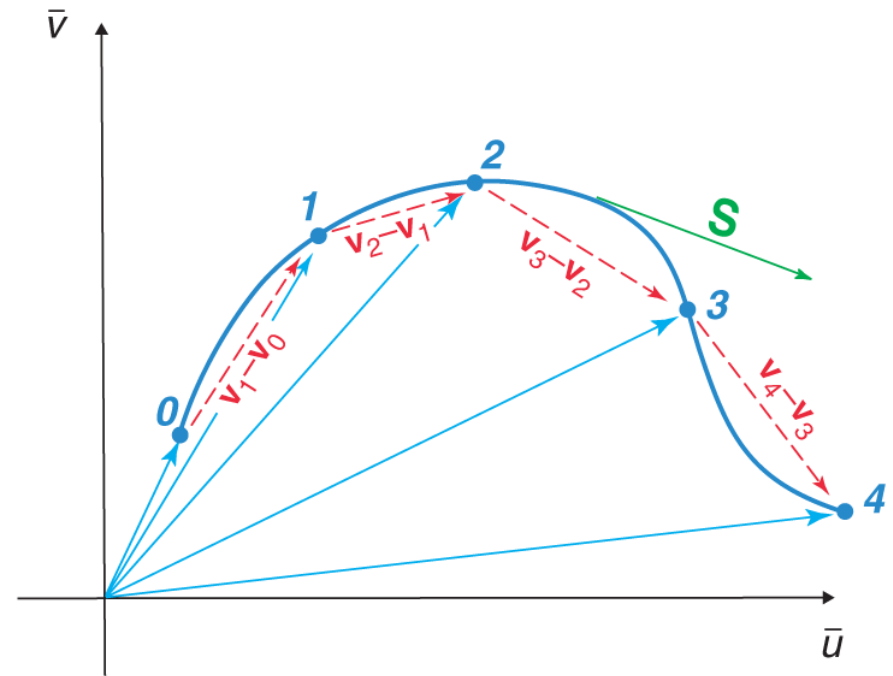
Available at <https://twister.caps.ou.edu/METR4403/lectures/HodographsDowsett.pdf>

Hodographs

- **Display of horizontal wind and its vertical variation**
- **Can determine vertical wind shear and horizontal vorticity vectors**
- **Useful for forecasting convective storm mode**
 - e.g., single cell, multicells, squall lines, supercells, tornadic storms
 - shape and length of hodograph are key characteristics

Hodograph Mechanics

- Plotted in $V-\theta$ (polar coordinates) or $u-v$ (Cartesian coordinates) space
- Connects tips of wind vectors plotted at different heights
- In example on right:
 - Plot zonal wind (u) on x -axis
 - Plot meridional wind (v) on y -axis
 - Indicate height of each observation



Dark blue curve: hodograph
Light blue arrows: horizontal wind
Vector wind difference between layers (bulk shear): red arrows
Shear vector at a point: green arrow (tangent to hodograph curve)

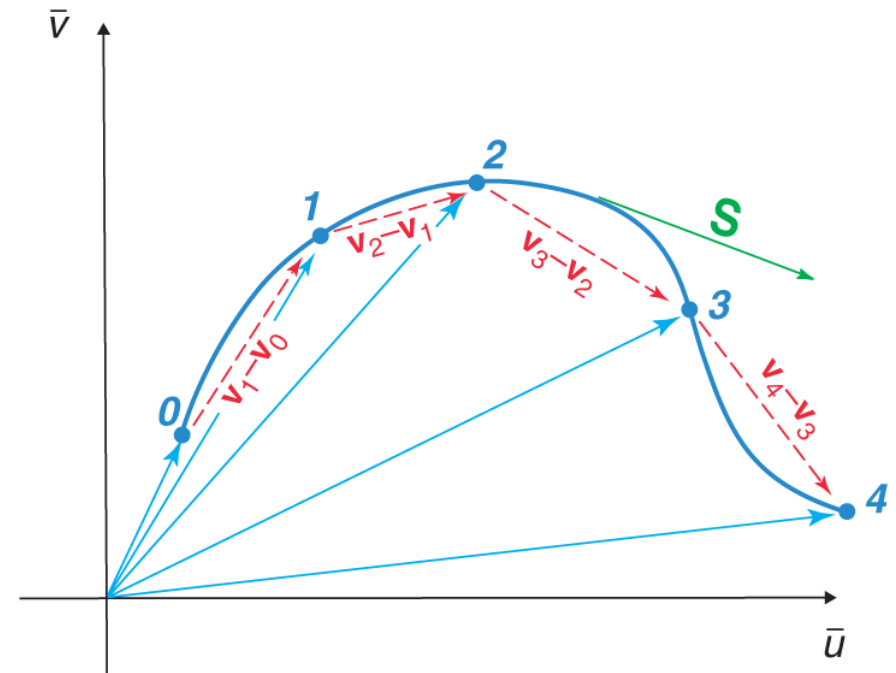
Hodograph Mechanics

- Vector subtraction of winds at different altitudes is called the **bulk wind shear**

- Actual vertical shear vector is tangent to the hodograph at every level and goes as

$$S = \frac{V_2 - V_1}{\Delta Z} = \frac{\partial V}{\partial z}$$

- Significant fraction of shear in hodograph results from background temperature gradient (thermal wind balance)
- At low-levels, surface drag produces shear also.



Dark blue curve: **hodograph**

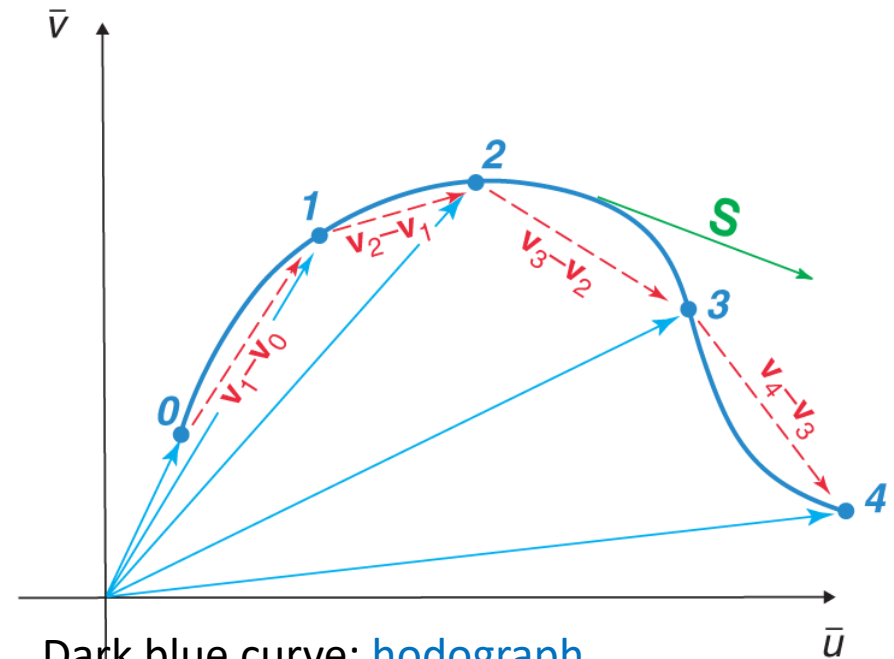
Light blue arrows: **horizontal wind**

Vector wind difference between layers (bulk shear): **red arrows**

Shear vector at a point: **green arrow** (tangent to hodograph curve)

Hodograph Mechanics

- Layer-mean wind can be estimated from hodograph
- Straight hodograph: mean wind lies on the hodograph
- Curved hodograph: mean wind lies on concave side of hodograph
- Storm motion:
 - Ordinary storms move with mean wind
 - Supercell storms move to the right (and slower) of the mean wind (Bunkers et al. 2000)



Dark blue curve: hodograph

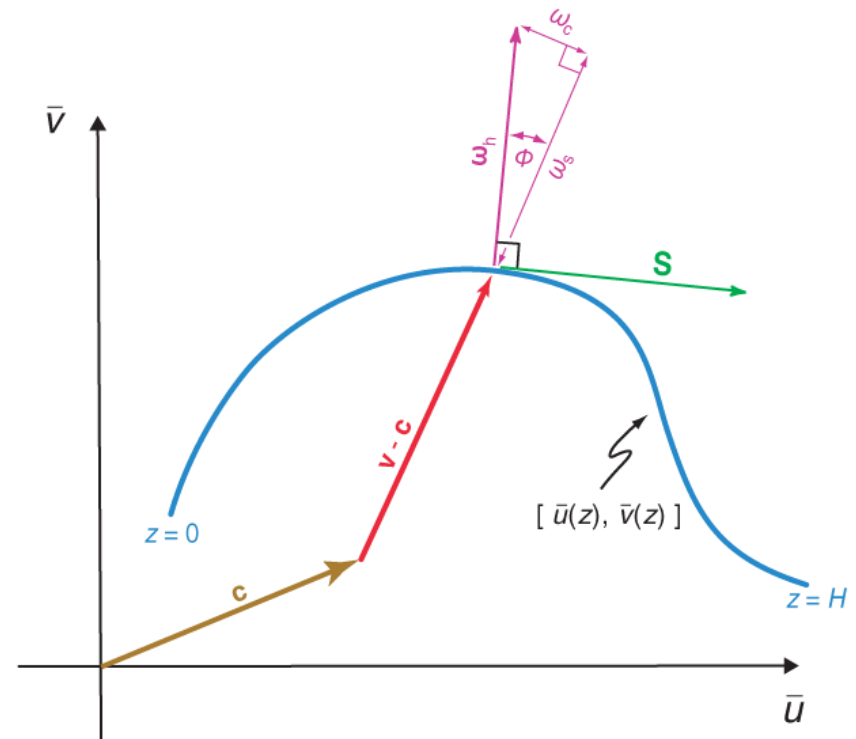
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Storm-Relative Winds

- Flow seen by observer moving with the storm
- Flow that the storm “feels” or “sees”
- Storm-relative winds are computed by subtracting storm motion vector (c) from the observed wind vector (v)
- Can also translate hodograph by $-c$ to get storm-relative hodograph



Dark blue curve: **hodograph**

Storm motion (c): **brown arrow**

Storm relative wind ($v-c$): **red arrow**

Shear vector at a point: **green arrow**

Horizontal vorticity components:

magenta arrows

Hodograph Mechanics

- Hodograph shape determines storm behavior
- Storm-relative winds are important and ground-relative winds are less meaningful
- Storm dynamics are determined by how shear vector changes with height
 - unidirectional shear vector: straight hodograph
 - directionally changing shear vector: curved hodograph

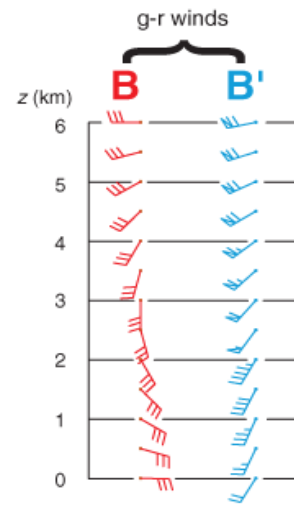
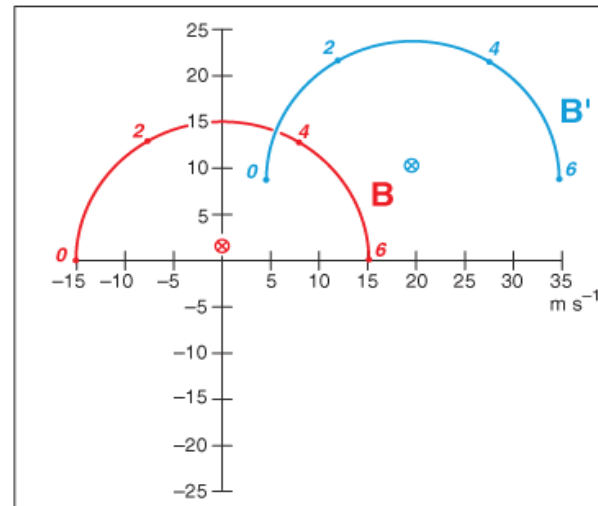
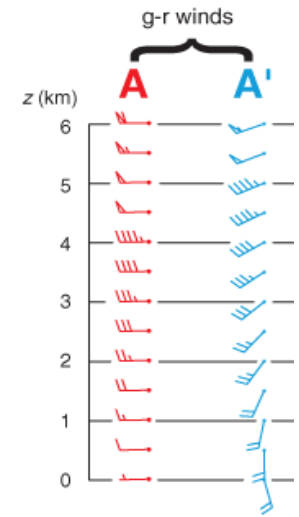
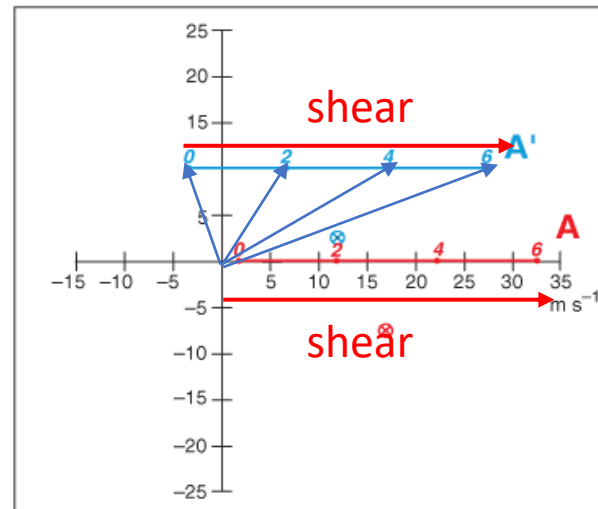
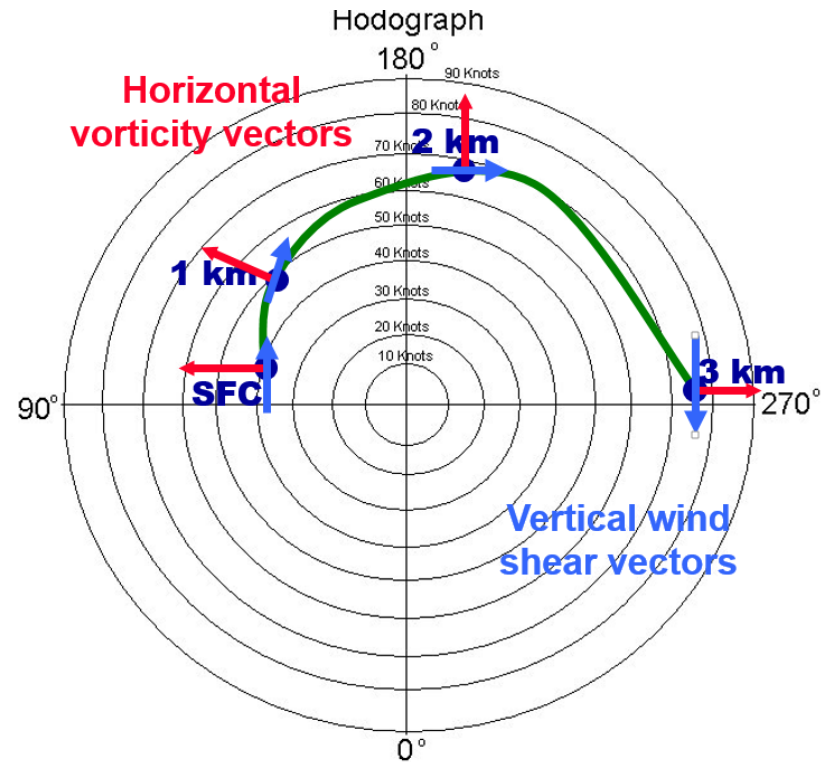
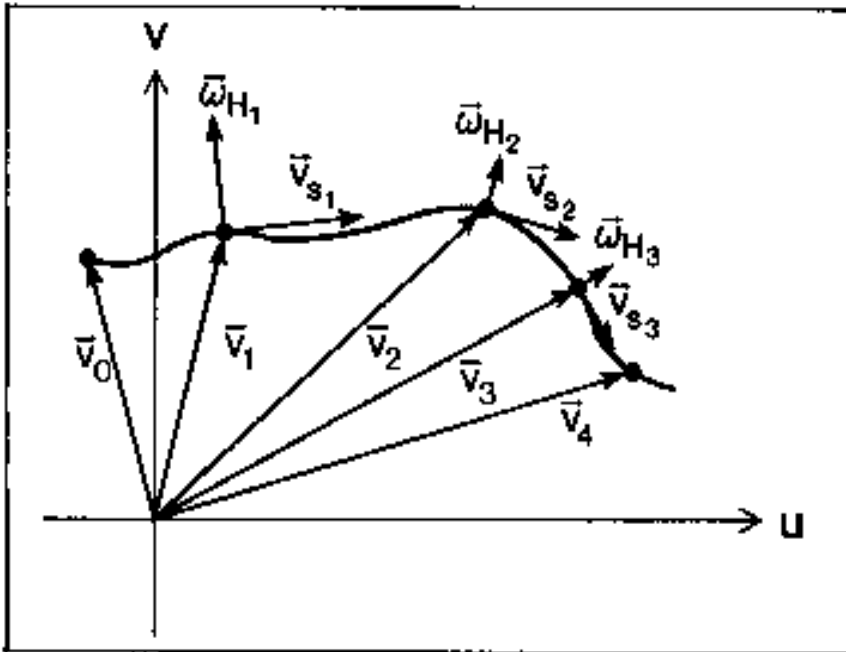


Fig. 2.13 MR (2010)

Horizontal vorticity $\vec{\omega}_H$ at various points on a hodograph

horizontal vorticity $\omega_h = (\xi, \eta) \approx \left(-\frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} \right) = \hat{k} \times S$

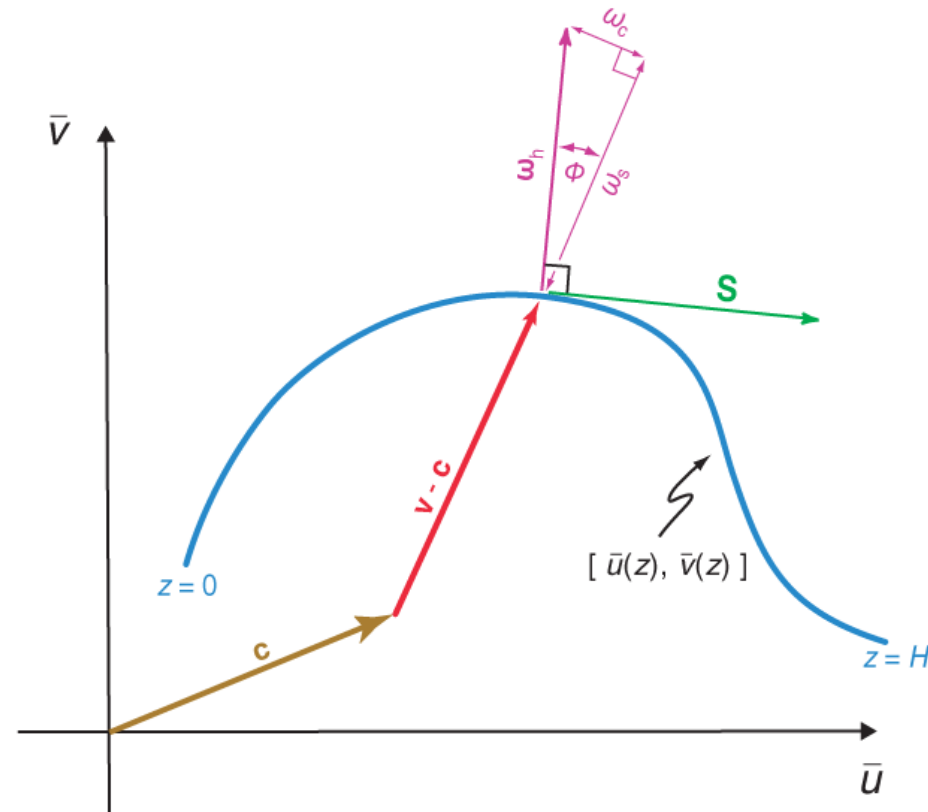
assumes $\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}$ small



Streamwise and Crosswise Vorticity

horizontal vorticity $\omega_h = (\xi, \eta) \approx \left(-\frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} \right) = \hat{k} \times S$
assumes $\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}$ small

- ω_h can be separated into parallel and perpendicular components relative to the storm-relative wind
- Component of ω_h parallel to storm-relative wind is **streamwise** vorticity (ω_s)
 - ω_h more streamwise for curved hodographs
- Component of ω_h perpendicular to storm-relative wind is **crosswise** vorticity (ω_c)
 - ω_h all crosswise for straight hodographs

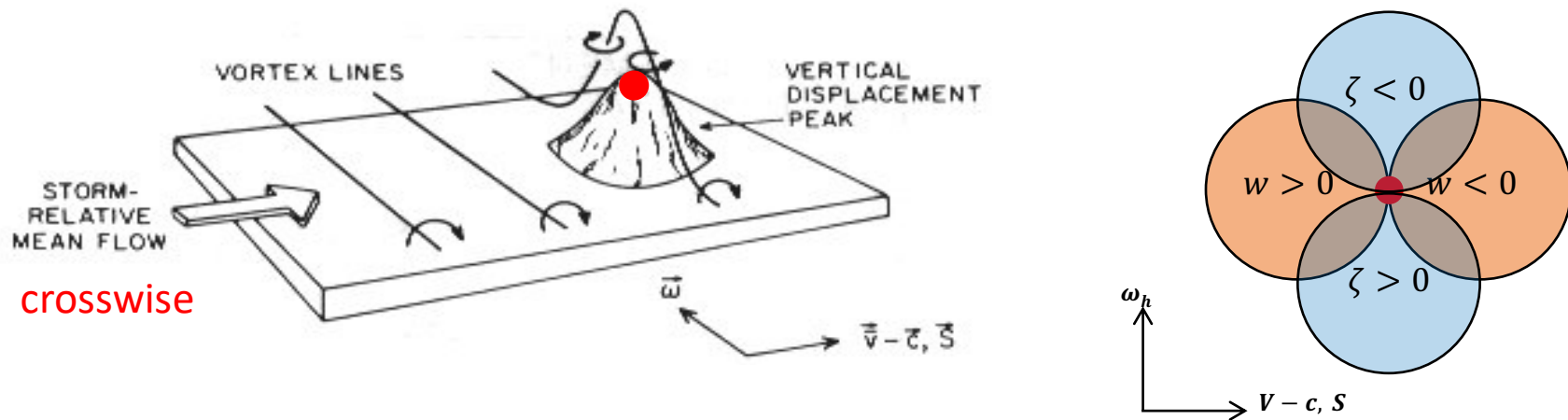


Dark blue curve: **hodograph**
 Storm motion (c): **brown arrow**
 Storm relative wind ($v-c$): **red arrow**
 Shear vector at a point: **green arrow**
 Horizontal vorticity components: **magenta arrows**

Situation where storm-relative inflow contains mostly crosswise vorticity

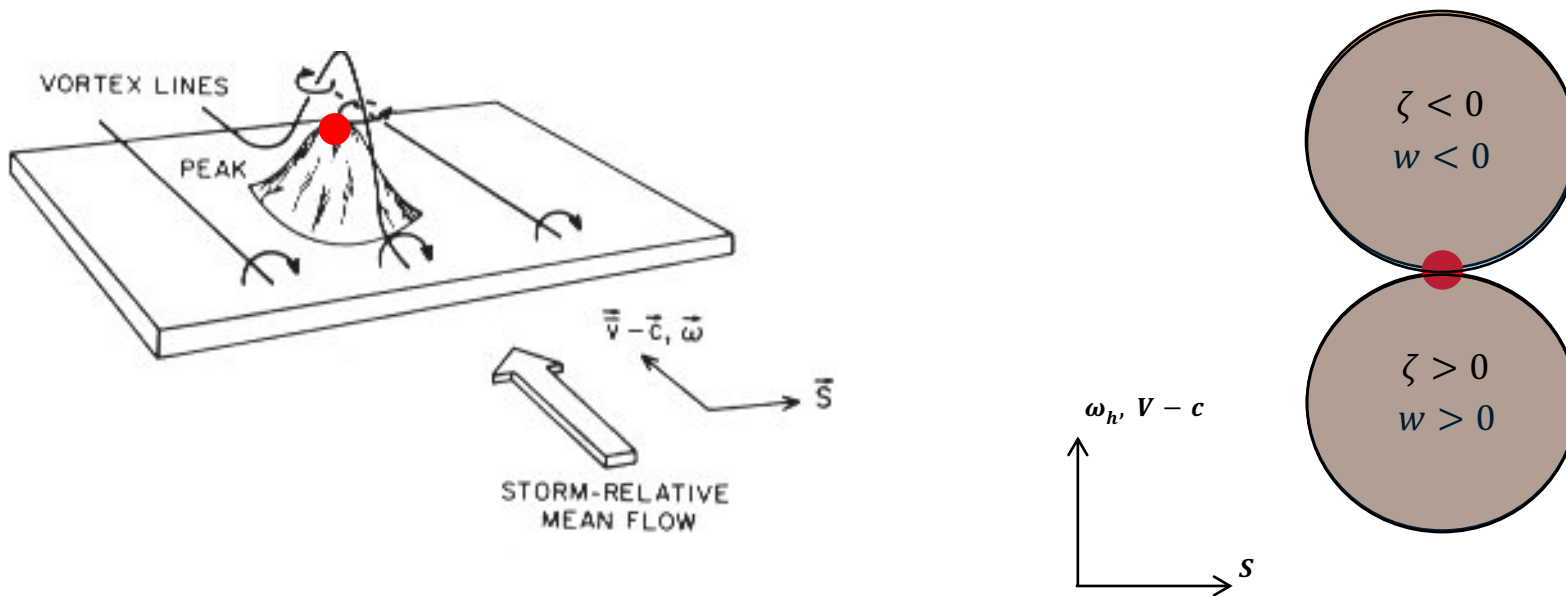
In the following figure, storm-relative low-level inflow is in the same direction as the vertical shear vector \rightarrow the flow is normal to the vorticity vector, therefore the vorticity is cross-stream.

Assume potential temperature is conserved below the cloud and θ_e is conserved in the cloud layer. Initially the θ (or θ_e) surfaces are flat. A 'bump' forms when there is an updraft. The storm-relative inflow turns upward on the upstream side of the bump and downward on the downstream side. Correspondingly on the right side of the bump is upward tilted vortex tube and the left side downward tilted vortex tube, corresponding to max/min vorticity. In this situation, the w and $\vec{\omega}_H$ are not correlated, there is no streamwise vorticity.



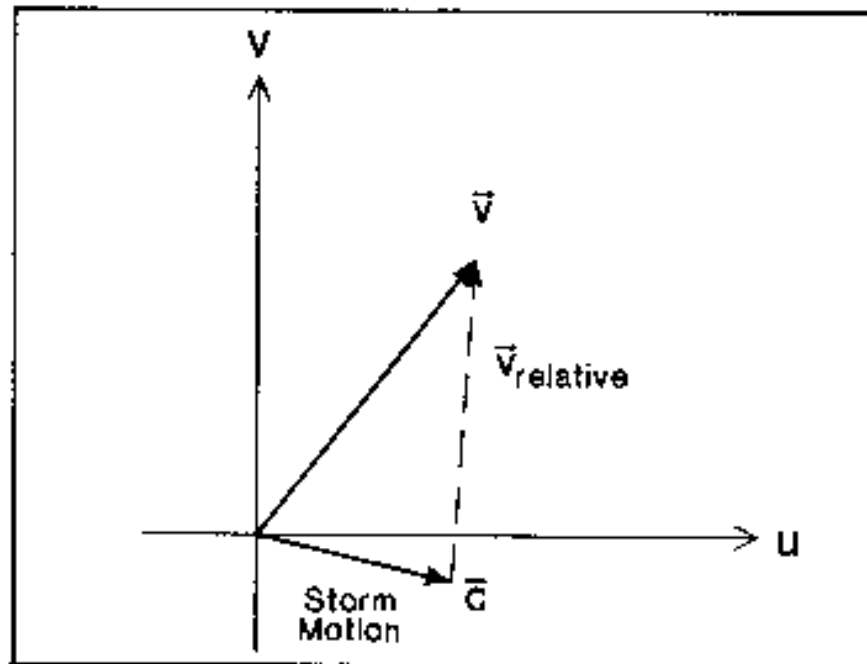
Situation where storm-relative flow contains mostly streamwise vorticity

In this case, storm-relative flow is normal to the vertical-shear vector, the flow is parallel to the horizontal vorticity vector. The max w coincides with max $\vec{\omega}_H$, and w and $\vec{\omega}_H$ are strongly (positively) correlated. There is large streamwise vorticity.

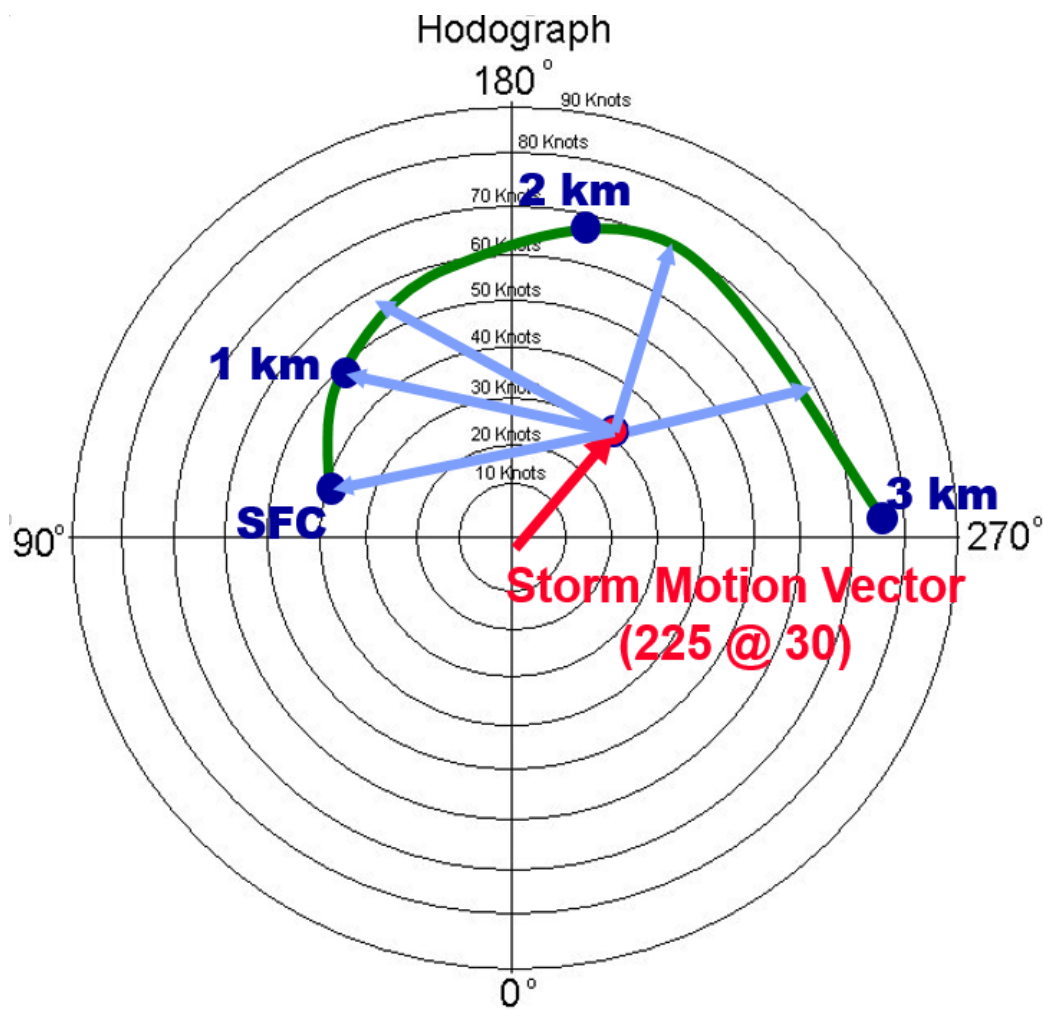


Importance of Storm-Relative Flow

Consider the effect of storm motion on the flow as seen by an observer moving along with the storm. Given a particular wind vector \mathbf{V} as shown below, for a storm moving with a vector velocity \mathbf{C} , the wind in a storm-relative framework can be obtained by subtracting out the storm motion; i.e., we define the relative flow \mathbf{V}_r to be $\mathbf{V} - \mathbf{C}$.

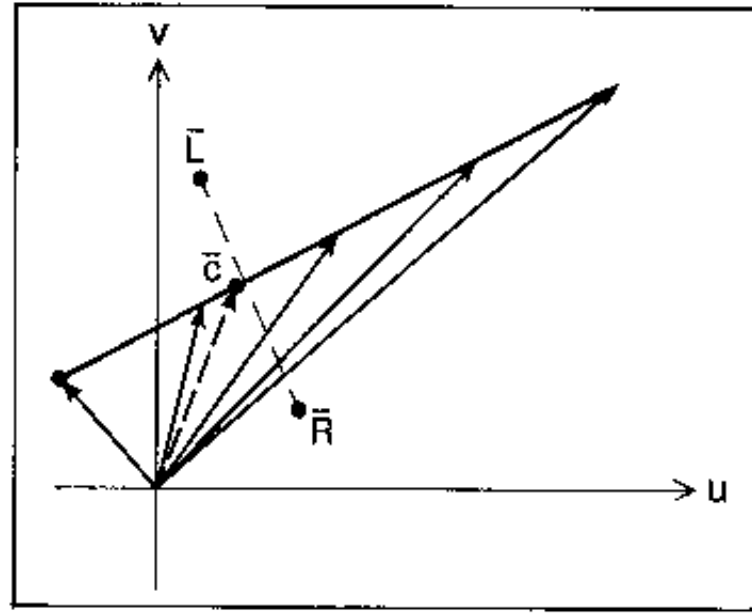


Storm relative wind vectors at different levels on the hodograph



What causes vorticity to be crosswise??

- when storm motion vector lies on hodograph!

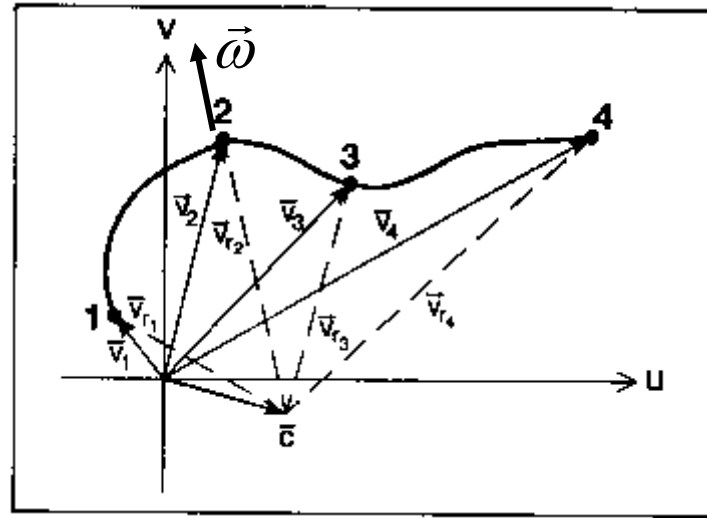


In the above example, the hodograph is a straight line and the shear is unidirectional (the flow is **not** unidirectional, however). If the storm-motion vector \vec{C} lies on the hodograph, then the storm-relative flow is always parallel to the shear vector therefore perpendicular to the vorticity vector \rightarrow the **vorticity is crosswise**.

For the above example, if the storm splits into the right and left mover. The storm motion vector for the right mover \vec{R} now lies on the right side of the hodograph. In this situation, one gets significant streamwise vorticity.

What hodograph causes large positive streamwise $\vec{\omega}$?

- when \vec{C} lies to the **right** of and away from the hodograph!



We see, for example at point 2, that the storm-relative velocity vector $\vec{V}_{r,2}$ is essentially parallel to the shear vorticity vector.

When the hodograph is curved, the storm motion is much more likely to lie somewhere off the hodograph. In this case, it can be shown that the simple average of the winds lies somewhere "inside" the curve of the hodograph. Since the advective part of the storm motion usually is considered to arise from the vertically-averaged winds in the storm-bearing layer, such a component of storm motion normally lies off the hodograph when it is curved.

A flow which only changes direction with no change in speed (i.e., a hodograph which is a segment of a circle centered on the origin) has *only* streamwise vorticity.

Helicity

We can compute the component of vorticity $\vec{\omega}$ in the direction of the storm-relative velocity $\vec{V} - \vec{C}$ as

$$\vec{\omega}_s = \frac{(\vec{V} - \vec{C}) \cdot \nabla \times \vec{V}}{|\vec{V} - \vec{C}|}$$

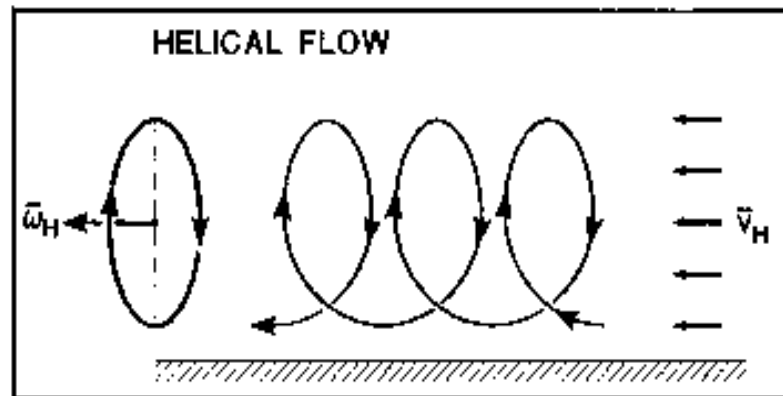
which we call the **streamwise vorticity**.

The numerator,

$$H = (\vec{V} - \vec{C}) \cdot \nabla \times \vec{V}$$

is known as **helicity**. It is the **product of storm-relative flow speed and the streamwise vorticity**.

In the case of streamwise vorticity, one can visualize the flow as being helical; a good mental image is a passed football rotating in a "spiral." Hence, the term **helicity** is associated directly with streamwise vorticity.



Storm Relative Helicity - SRH

However, $(\vec{V} - \vec{C}) \cdot \nabla \times \vec{V}$ is **not** Galilean invariant, i.e., it is dependent on the coordinate system (following the storm motion) chosen, i.e., it is a quantity in a storm-relative coordinate.

Therefore there is no single value of helicity for a given sounding (unlike CAPE) – it depends on the value of storm-motion vector \vec{C} and requires an estimate of \vec{C} before the helicity can be calculated.

SRH in the lowest two or three kilometers of the atmosphere is most relevant to the likelihood of supercell behavior with storms in that environment.

Therefore, helicity integrated over the lowest 3 km, i.e.,

$$\bar{H} = \int_0^{3km} (\vec{V} - \vec{C}) \cdot (\hat{k} \times \frac{\partial \vec{V}}{\partial z}) dz$$

is often calculated and used as a guidance for thunderstorm forecast. Here we assume the vorticity is given by the environmental wind vector \vec{V} . This quantity is called **Storm-Relative Environmental Helicity (SREH)**.

To do that one also has to estimate the storm motion vector \vec{C} . A crude way is to use the pressure-weighted mean wind in the lowest 5 to 6 km.

Recent research has shown that lower-level SRH (e.g., that in the lowest 1 km or 500m) has higher correlation with tornado potential.

SRH on Hodographs

The vertical integrated helicity is equal to minus twice the signed area swept out by the storm-relative winds between the surface and height Z , i.e.

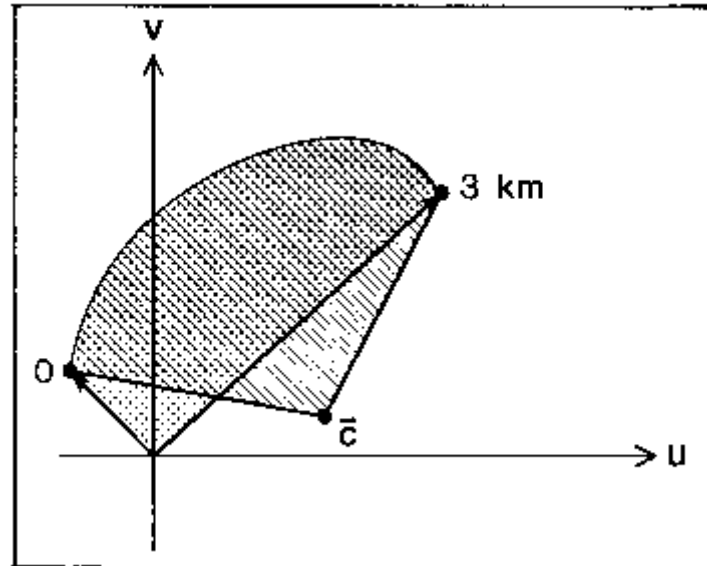


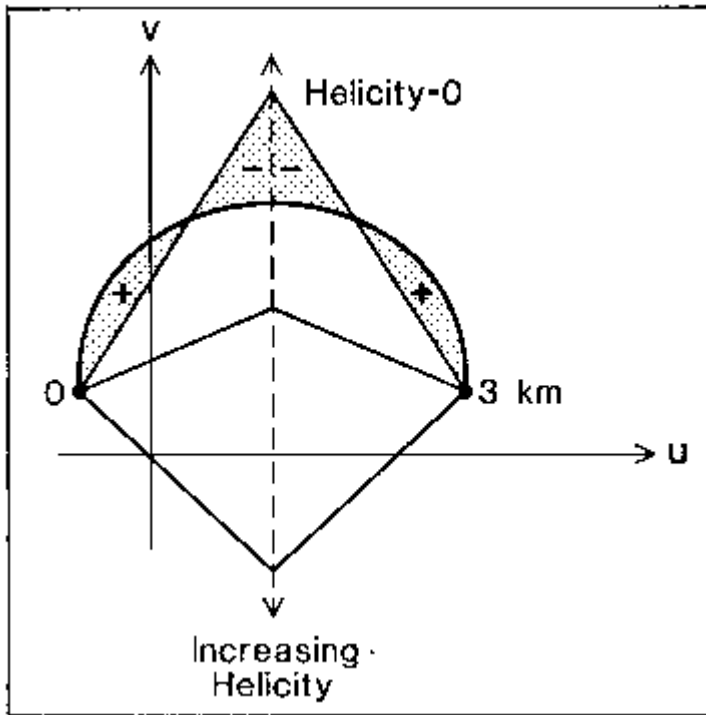
Illustration of the area (stippled) swept out by the ground-relative wind vectors along the hodograph from 0 to 3 km. Also shown is the area swept out by the storm-relative wind vectors (hatched).

Therefore, the storm motion can increase or decrease the storm-relative helicity associated with a given hodograph, including change its sign.

The farther is the tip of storm-motion vector from the hodograph, usually the larger is the helicity.

Effect of Storm Motion on SRH

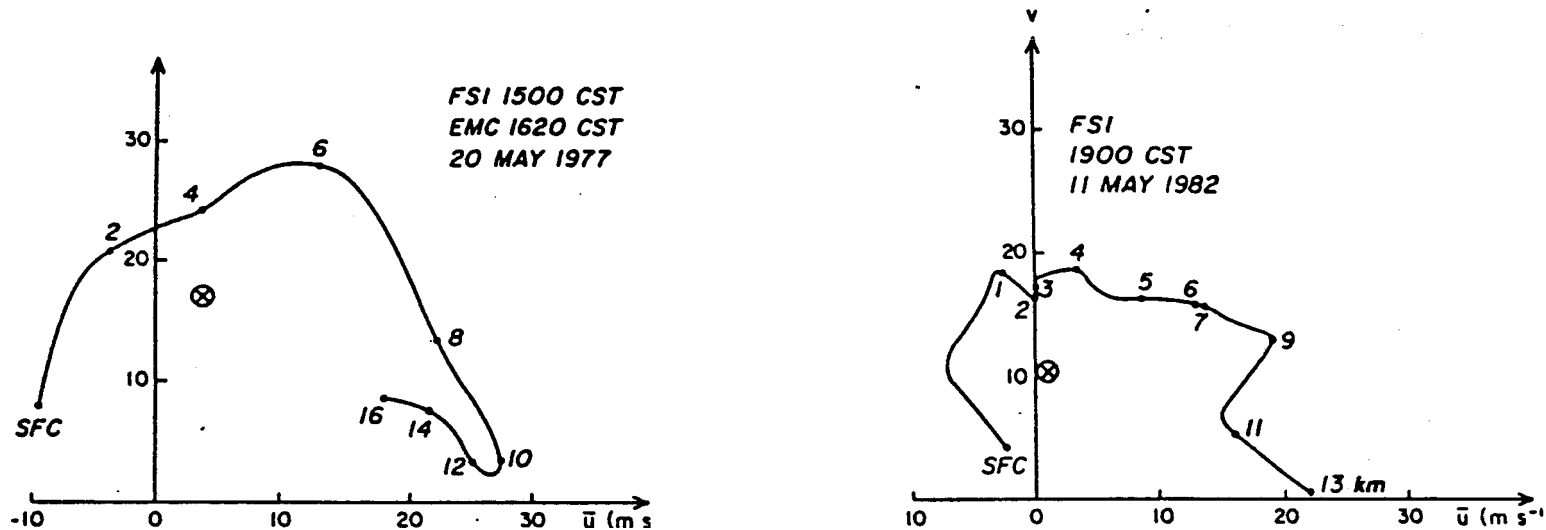
There are storm motions that can make the storm-relative helicity (averaged over some fixed layer) vanish; one example is shown below.



An example of the changes in storm-relative helicity, integrated over the layer from 0 to 3 km, as a function of the storm motion (C). In this example, changes in C are limited to changes in the north-south component, so C moves only upward or downward along the dashed line. There is a point (indicated) where the negative and positive areas cancel and the resulting total storm-relative helicity averaged over the layer vanishes.

Examples of Tornadic Hodographs

Famous 1977 Del City Supercell Storm:



Early results indicate that for tornado producing environment, SRH ranges from approximately $150 \text{ m}^2 \text{ s}^{-2}$ to upwards of $1000 \text{ m}^2 \text{ s}^{-2}$.

Davies-Jones (1990) examined the results of 28 tornado cases with the following categories for H (SREH):

$150 < H < 299$	weak tornadoes
$300 < H < 449$	strong tornadoes
$H > 450$	violent tornado

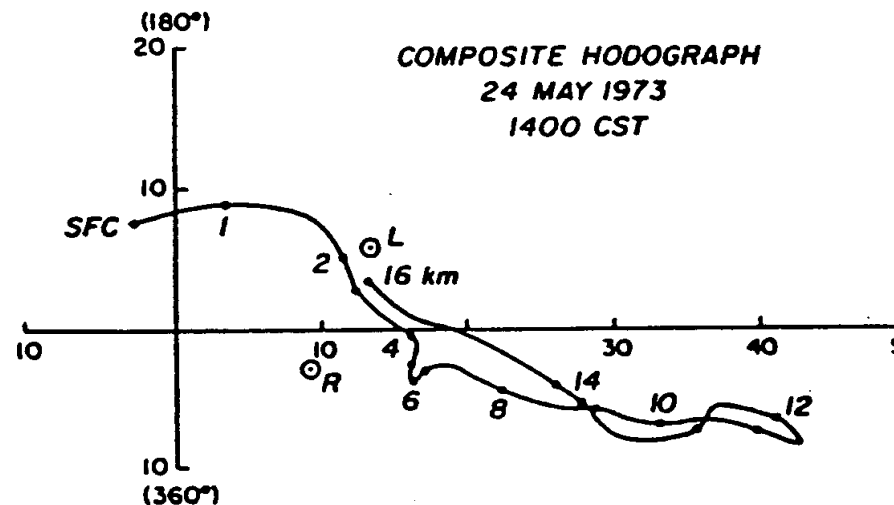
It is important to point out that helicity will not determine whether or not storms will develop, but instead indicates how a particular storm (or storms) may evolve given the ambient shear.

Strongly veering curved hodograph versus straight hodographs

At first glance, tornadic storms would be most likely only when the hodograph shows vertical shear that veers strongly with height, such storms are also possible when the hodograph is relatively straight.

Considerable streamwise vorticity may be present with a straight hodograph if storm motions lie significantly to the right of the hodograph. This would occur when split cells move sideways away from the “steering level” flow that originally lies on the straight hodograph.

An example of a hodograph which was approximately straight from 2 km to 11 km of altitude, along with the observed storm motions. (L = left-mover, R = right-mover) for a splitting storm pair, are shown below. Also shown are the tracks of splitting storms observed by radar, and the corresponding tracks of storms simulated numerically for similar environmental conditions.



Hodograph for the Union City, Oklahoma, splitting storm. The right mover produced the Union City tornado.

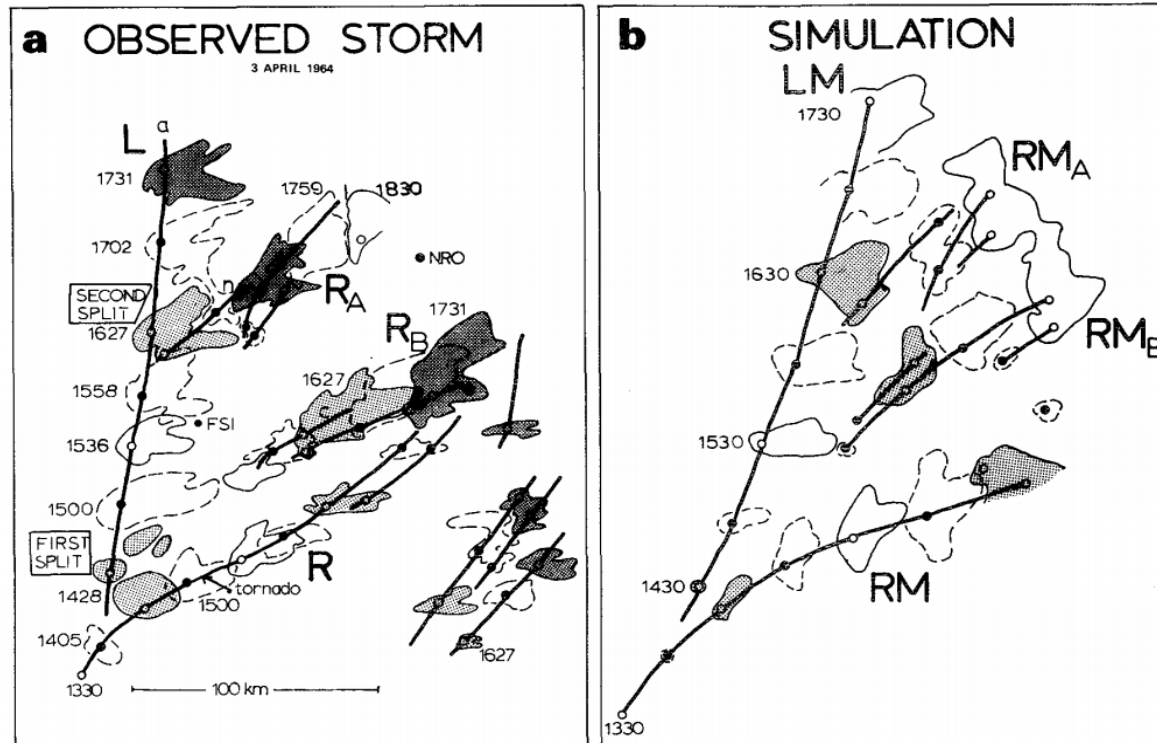


FIG. 1. The (a) observed and (b) modeled storm development on 3 April 1964. Observed reflectivities > 12 dBZ at 0° and modeled rainwater contents $> 0.5 \text{ g kg}^{-1}$ at $z = 0.4 \text{ km}$ are enclosed by alternating solid and dashed contours about every 30 min. Maxima in these fields are connected by solid lines. The storms are labeled and at several times the contoured regions are stippled for better visualization of the storm development. Labels for the modeled storms are the same as the corresponding observed storms except for the inclusion of M. The scale shown in (a) applies in (b).

The observed and modeled storm development on 2 April 1964, which has similar environment as the Union City OK storm. The storms are labeled and are several times the contoured regions are stippled for better visualization. (From Wilhelmson and Klemp. 1981.)