# Parcel Theory

METR 4403/5403 – Spring 2023

Based on materials originally prepared by Tom Galarneau

# Buoyancy

- **Buoyancy is the upward force arising from the displacement of a fluid by another fluid or object (Archimedes' Principle)**
  - **The upward force is equal to the weight of the displaced fluid**
  - **Buoyancy is the key force for convection! (Supercells are more complicated…)**
- **Vertical momentum equation for convective scales goes as:**

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p'}{\partial z} - g\frac{\rho'}{\rho} = -\frac{1}{\rho}\frac{\partial p'}{\partial z} + B \qquad B = -g\frac{\rho'}{\rho} = buoyancy$$

$$-\frac{\partial \bar{p}}{\partial z} - \bar{\rho} g = 0$$

Note hydrostatic equation used

If $\rho' > 0$, parcel is more dense than environment. $\therefore B < 0 \rightarrow \frac{dw}{dt} < 0$

If $\rho' < 0$, parcel is less dense than environment. $\therefore B > 0 \rightarrow \frac{dw}{dt} > 0$

- **B controls the parcel acceleration. So, rising parcels can continue to rise for some time after becoming negatively buoyant (like overshooting top!)**

# Buoyancy

- **Write buoyancy in terms of temperature since we measure that**

Using ideal gas law: $B = -g\dfrac{\rho'}{\bar{\rho}} = g\left(\dfrac{T_v{}'}{\overline{T_v}} - \dfrac{p'}{\bar{p}}\right)$

For small mach number: $\left|\dfrac{p'}{\bar{p}}\right| \ll \left|\dfrac{T_v{}'}{\overline{T_v}}\right|$ $\qquad \boxed{\therefore B = g\dfrac{T_v{}'}{\overline{T_v}} = g\dfrac{\theta_v{}'}{\overline{\theta_v}}}$ $\qquad B = g\dfrac{\theta'}{\bar{\theta}}$ Neglecting water vapor

- **Reference state temperature is environment (temperature line on sounding)**

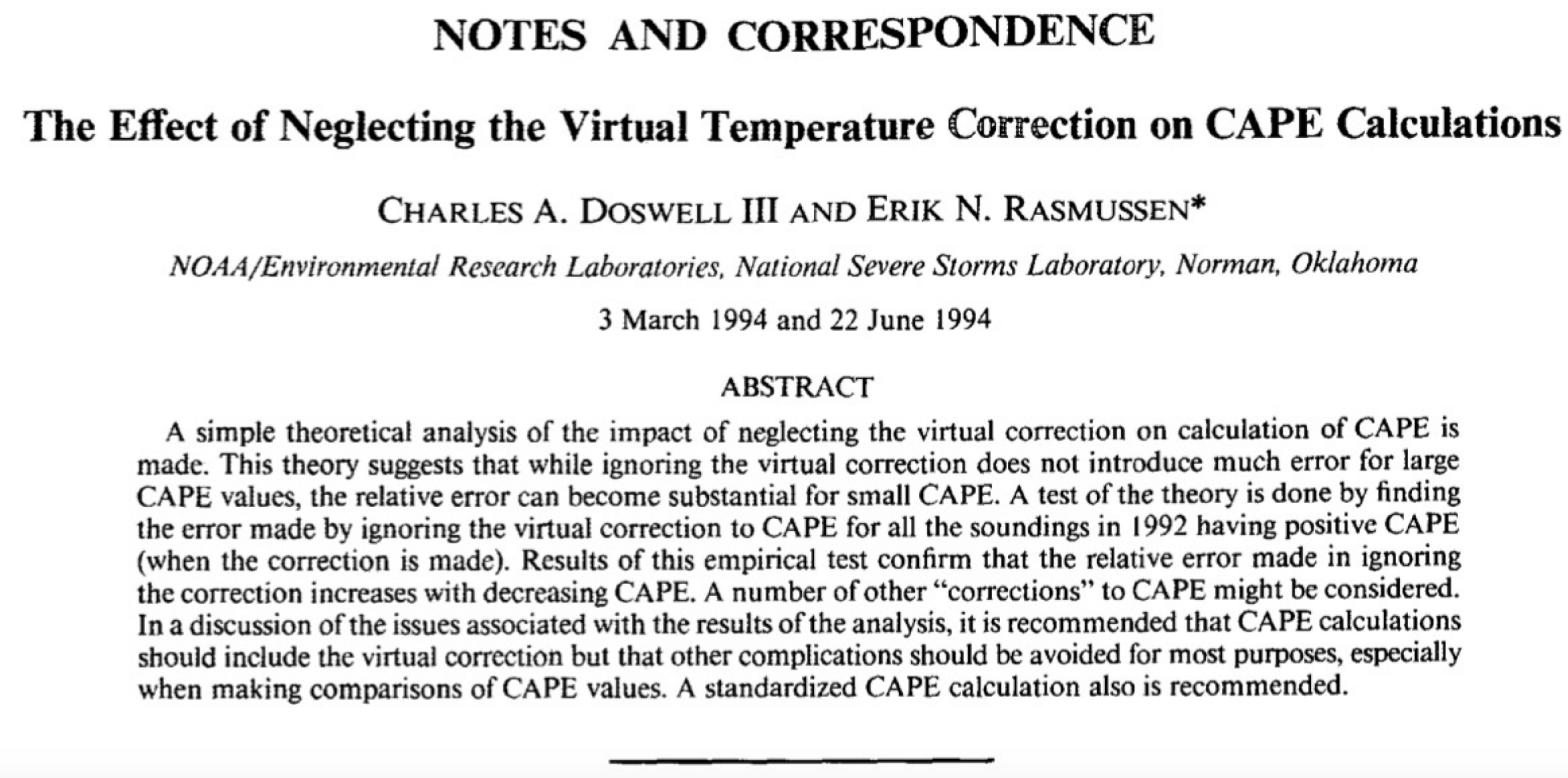
$$\therefore B = g\left(\frac{T_{vp} - T_{venv}}{T_{venv}}\right)$$

$T_{vp}$ is virtual temperature of air parcel
$T_{venv}$ is virtual temperature of ambient environment

If $T_{vp} < T_{venv}$, parcel is colder than environment. $\therefore B < 0 \rightarrow \dfrac{dw}{dt} < 0$

If $T_{vp} > T_{venv}$, parcel is warmer than environment. $\therefore B > 0 \rightarrow \dfrac{dw}{dt} > 0$

( $T_v = T(1 + \varepsilon q_v)$, $\varepsilon = 0.608$ $\qquad p = \rho R T_v$ $\qquad \ln p = \ln\rho + \ln R + \ln T_v$ $\qquad \dfrac{\delta p}{p} = \dfrac{\delta\rho}{\rho} + \dfrac{\delta T_v}{T_v}$ )



## NOTES AND CORRESPONDENCE

# The Effect of Neglecting the Virtual Temperature Correction on CAPE Calculations

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3 March 1994 and 22 June 1994

## ABSTRACT

A simple theoretical analysis of the impact of neglecting the virtual correction on calculation of CAPE is made. This theory suggests that while ignoring the virtual correction does not introduce much error for large CAPE values, the relative error can become substantial for small CAPE. A test of the theory is done by finding the error made by ignoring the virtual correction to CAPE for all the soundings in 1992 having positive CAPE (when the correction is made). Results of this empirical test confirm that the relative error made in ignoring the correction increases with decreasing CAPE. A number of other "corrections" to CAPE might be considered. In a discussion of the issues associated with the results of the analysis, it is recommended that CAPE calculations should include the virtual correction but that other complications should be avoided for most purposes, especially when making comparisons of CAPE values. A standardized CAPE calculation also is recommended.

https://doi.org/10.1175/1520-0434(1994)009<0625:TEONTV>2.0.CO;2

Inclusion of water vapor impacts CAPE/CIN calculation

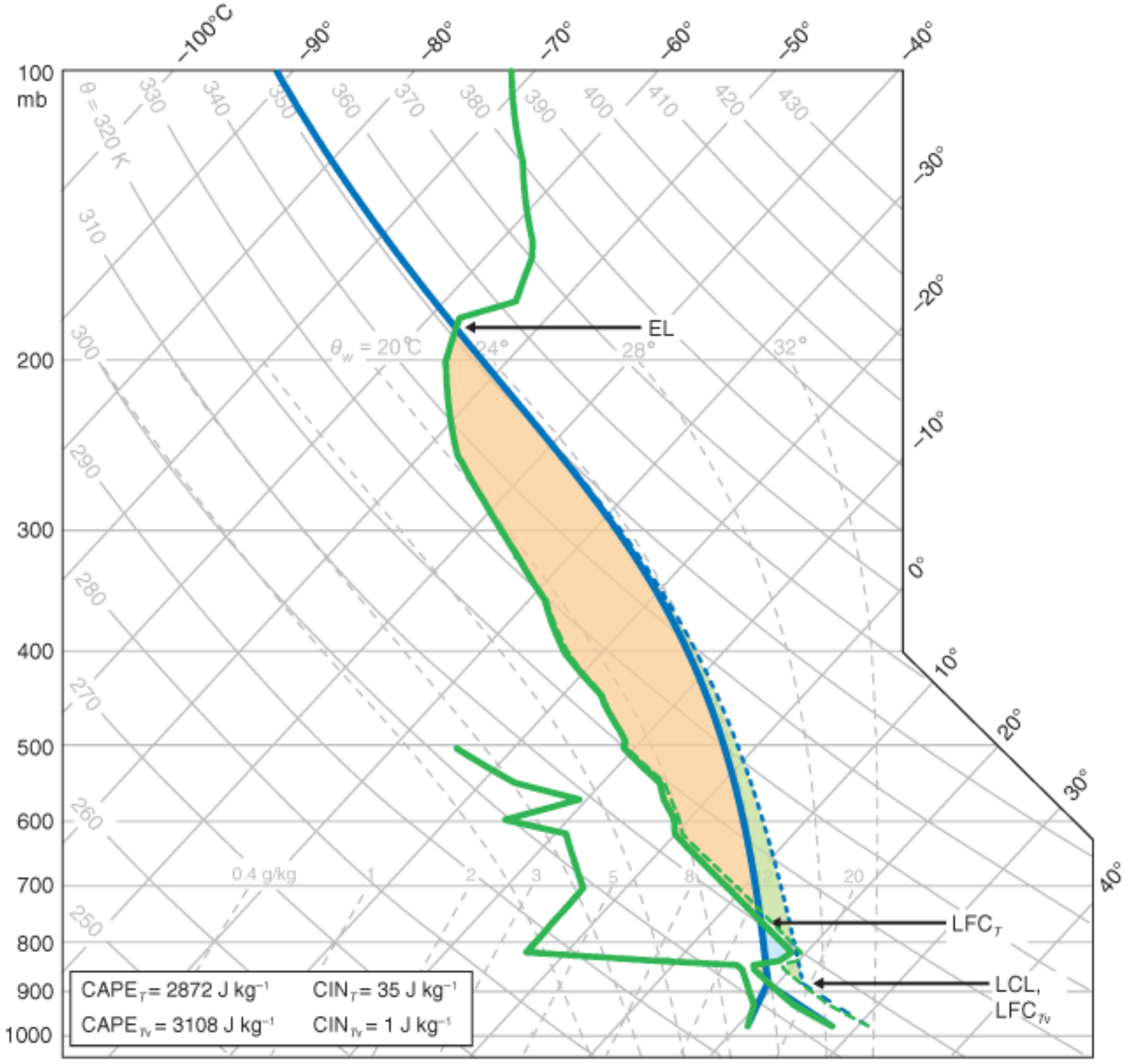


Figure 2.9

Fig. 2.9 MR 2010

# Parcel theory

- **Vertical accelerations are due to parcel buoyancy alone**

- **Vertical momentum equation for convective scales goes as:**

$$\frac{dw}{dt} = B$$

- **Parcel theory neglects vertical pressure gradient force, viscosity, entrainment, hydrometeor loading, environment modifications**

# Parcel theory

- **We need to be able to determine whether a lifted parcel has buoyancy**

- **Convective available potential energy (CAPE) tells us the kinetic energy a parcel may gain due to buoyant acceleration**

$$CAPE = \int_{LFC}^{EL} B\,dz$$

Vertical integration of buoyancy from LFC to EL.

Caveats: CAPE>0 does not guarantee convection.
Not all parcels have an LFC.

- **Convective inhibition (CIN) is the work done by a parcel against stable stratification to reach its LFC**

$$CIN = -\int_{0}^{LFC} B\,dz$$

Vertical integration of buoyancy from ground to EL.

Need to overcome CIN to trigger convection.

# Theoretical Maximum Updraft Speed

- **Parcel theory can be used to estimate $w_{max}$ from buoyancy alone**

- **Manipulate vertical momentum equation ($\frac{dw}{dt} = B$) for parcel theory**

$$w_{max} = \sqrt{2 * CAPE}$$

1000 J $kg^{-1}$ CAPE → 45 m $s^{-1}$ updraft (??)

$$dz\,\frac{dw}{dt} = B\,dz \rightarrow wdw = B\,dz \rightarrow$$

$$d\left(\frac{w^2}{2}\right) = B\,\mathrm{dz}$$

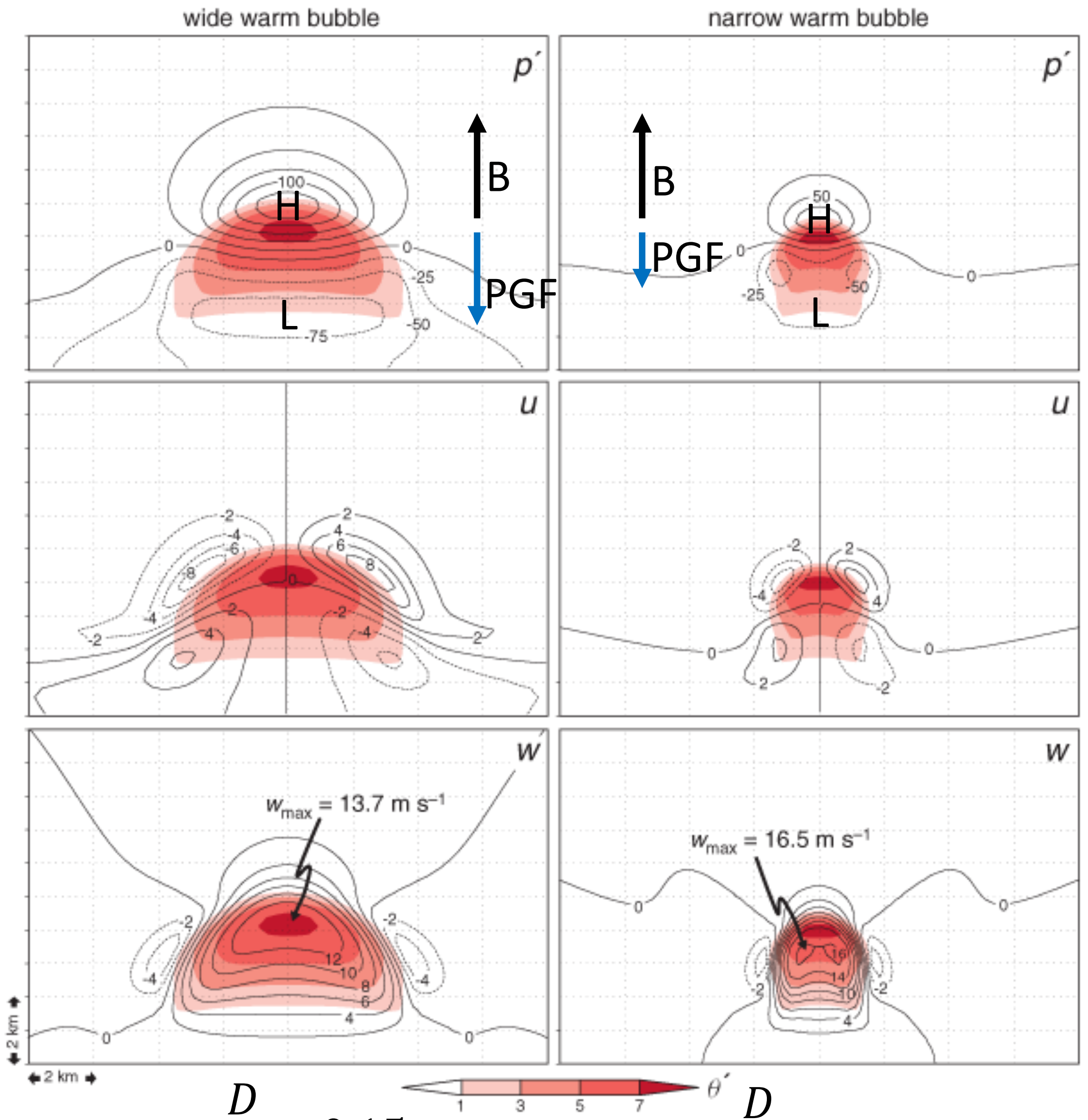
- **Theoretical updraft speeds based on CAPE seem large – what factors counteract buoyant accelerations for air parcels?**

## 1. Vertical PGF

- Wider thermal has larger PGF compared to narrow thermal (more air needs to be moved out of the way)
- As thermal becomes wider the scenario approaches hydrostatic where PGF offsets buoyancy (dw/dt=0)
- **Narrow updrafts more favorable for thunderstorm development**

## 2. Entrainment

- Mixing of environment air into rising thermal
- If env is cooler/drier, evaporation cools thermal, reduces B
- **Updraft dilution more detrimental for narrow or tilted updrafts (wider is better)**



$\frac{D}{L} \approx 0.15$

$\frac{D}{L} \approx 0.5$

Figure 3.1

Fig. 3.1 MR 2010

# Summary

- **Parcel theory overestimates updraft speed**
  - Vertical PGF limits updraft speed; significant for wide updrafts
  - Entrainment limits updraft speed; significant for narrow/tilted updrafts
  - Hydrometeor loading also limits updraft speed

- **Parcel theory does not account for layer lifting and development of moist absolutely unstable layers (MAULs)**

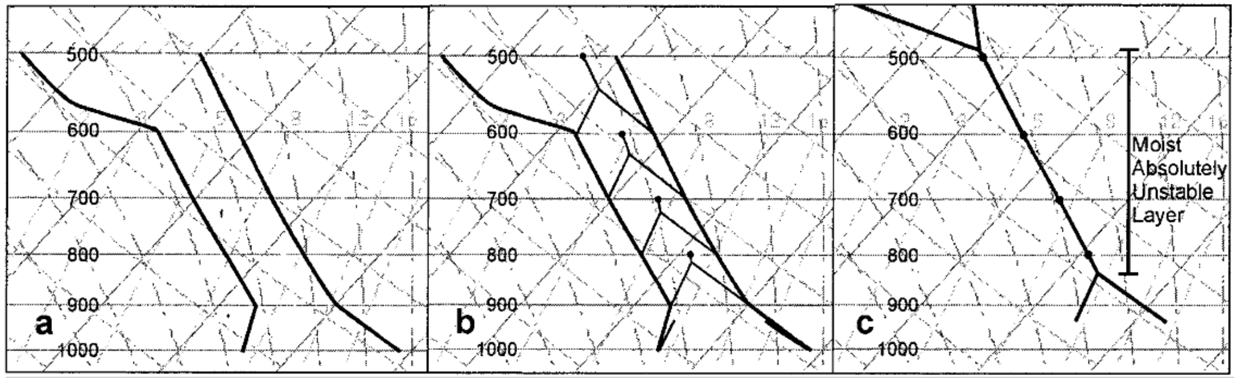


FIG. 1. Idealized sounding: (a) initial conditionally unstable sounding, (b) layer lifting is applied (thin black lines represent the path of selected parcels within the lifted layer), and (c) resulting structure after lifting, which features a deep moist absolutely unstable layer.

Bryan and Weisman 2000

## Layer Lifting

- Layer lifting can lead to convectively unstable conditions
- Unsaturated layer lifted, bottom reaches saturation before top of layer
- Bottom of layer cools at $\Gamma_m$
- Top of layer cools at $\Gamma_d$
- **Layer destabilizes!**

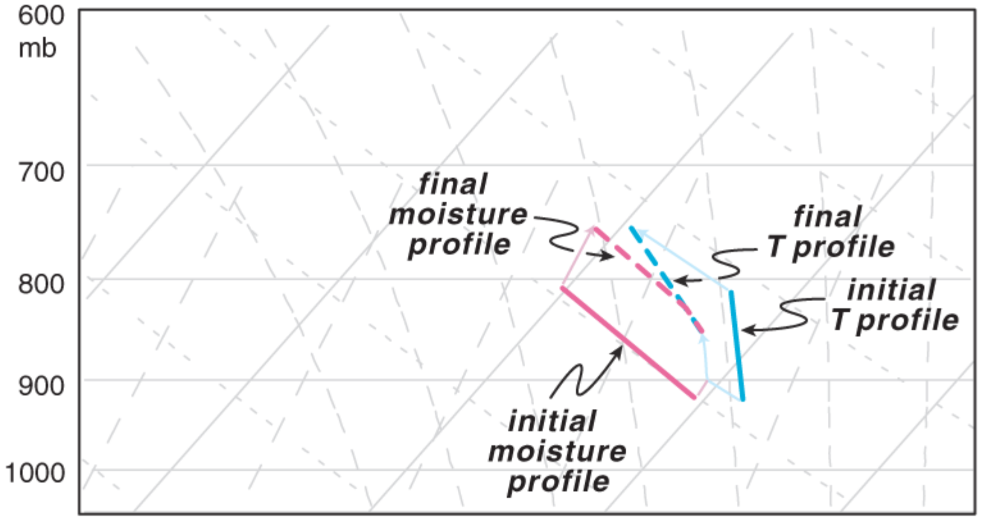


Figure 3.3

Illustration of the concept of potential instability. A potentially unstable layer initially spanning the pressure range of 910–810 mb has been lifted to 850–750 mb. Although destabilization of the layer has occurred, lifting would have cooled the layer (and therefore reduced CIN) regardless of whether or not lifting led to saturation at the bottom of the layer.

Fig. 3.3 MR 2010

# Skew-T Diagram Basics

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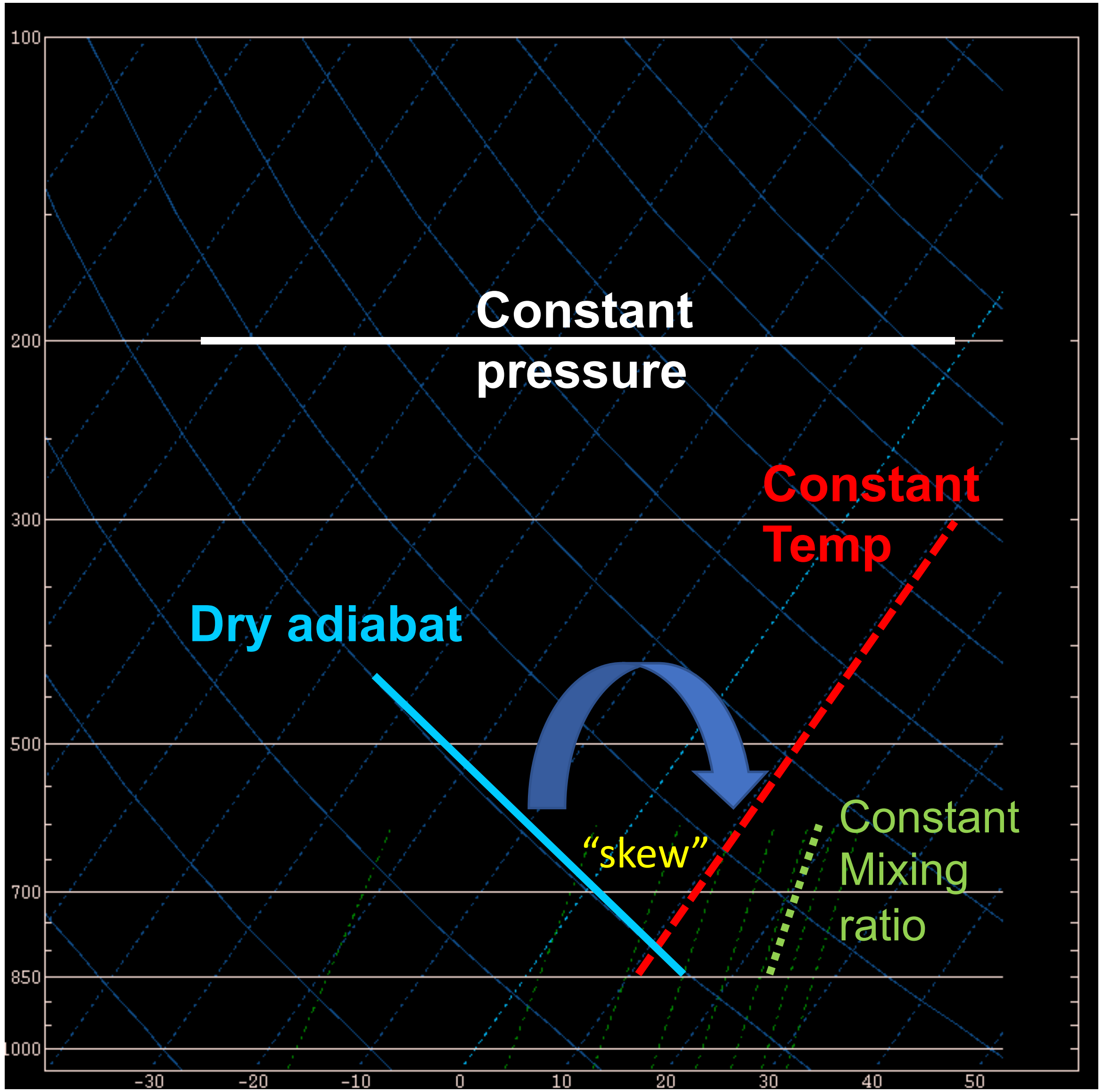
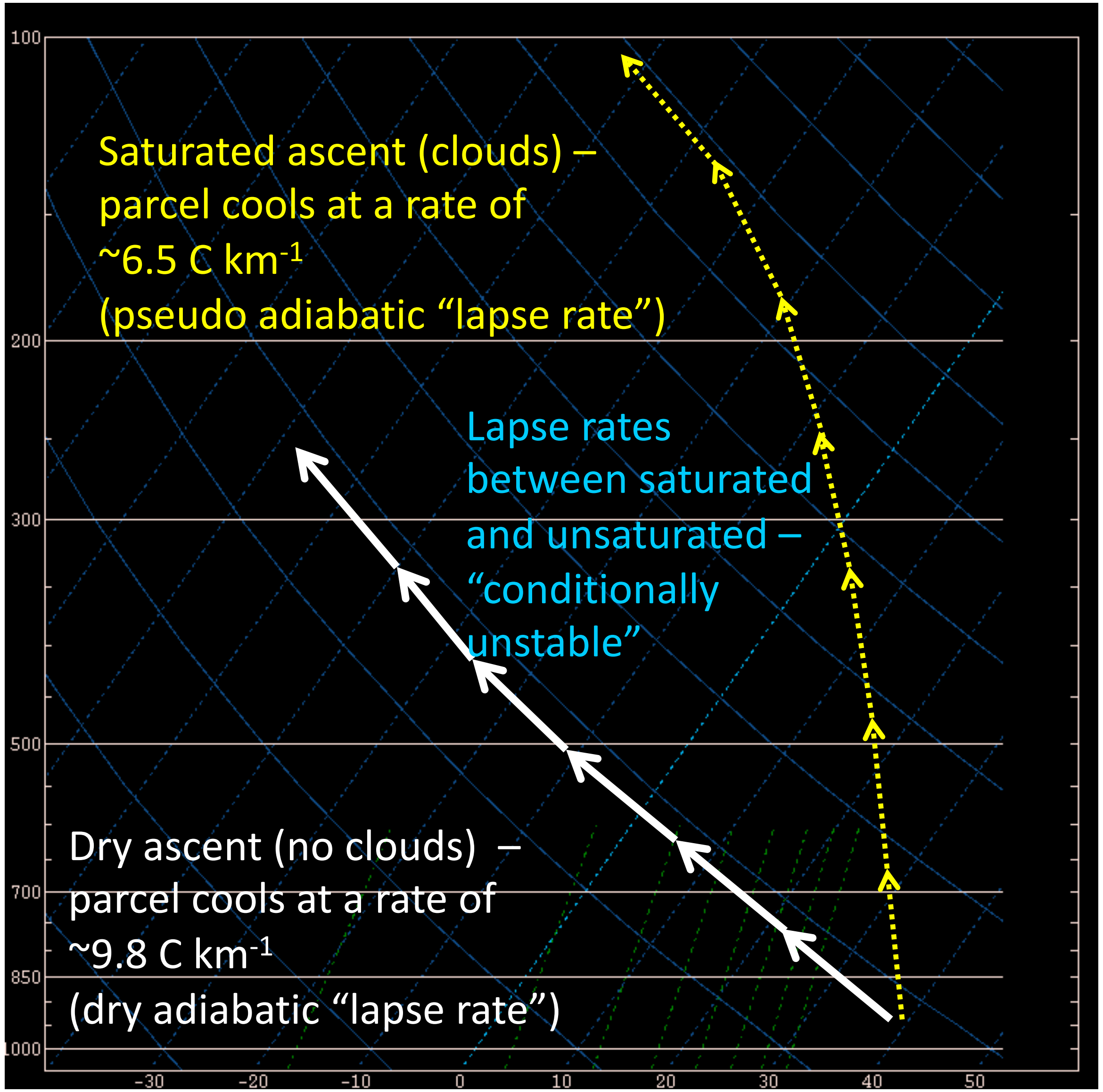
Material originally prepared by Rich Thompson

# Raw SkewT-log P diagram



# Features of note in SkewT log P

- Temperature is skewed about 90$^{o}$ from the dry adiabats
- Pressure decreases as a logarithm of height (faster at bottom than top)
- Mixing ratio crosses over temperature lines
- One thing missing is a plot of saturated parcel ascent



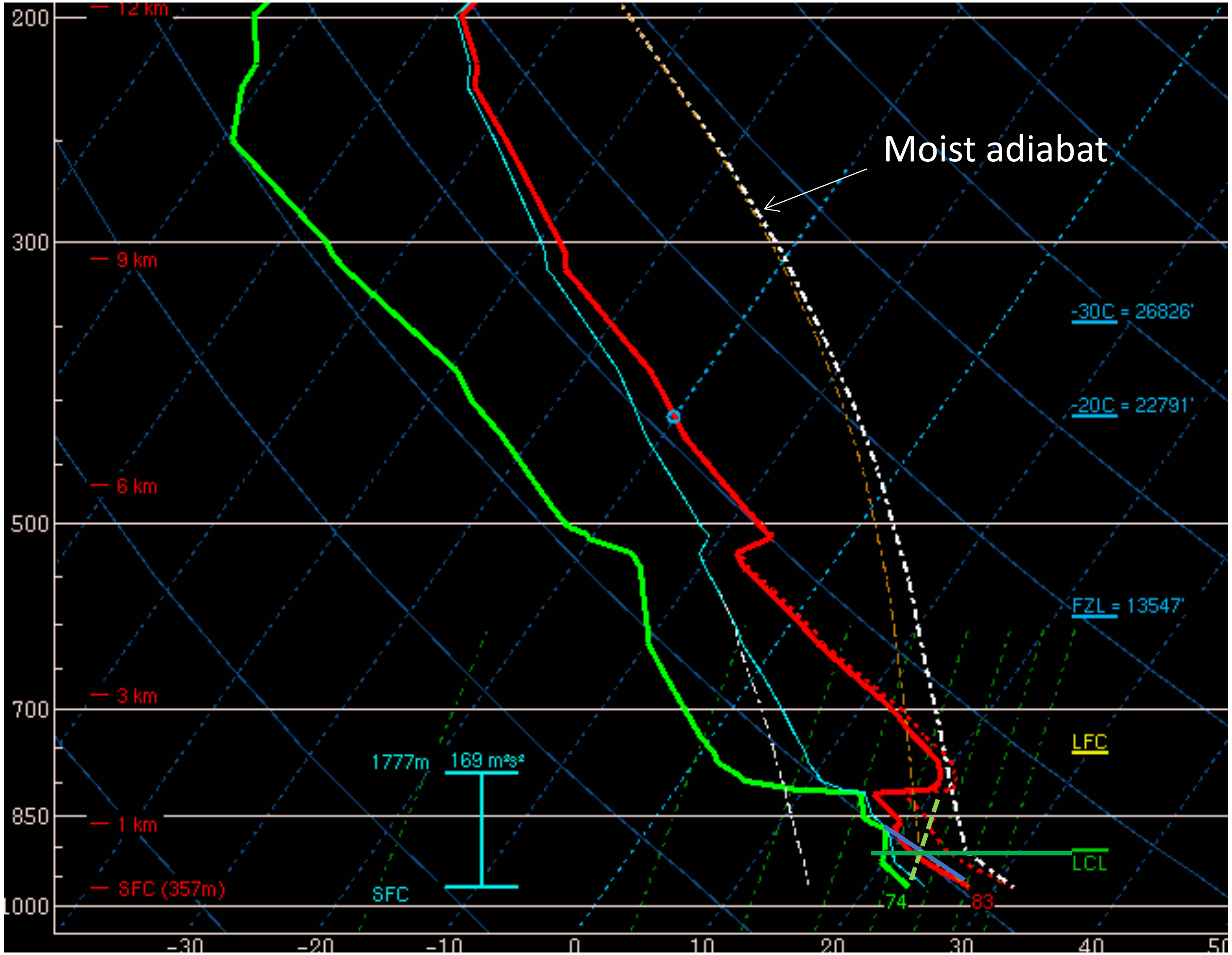
Saturated ascent (clouds) – parcel cools at a rate of ~6.5 C km$^{-1}$ (pseudo adiabatic “lapse rate”)

Lapse rates between saturated and unsaturated – “conditionally unstable”

Dry ascent (no clouds) – parcel cools at a rate of ~9.8 C km$^{-1}$ (dry adiabatic “lapse rate”)

# Lifted Parcel (chunk of air)

- Begin at lifted parcel level (ground)
- Rise “dry adiabatically” until saturation
  - Where dry adiabat crosses mixing ratio
  - We call this the “lifting condensation level” or **LCL**
  - First guess at cloud base



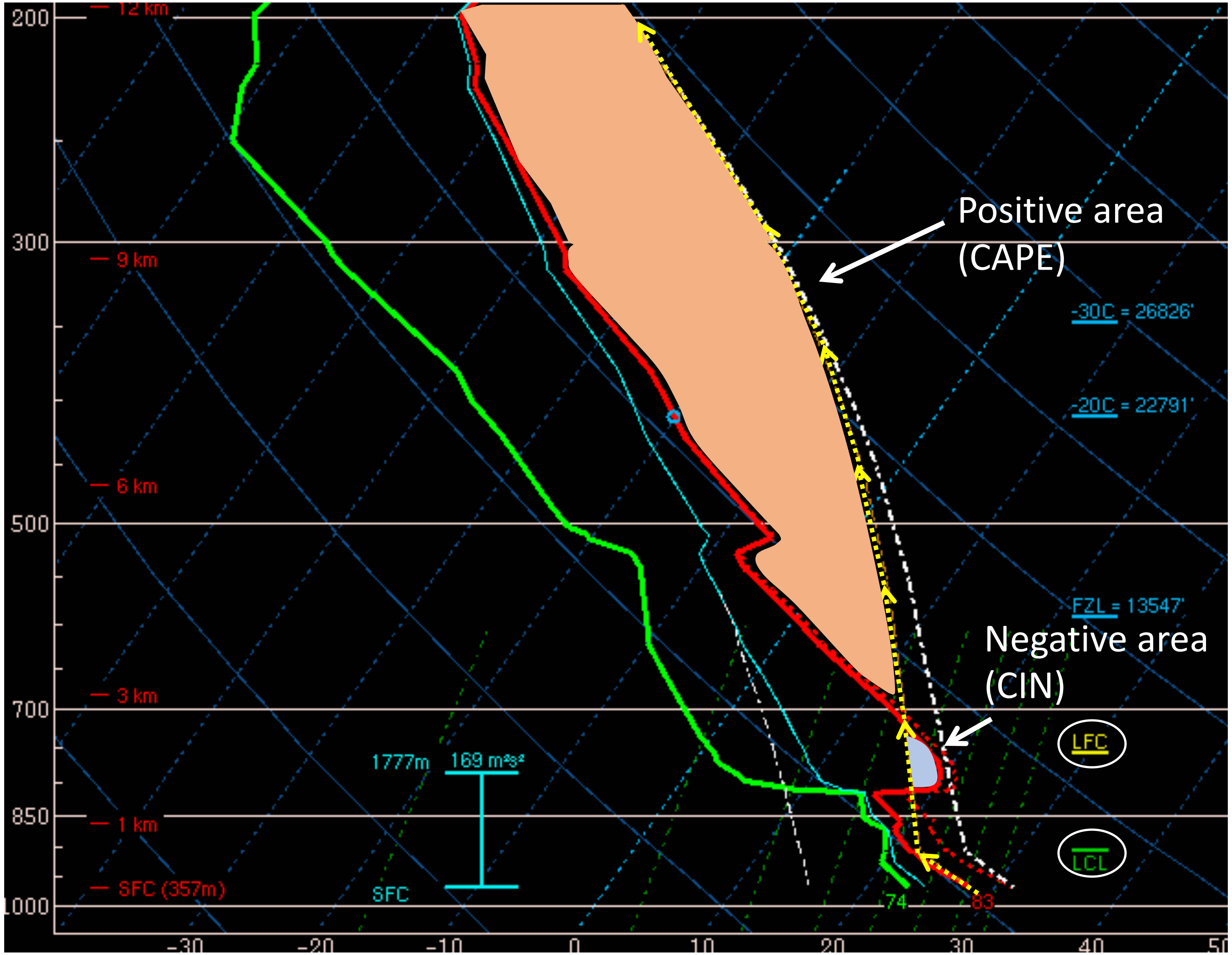
Moist adiabat

# Illustration of lifted parcel

- From LCL, rise “moist adiabatically” to “level of free convection” or LFC
  - “Free convection” begins where lifted parcel becomes warmer than environment
  - Energy resisting lift below LFC is known as “convective inhibition” or CIN
- From LFC, continue up to the “equilibrium level” or EL. Accumulated area (energy) from LFC to EL is known as CAPE
  - “Overshoot” above EL

# Common Sounding Terms

- Lapse rate – change in temperature with height
  - Dry adiabat ≈ 9.8 C km$^{-1}$
  - Moist adiabat ≈ 6.5 C km$^{-1}$
- Conditional instability – lapse rate between dry and moist adiabatic
- LCL – lifting condensation level
- LFC – level of free convection
- EL – equilibrium level
- CAPE – buoyancy (positive area)
- CIN – convective inhibition (negative area)

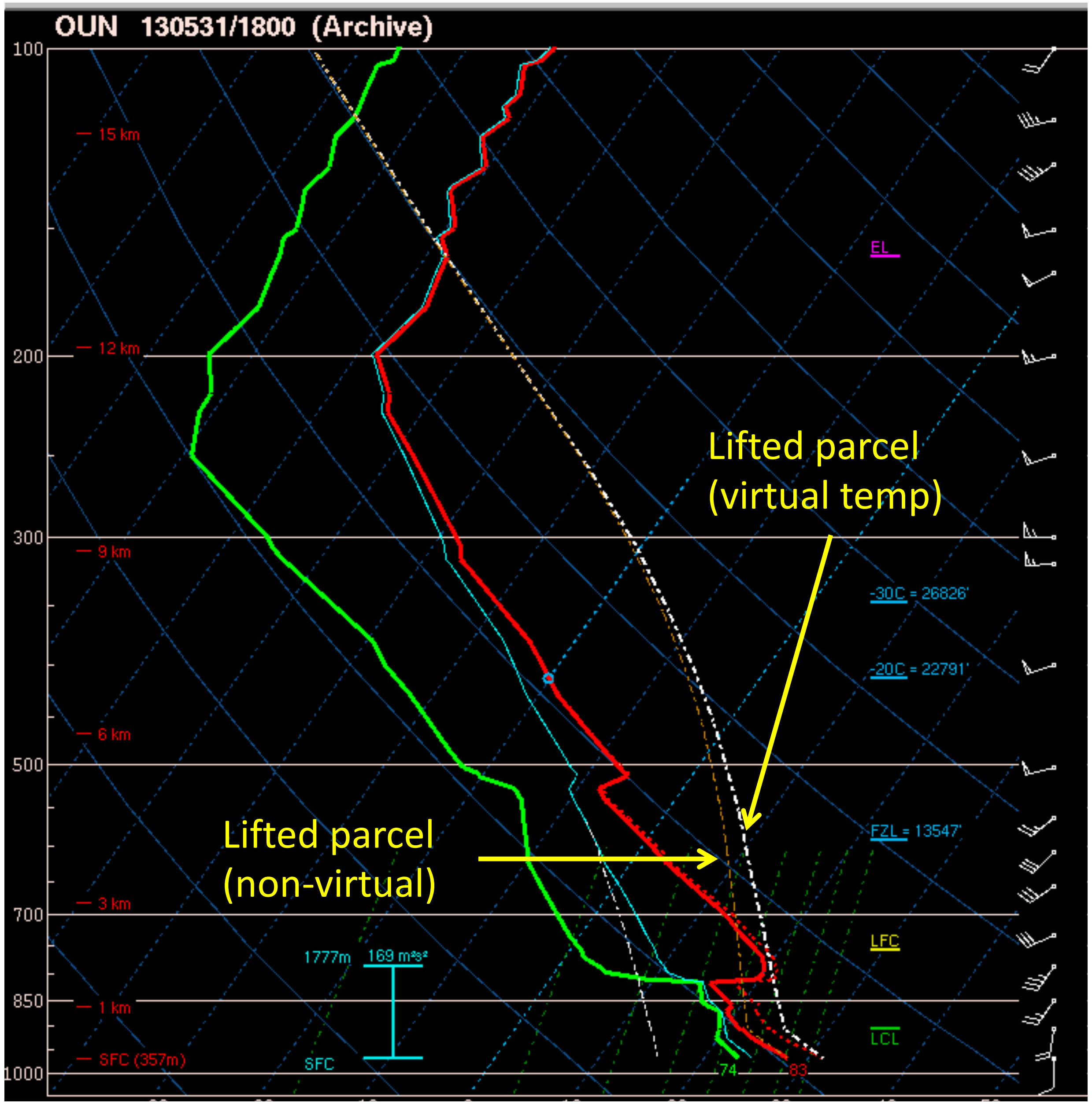


Positive area (CAPE)

Negative area (CIN)

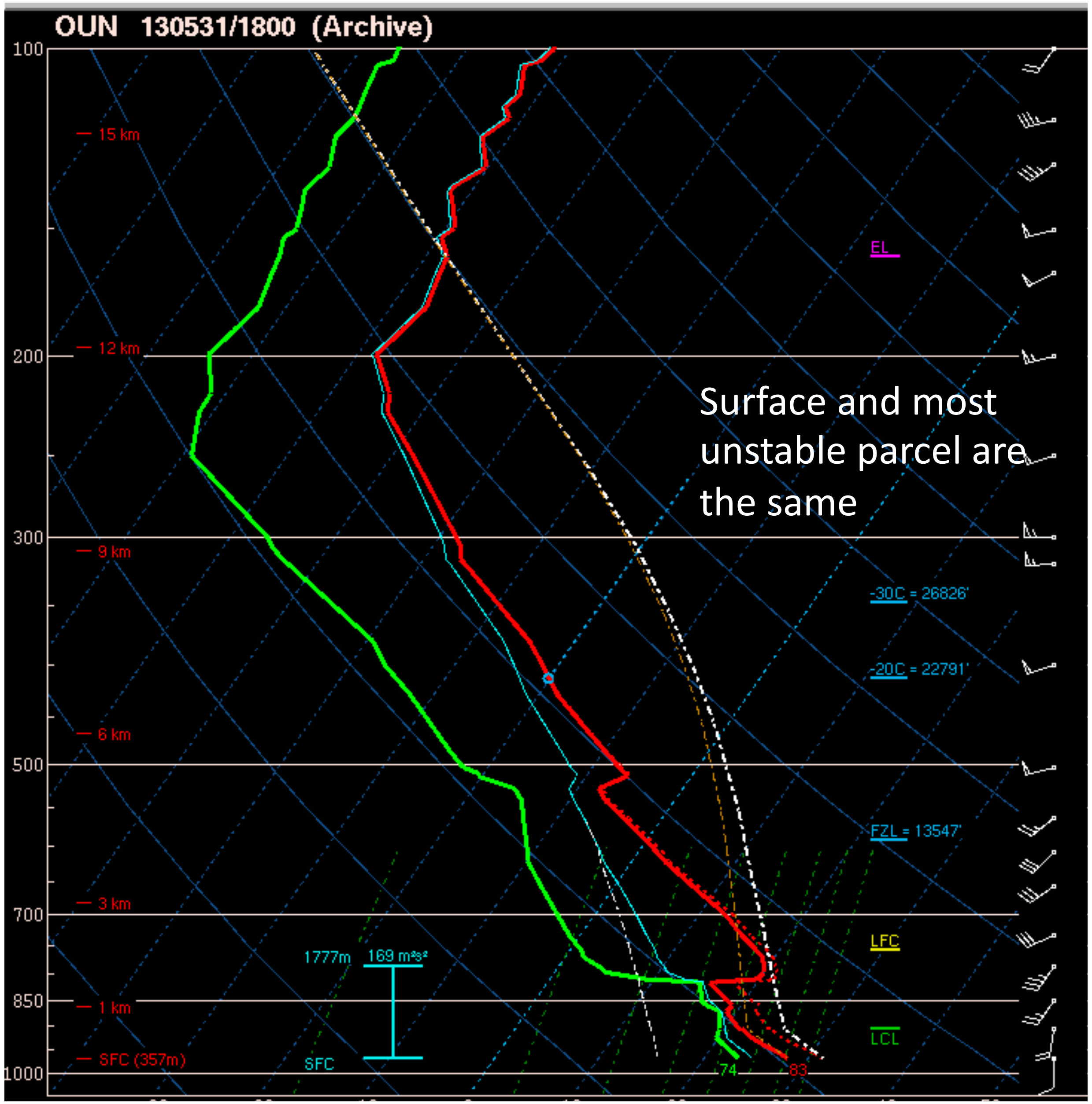
# What’s the difference between the lifted parcels?

- Virtual temperature accounts for moisture
    - Warmer than measured temperature
    - Makes most difference with tropical moisture
- Virtual temperature correction increases CAPE and reduces CIN
- Which chunk of air to lift?
    - Some sort of averaging is usually more representative
    - Surface vs. “mixed layer” or “most unstable”
    - Leading to different CAPE values – mixed layer CAPE, most unstable CAPE (MCAPE), sfc-based CAPE.

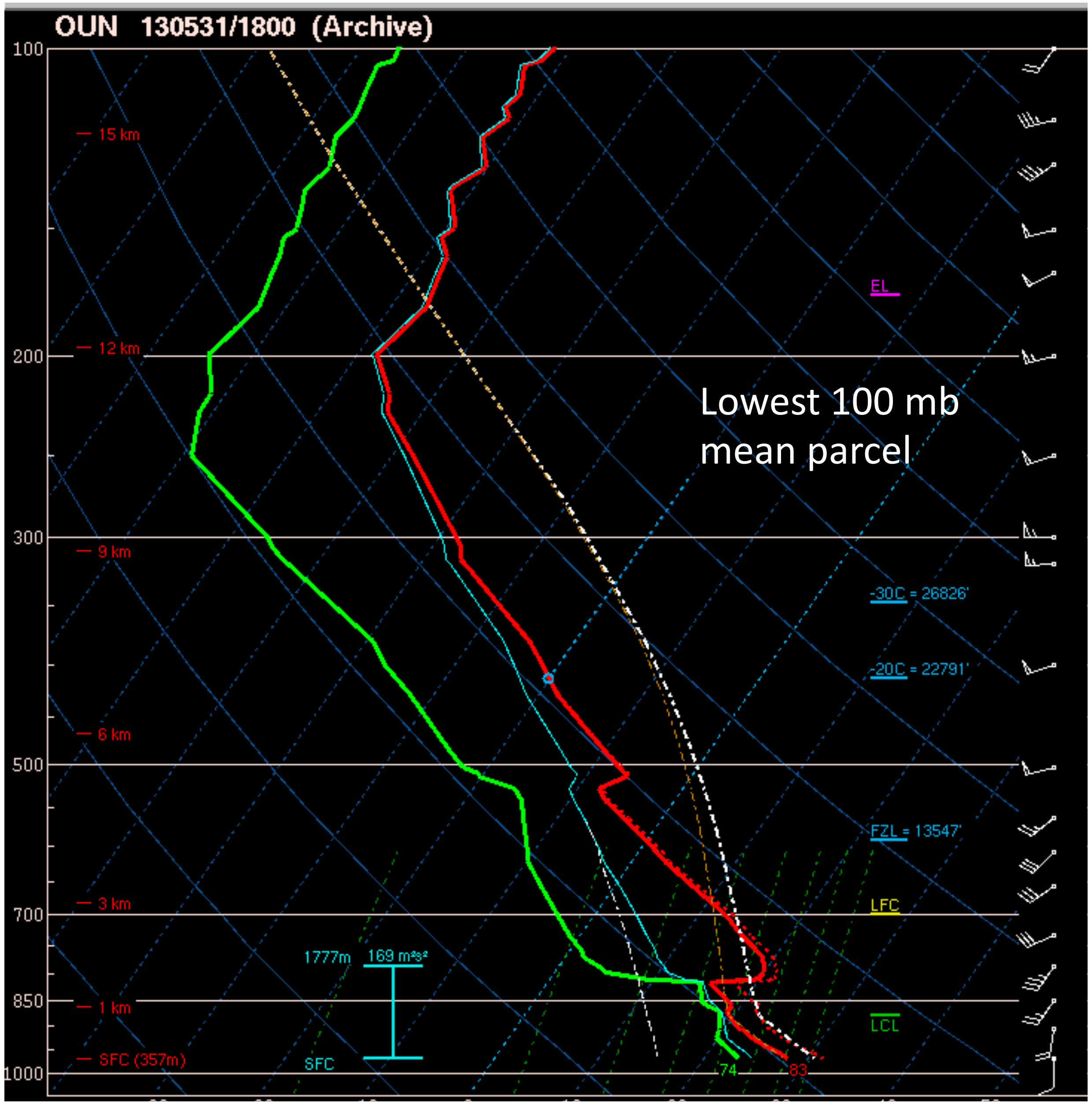


Lifted parcel
(virtual temp)

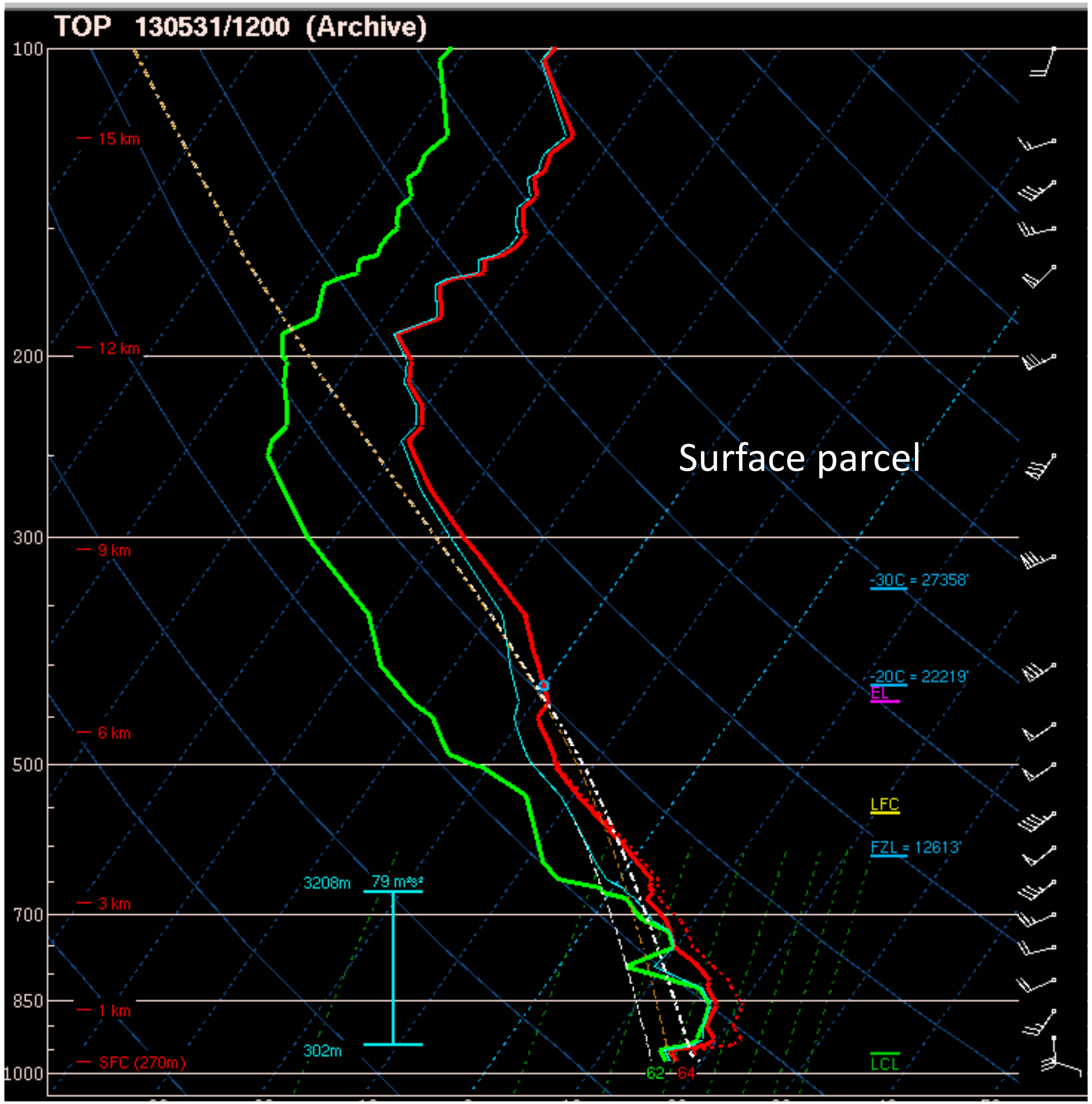
Lifted parcel
(non-virtual)



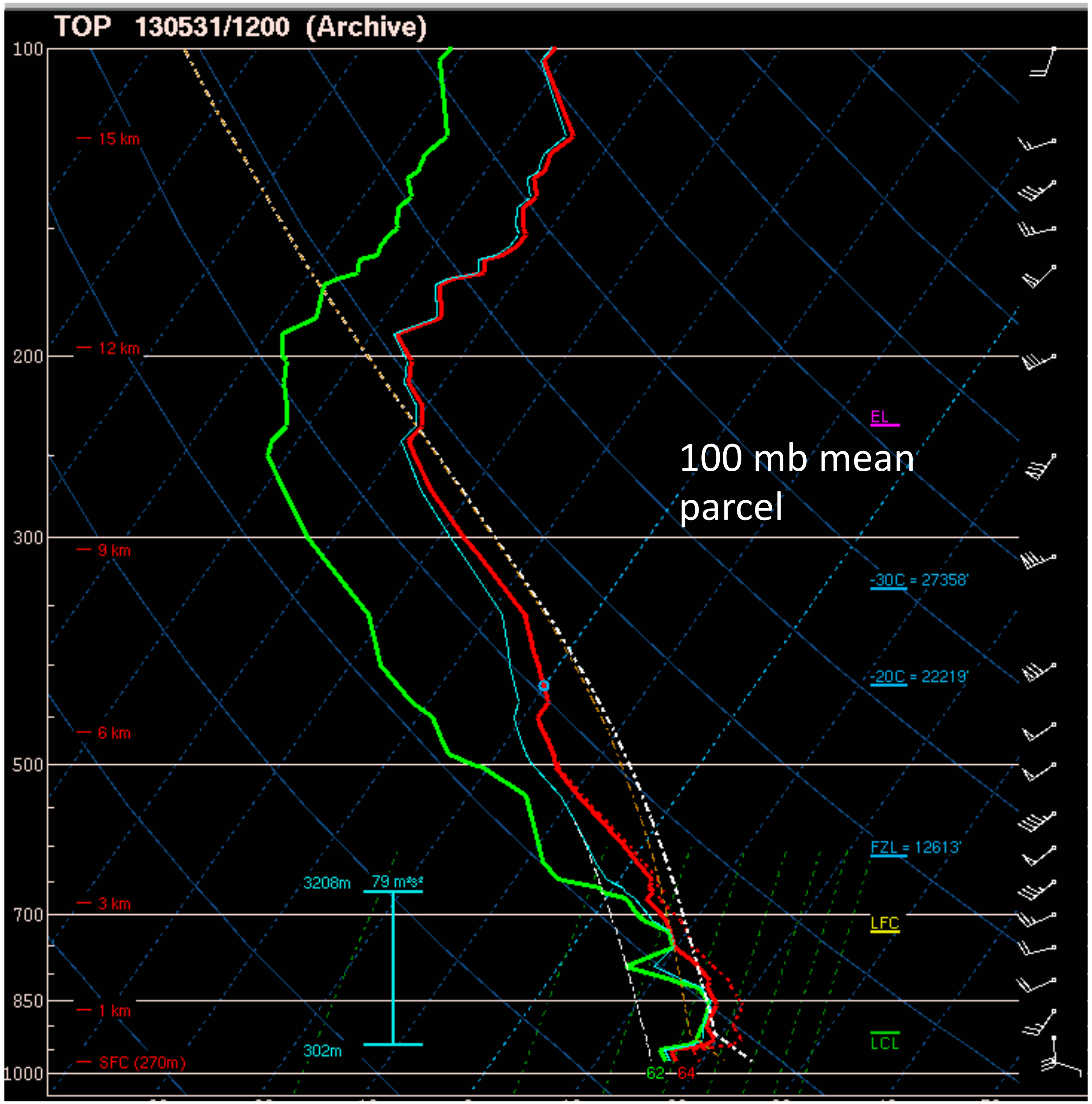
Surface and most unstable parcel are the same



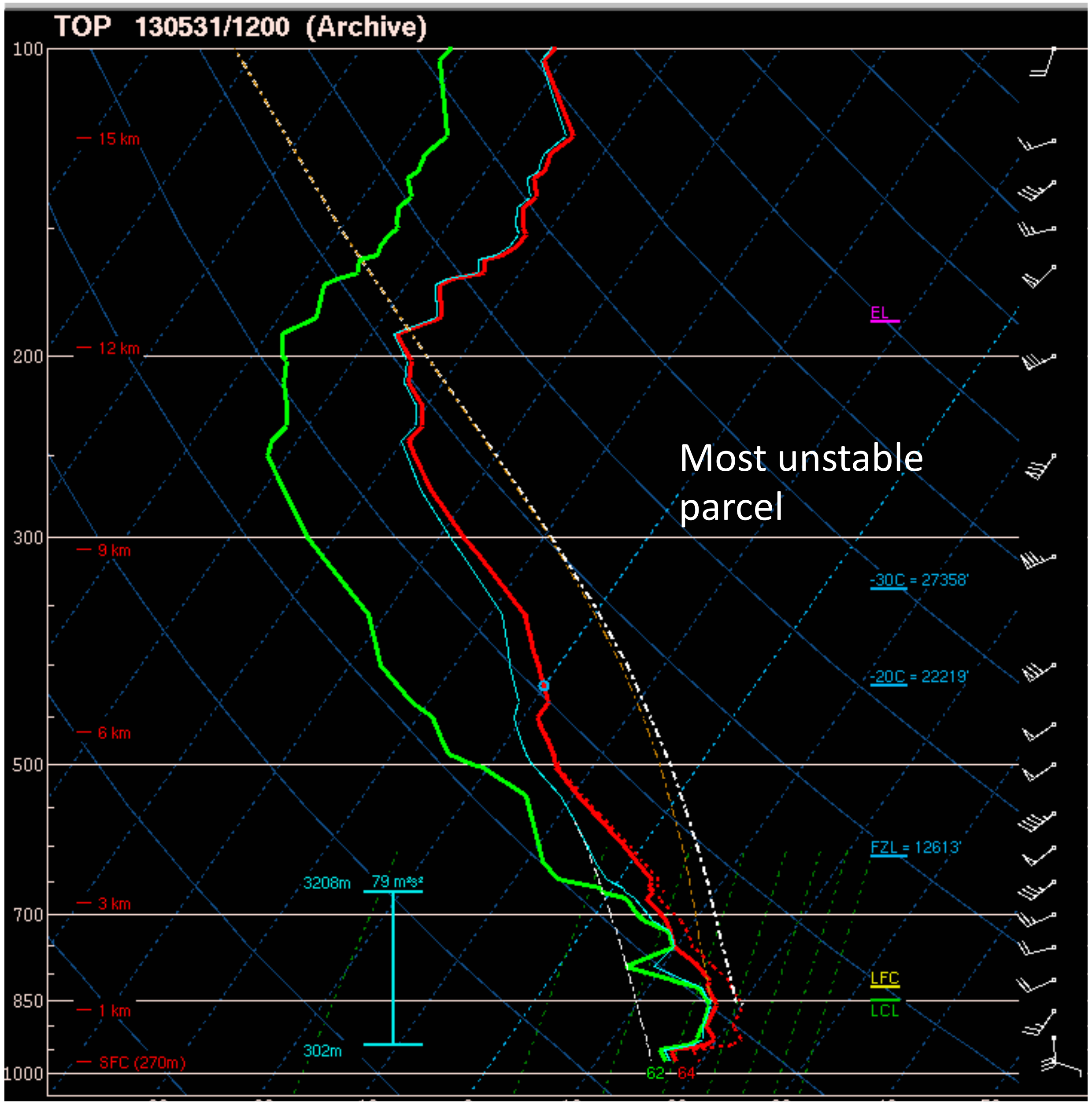
Lowest 100 mb
mean parcel



Surface parcel



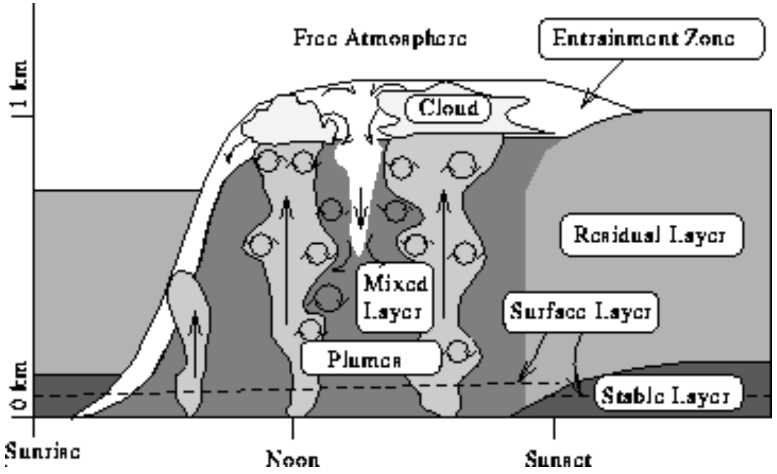
100 mb mean parcel



Most unstable parcel

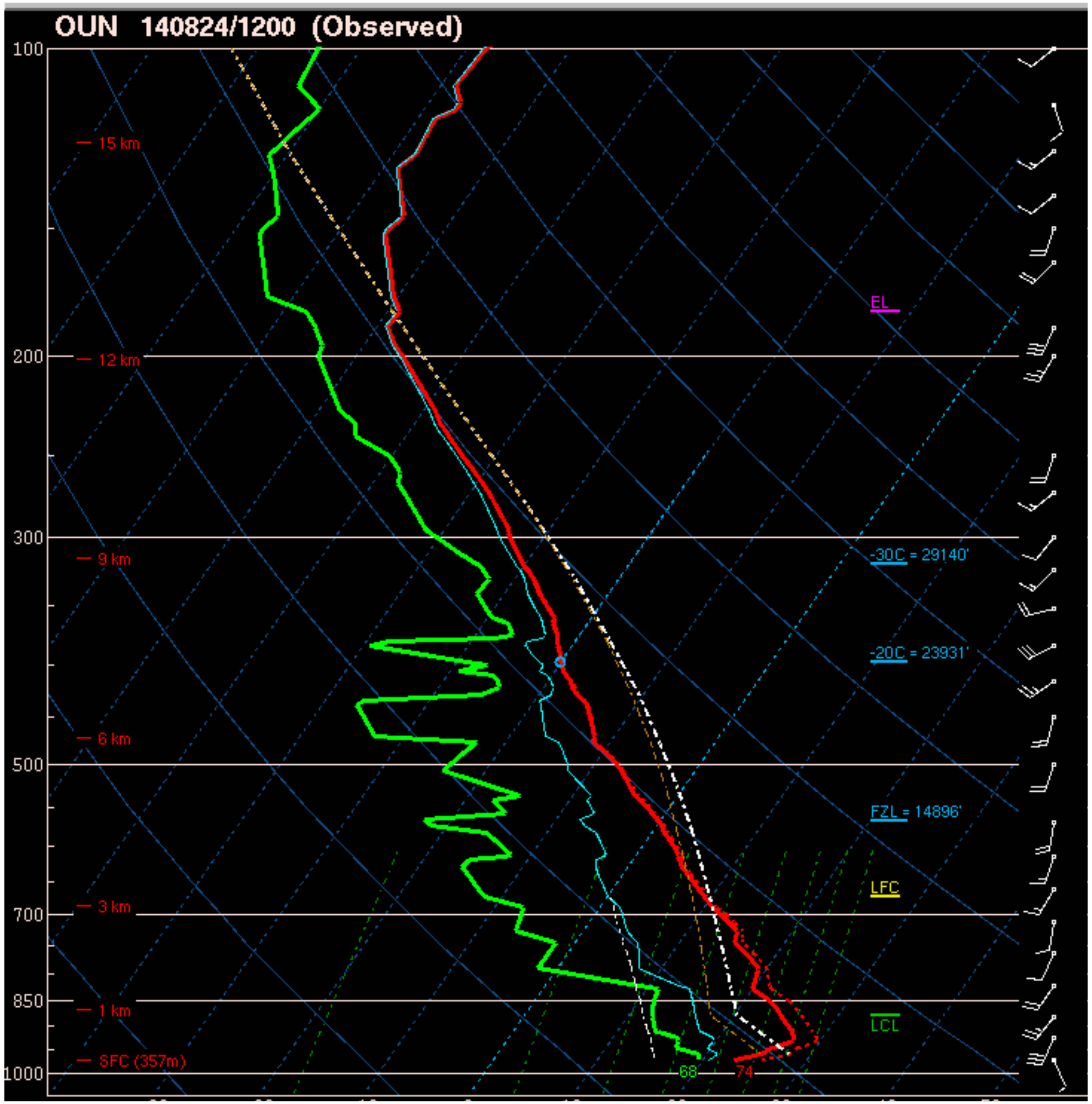
# Boundary layer vertical mixing

- During daytime, surface heating drives thermals and mixing, which take heat and moisture both upward (from surface) and downward (from aloft)
- Usually see surface dew point drop in afternoon if not offset by moisture advection (bringing in greater moisture from somewhere else)



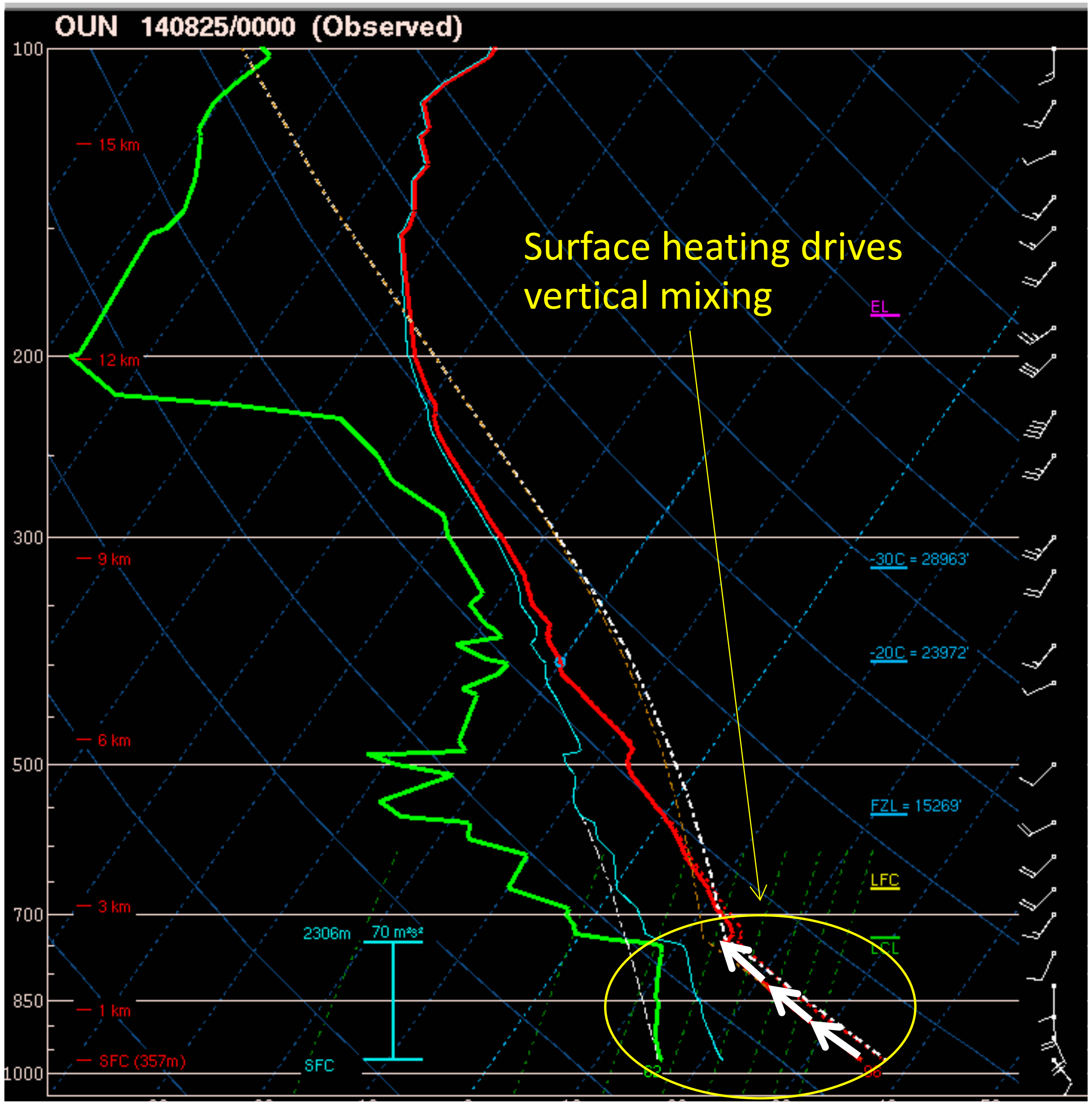
Norman OK sounding

Morning



Norman OK soounding

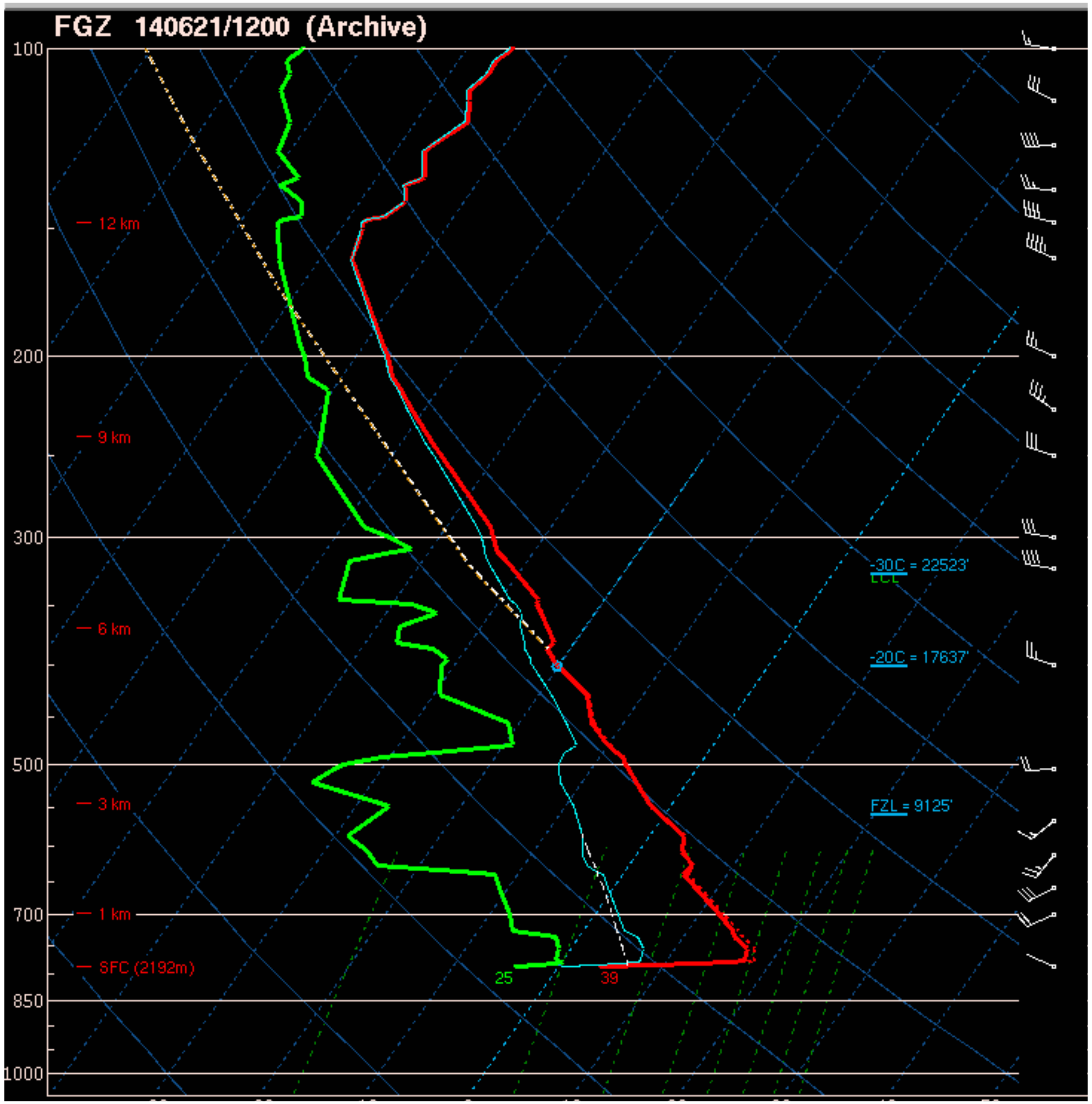
Late afternoon



Surface heating drives vertical mixing

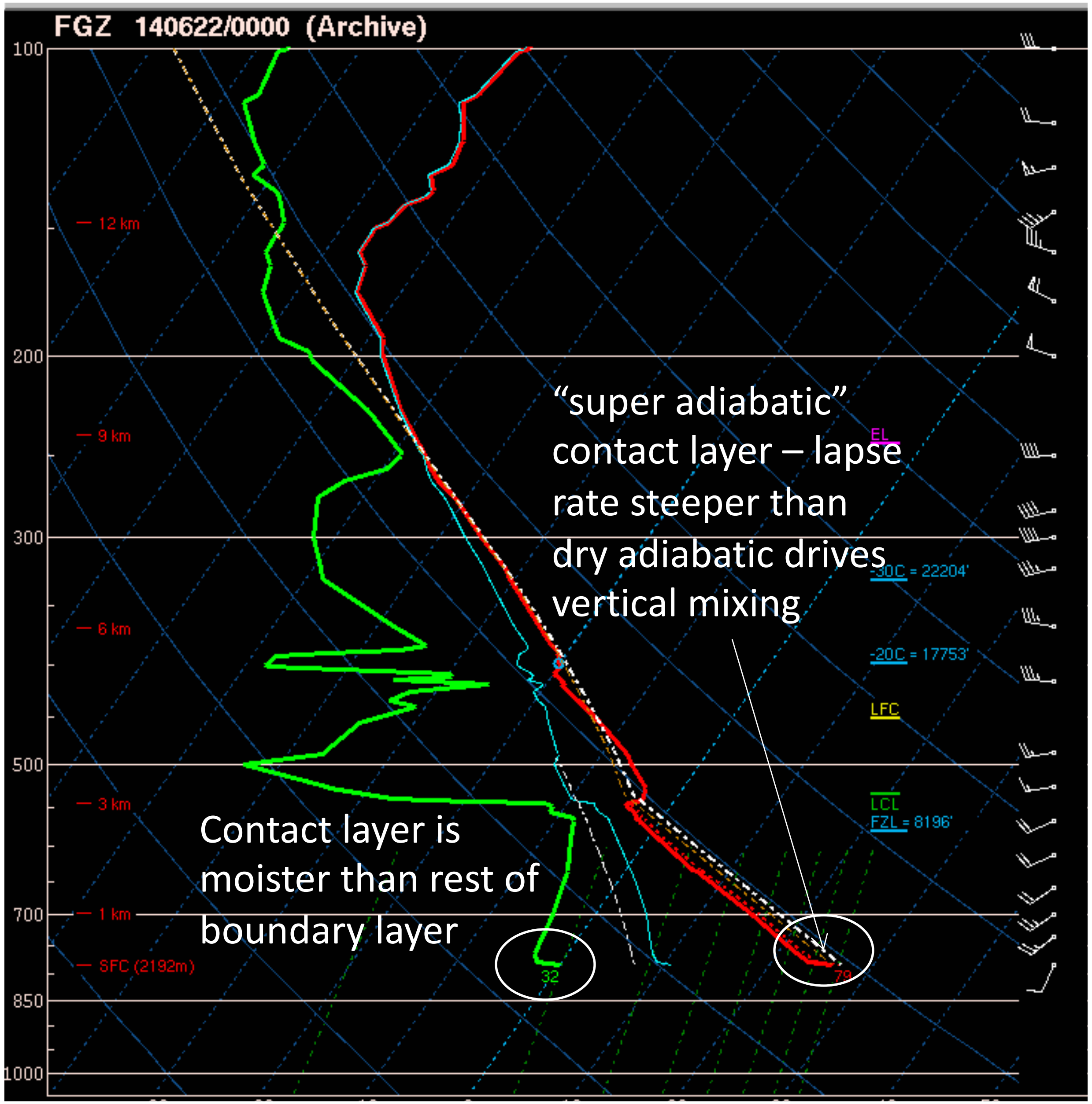
Flagstaff, AZ sounding

Morning



Flagstaff, AZ sounding

Late afternoon



“super adiabatic” contact layer – lapse rate steeper than dry adiabatic drives vertical mixing

Contact layer is moister than rest of boundary layer

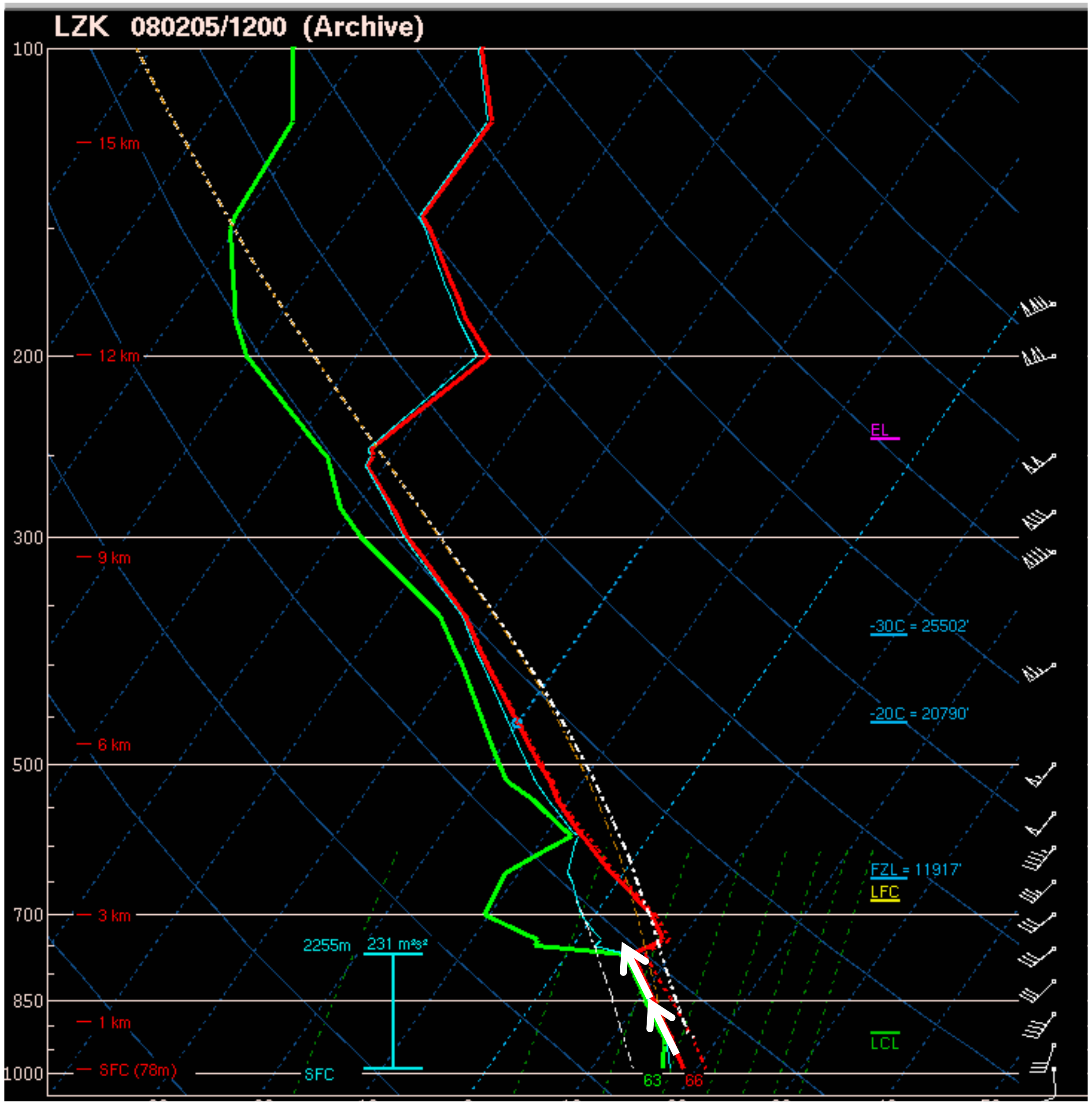
# Impact of ascent and moisture advection

- See moist layer deepen faster than you would expect with just surface heating and mixing
- “Deep” moist layer and horizontal moisture advection both combat vertical mixing driven by surface heating
    - Can see moist layer deepen while dew points increase near surface

Little Rock, AL sounding

Morning

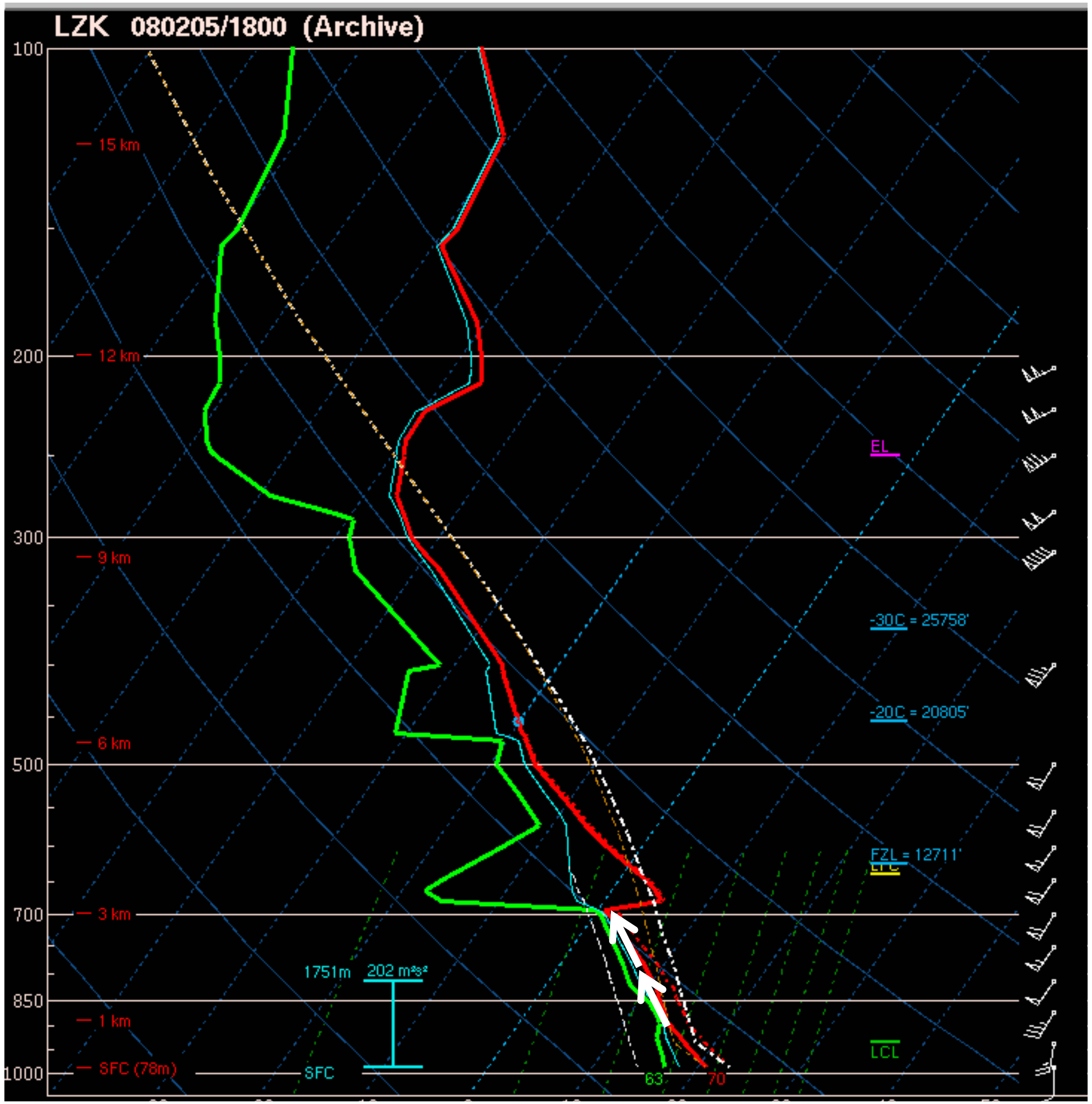
Nearly saturated and close to moist adiabat



Little Rock, AL sounding

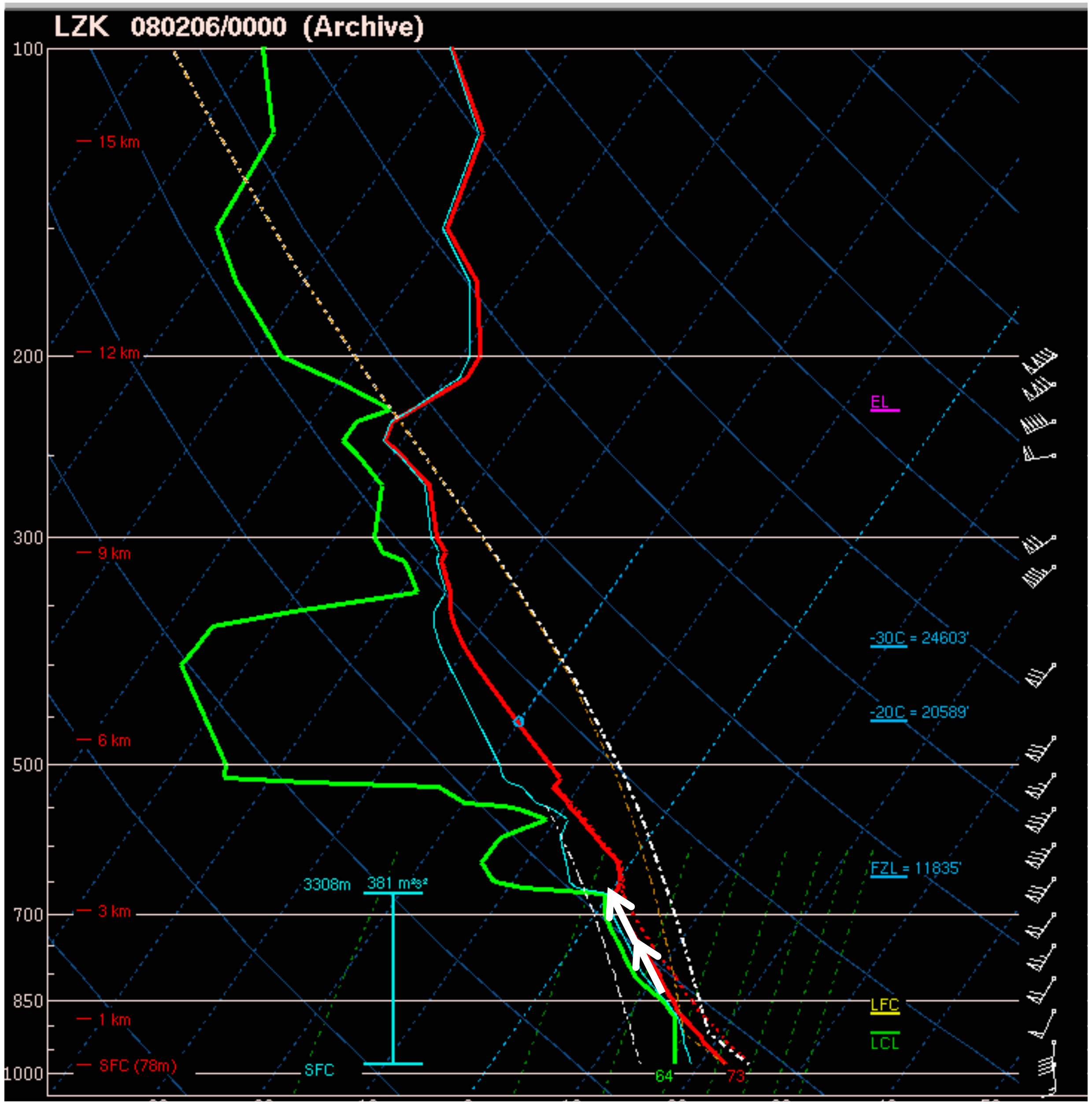
Noon

Easy to develop mixing

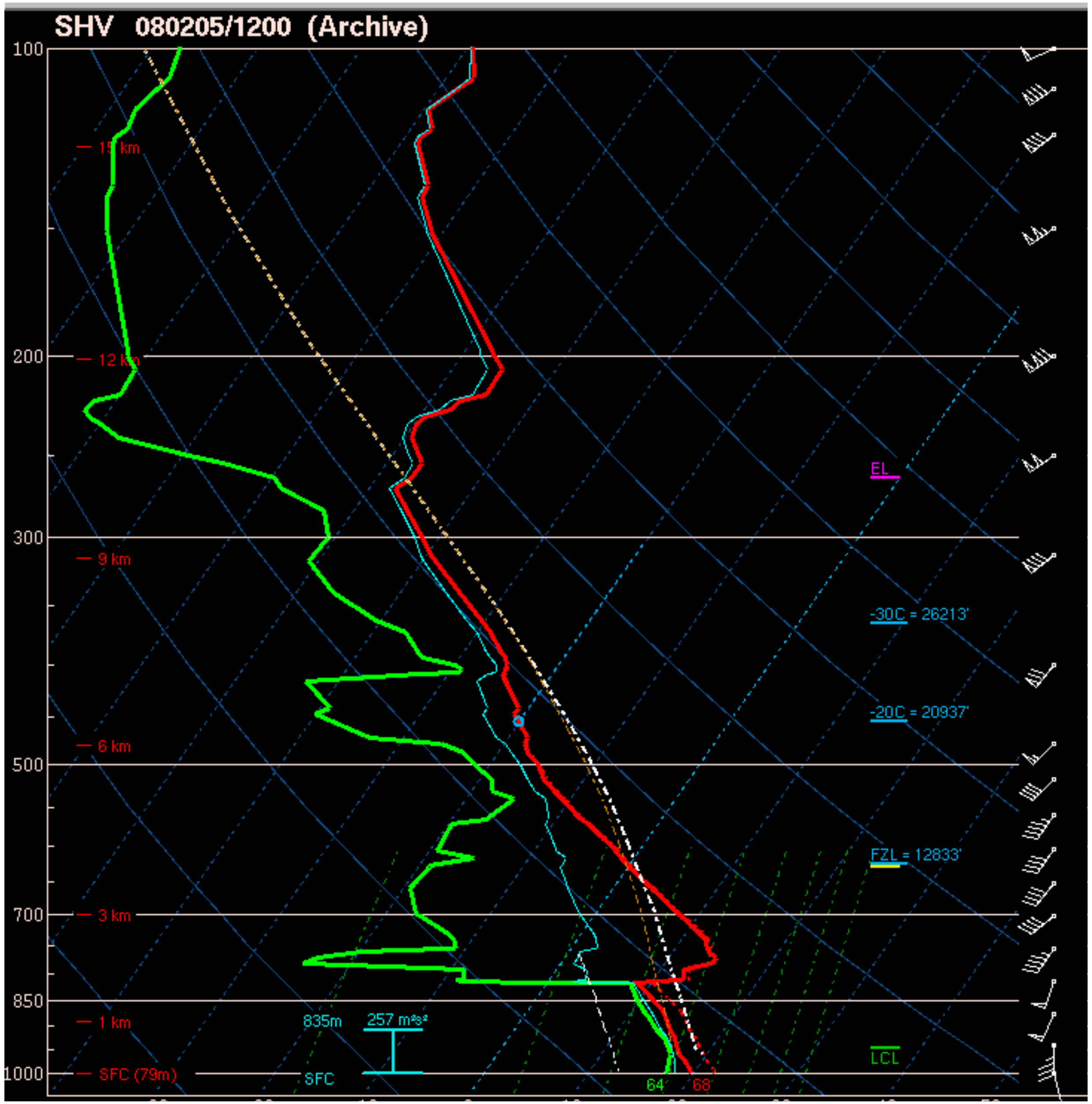


Little Rock, AL sounding

Late afternoon



Shreveport, LA sounding



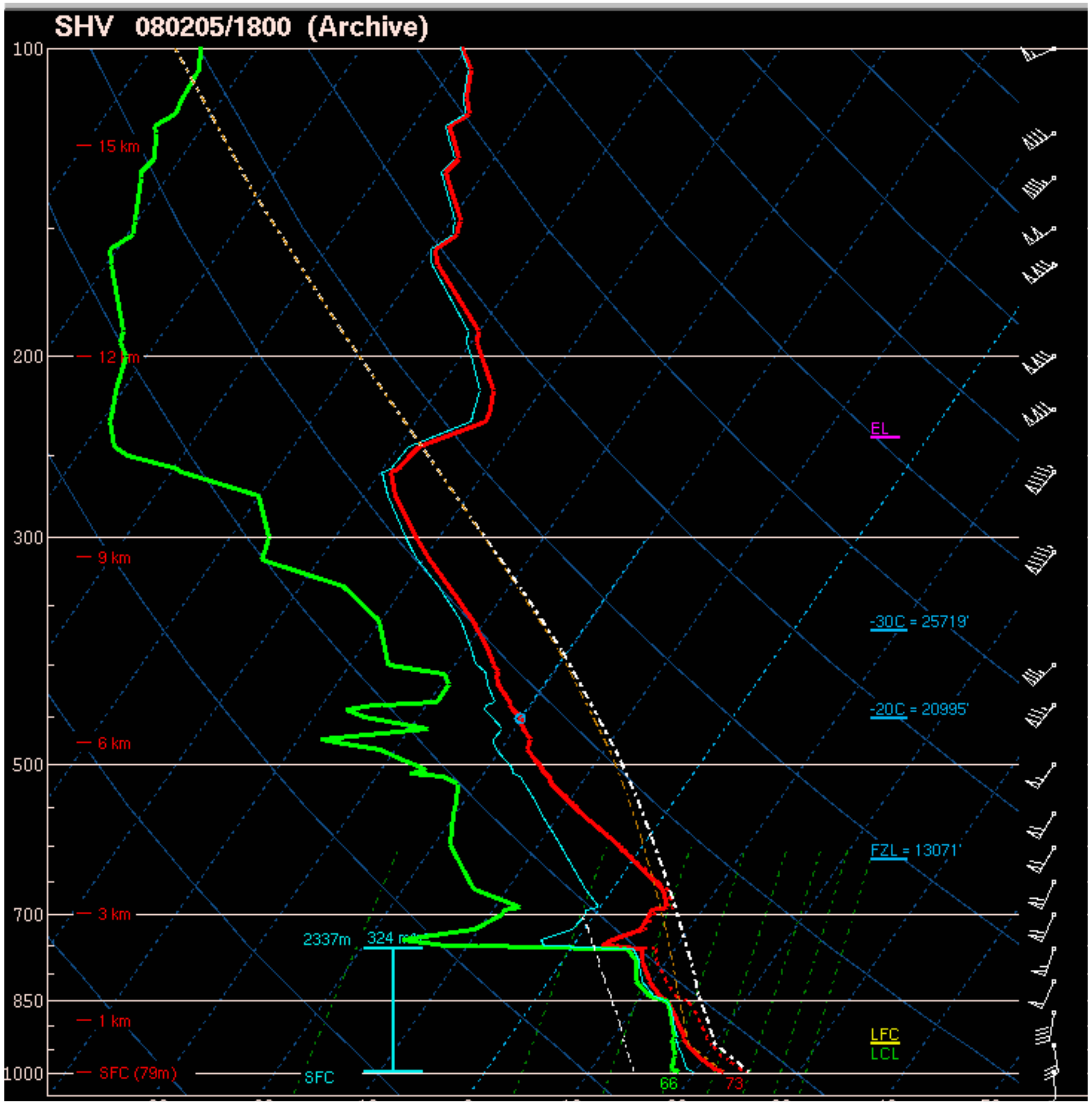
Morning

Strong inversion and large CIN in the morning

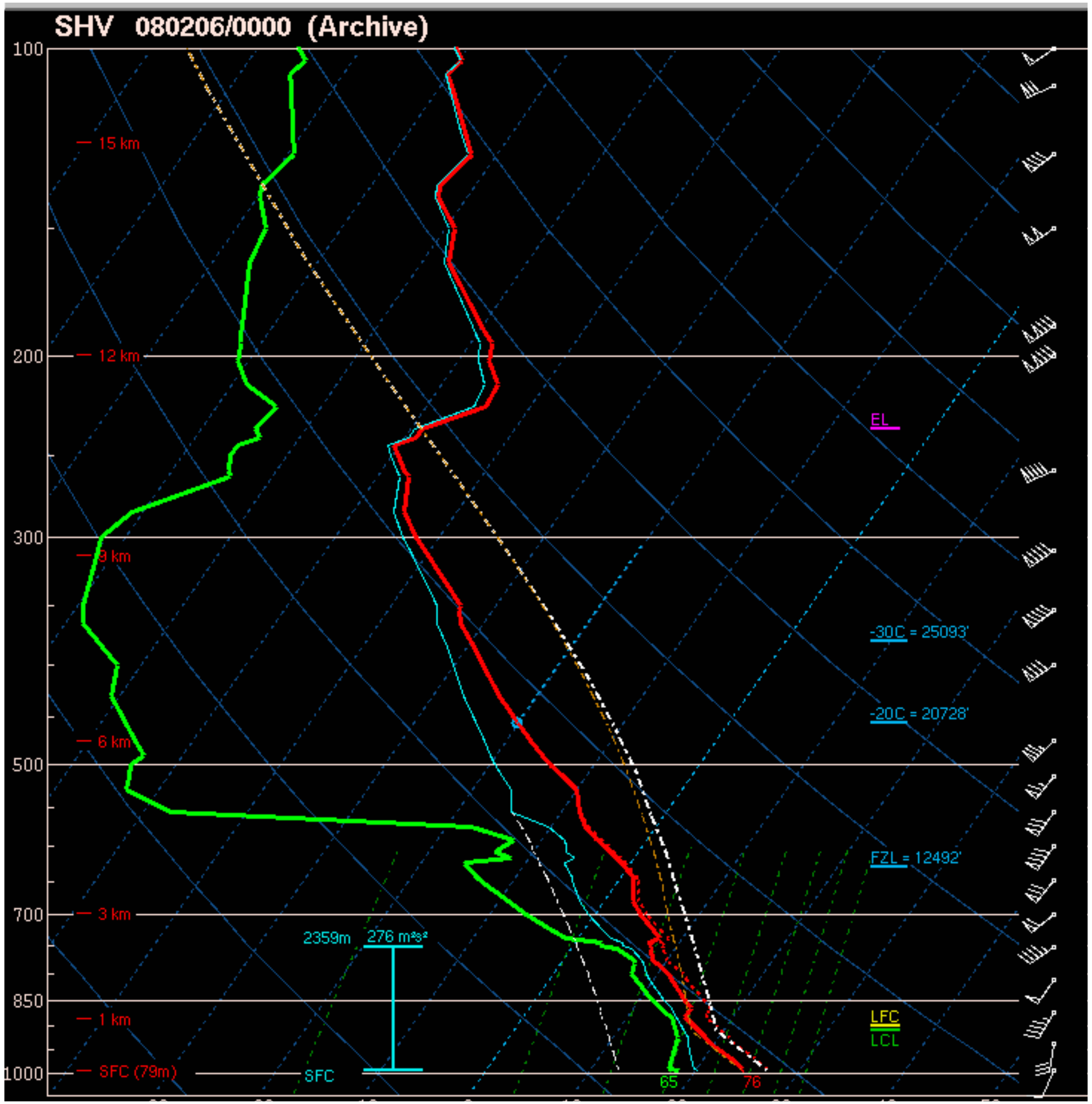
Shreveport, LA sounding

Noon

CIN is gone by noon due to surface heating and upward shifting of inversion



Shreveport, LA sounding



Late afternoon

Convection can easily develop

# Sounding diagrams are used for…

- Moisture and temperature profiles
- Estimates of CAPE, CIN, Lifted Index, etc.
  - Will storms form?
- Vertical wind shear (material on hodographs later!)
  - What kind of storms will form?
- Many of your favorite thunderstorm parameters are based in these diagrams, and subject to the same errors and concerns!