An Improved Force-restore Model for Land-surface Modeling

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ABSTRACT

To clarify the definition and to improve the forecast of the 'deep layer' temperature in the soil-temperature force-restore model, the prediction equations in such a model are re-derived. The derivation led to a deep-layer temperature, commonly denoted $T_2$, that is defined as the soil temperature at depth $\pi d$ plus a transient term where $d$ is the e-folding damping depth of soil temperature diurnal oscillations. The corresponding prediction equation for $T_2$ has the same form as the commonly used one except for an additional term involving the lapse rate of the 'seasonal-mean' soil temperature and the damping depth $d$. A term involving the same also appears in the skin temperature prediction equation, which also includes a transient term. The impact on the soil temperature prediction by these additional terms are tested against the Oklahoma Atmospheric Surface-layer Instrumentation System Project (OASIS) observations for four week-long periods selected from four seasons in 2000.

Clear improvements in the prediction of both skin and deep layer temperature were obtained using the revised formulation, and improvement in the latter is much more dramatic. The inclusion the lapse-rate related terms is most significant while the transient terms can be neglected. The addition of the lapse-rate related terms removes a drift found in the predicted deep-layer temperature towards the mean of skin temperature. Practical application of the revised system is also discussed.
1. Introduction

Land surface models (LSMs) deal with evolution of land surface and deep soil layer conditions and the exchanges of moisture and thermal energy between the land surface and the atmosphere. Land surface modeling is important for the studies of climatic predictions (e.g., Dickinson and Henderson-Sellers 1988), hydrology (e.g., Milly and Dunne 1994) and numerical weather prediction (e.g., Betts et al. 1997; Chen and Dudhia 2001).

Among land surface models of various complexities, a so-called force-restore (FR) model for soil temperature prediction is rather popular (e.g., Deardorff 1978, D78 hereafter; Noilhan and Planton 1989, NP89 hereafter; Mahfouf et al. 1995) because of its computational efficiency and reasonable physical foundation. It employs a minimum number of prognostic variables yet captures the most important physical processes. The model was originally developed by Bhumralkar (1975, B75 hereafter), and Blackadar (1976, B76 hereafter), and adopted by D78 and used in models such as ISBA (Interactions between Soil, Biosphere, and Atmosphere, NP89; Mahfouf et al. 1995).

The force-restore model, as applied to soil temperature, usually involves two prognostic equations, one for the surface or skin temperature ($T_s$) representing the temperature of both canopy and soil surface, and the other for a temperature ($T_2$) towards which $T_s$ is relaxed. $T_2$ is defined as the mean surface temperature averaged over one day by B75. In some other modeling studies (e.g., D78), $T_2$ is treated as deep soil temperature, however.

In this study, the soil model in the Advanced Regional Prediction System (ARPS, Xue et al. 1995; Xue et al. 2000; Xue et al. 2001) is tested against observed soil temperature data in week-long periods during four different seasons. The model is based on NP89 and some of its later modifications. Drift on the order of 5 K is found in the deep soil temperature, variable $T_2$, ...
predicted by the model. Modifications to the temperature equations are found through re-
derivation of the equations, taking into account the change in the seasonally averaged soil
temperature with depth. In section 2, the equations as used by B75 and NP89 are first briefly
reviewed followed by derivation of a revised version. The physical meaning of $T_2$ will also be
discussed. The Oklahoma Atmospheric Surface-layer Instrumentation System Project (OASIS,
Brotzge et al. 2001) observational data will be described in Section 3. Land surface scheme in
ARPS and the design of the numerical experiments will be discussed in Section 4. The numerical
results will be presented in Section 5 and compared with observational data. Finally, conclusions
are given in Section 6.

2. The revised force-restore model for soil temperature

a) Prognostic equations for soil temperature and the force-restore model

The equation describing the time evolution of soil temperature, $T$, can be written in terms
of the vertical flux convergence, assuming the effect of horizontal heat exchange can be
neglected:

$$C \frac{\partial T}{\partial t} = - \frac{\partial G}{\partial z}, \quad -\infty < z \leq 0,$$

where $G$ is the heat flux given by the following:

$$G(z,t) = \begin{cases} 
G_0, & z = 0 \\
-K_T C \frac{\partial T}{\partial z}, & z < 0
\end{cases}$$

Here, $C$ is the volumetric soil heat capacity ($J \cdot m^{-3} \cdot K^{-1}$), $t$ the time, $z$ the vertical coordinate
(positive upward and zero at the ground surface), $K_T$ the soil thermal diffusivity ($m^2 \cdot s^{-1}$). $K_T$ is
related to soil thermal conductivity $\lambda$ ($W \cdot m^{-1} \cdot K^{-1}$) by soil heat capacity, i.e., $K_T = \lambda/C$. $G_0$ in
Eq.(2) is the net heat flux at the ground given by
\[
G_0 = LE + H - R_n,
\]  

where \( R_n \) is the net radiation flux at the surface which includes net short and long wave radiation. \( LE \) is the latent heat and \( H \) the sensible heat fluxes leaving the ground surface.

For the interior of the soil, Eq.(1) becomes

\[
\frac{C}{\partial t} \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} (\lambda \frac{\partial T}{\partial z}).
\]  

If \( \lambda \) is assumed constant,

\[
\frac{C}{\partial t} \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial z^2}.
\]  

Numerical integration of Eq. (4) using multiple vertical layers would allow for variable \( \lambda \), realistic initial soil temperature profiles and boundary conditions. In the case where the vertical variation in thermal conductivity is relatively small, or little knowledge is known of its vertical variation, a simpler and more economical model derived from Eq.(4), with \( \lambda \) set as constant, can be rather accurate. The force-restore model is such a model and it was originally developed by B75 and B76 and popularized by D78. It is used in land surface models such as ISBA (NP89; Mahfouf et al. 1995).

In B75, the temperature at the surface is assumed to be sinusoidal with a 24-hour diurnal cycle and is expressed as

\[
T(0,t) = \bar{T} + A \sin(\omega t)
\]  

where \( \omega \) is the frequency of oscillation \( (\omega=2\pi/\tau, \text{ with } \tau \text{ being the period of oscillation and equal to 24 hours)} \). \( \bar{T} \) is defined in B75 as the daily average temperature of the soil assumed to be the same at all depths, and \( A \) is the amplitude of temperature wave at the surface. An equation for the mean temperature in a thin surface layer of depth \( d_f \) (taken to be 1 cm) was obtained by B75 as follows,
\[ \frac{c \, \partial T_s}{\partial t} = G_0 - \left( \frac{\omega C \lambda}{2} \right)^{1/2} (T_s - \bar{T}), \]

where \( T_s \) is the ground surface temperature, \( c = d_i C + \sqrt{\lambda C / 2\omega} \), and \( G_0 \) is the net surface heat flux given in Eq. (3). It is this equation or a variation of it that is used in land surface models like that of NP89. In NP89, the definition of \( c \) is approximated by dropping the first term on the right hand side (rhs) of its definition, i.e. \( \text{term} 1 \frac{d_i C}{d_i C} \). The approximation actually originated from and was discussed in D78, and is equivalent to assuming that the heat capacity of this thin layer is so small that the surface heat flux into this thin layer is roughly balanced by the ground heat flux into the deep layer.

With the above approximation or consideration, Eq. (7) becomes

\[ \frac{\partial T_s}{\partial t} = C_g G_0 - \frac{2\pi}{\tau} (T_s - T_2). \]

where \( C_g \equiv \sqrt{2\omega \lambda C} \). When vegetation is present, this coefficient will be modified to include the effect of vegetation on heat capacity (see, e.g., NP89). In the above equation, the daily mean temperature, \( \bar{T} \), has been replaced by a new symbol, \( T_2 \), because variable \( \bar{T} \) has become a time dependent variable to be predicted by another equation to be discussed next. Symbol \( \bar{T} \), with an over-bar denoting time averaging, can be misleading. Equation (8), in a slightly different form, was also tested by D78.

In NP89, a second equation is given for \( \bar{T} \), now named \( T_2 \) but still defined as the daily mean temperature. This equation is

\[ \frac{\partial T_2}{\partial t} = \frac{1}{\tau} (T_s - T_2). \]

No detail on the derivation of this equation was, however, given in NP89, except for references to B75 and B76. Interestingly, neither B75 nor B76 provided the equation for the prediction of
In B76, it is said that "the value to be used for $\theta_m$ (our $T_2$) is the mean temperature of the surface air during the most recent 24 hours".

The definition of $T_2$ has not always been consistent in the literature. B75 defines it, when it appears in the equation for surface temperature, as "the daily mean temperature at the surface assumed to be the same at all depths". NP89 defines it in a similar way, as the mean value of $T_s$ over one day. D78, however, calls it the deep soil temperature, and predicts it using an equation that is analogous to Eq.(8) but without the restore term on the rhs. In the forcing term of that equation that corresponds to the first term on the rhs of Eq.(8), a frequency that corresponds to annual instead of daily cycle is used. In another word, a different equation for the prediction of $T_2$ is used in D78 than that in NP89.

The definition of $T_2$ has been vague even in the same ISBA community; daily mean soil temperature is used in some papers (e.g., NP89, Mahfouf and Noilhan 1991; Noilhan and Lacarrére 1995; Noilhan and Mahfouf 1996; Calvet et al. 1998) and while deep soil temperature in others (e.g., Bouttier et al. 1993; Boone et al. 2000). Different definitions can even be found in the same article (Bouttier et al. 1993). When $T_2$ is considered deep soil temperature, different depths had been used. For example, Bouttier et al. (1993) considered $T_2$ the temperature at 1 m depth, while Calvet et al. (1998) and Mahfouf and Noilhan (1991) the temperature at 0.81 meter. In Mahfouf and Noilhan (1991), even though $T_2$ is defined as the mean daily temperature (page 1357) but measurement at 81 cm depth is used to initialize $T_2$ (page 1358). These differences in definition often lead to confusion in interpreting and verifying the model results.

One reason for the seemingly interchangeable use of daily average temperature and the deep soil temperature may have risen from the original assumption of B75, who assumed the daily mean temperature is the same at all depths. This assumption may be valid in the spring and
fall seasons, but is certainly not true in most parts of the world at summer and winter (de Vries 1963). Figure 1a shows the OASIS measurements of soil temperature at ground level, and those at 25 and 60 cm depth for a 7-day summer period starting from 12 August 2000. The means of these three temperature time series are respectively 306.7, 304.2 and 301.2 K, indicating a decreasing trend with depth. The difference between the mean temperatures of skin and at 60 cm is as large as 5.5 K for this period. A reverse trend is apparent for the winter period (a 7-day period starting from 5 January 2000) as shown in Fig.1b, where the soil temperatures at 5, 25 and 60 cm have respective means of 280.0, 281.1 and 283.4 K. The temperature increases with depth in this case. The above behavior appears to be also observed by Mahfouf et al. (1995) who found for climate simulations a spurious drift towards too low temperatures, especially over continental areas during the polar nights. Their solution to this problem with a force-restore model is to add another term to the \( \text{rhs} \) of \( T_2 \) equation that relaxes \( T_2 \) towards a climatological deep soil temperature. While it appears to work, the physical basis for doing so is not entirely clear.

In the following section, we re-derive the two equations for soil temperature and attempt to establish a clear definition for \( T_2 \) and at the same time obtain a set of equations for improved prediction of \( T_s \) and \( T_2 \), especially of the latter.

\textit{b) Revised Force-Restore Model for Soil Temperature}

Here we define the seasonal mean soil temperature as the running mean of temperature over 1-2 weeks, a period long enough to remove diurnal temperature changes while retain the seasonal variations. In general, this temperature increases (decreases) downward in winter (summer). Seasonal mean temperature can be considered the background upon which the diurnal oscillations are superimposed. This temperature is similar to the mean temperature defined by B75 except that we do not assume it constant at all depths. We will see that taking into account
of the vertical variation in this mean temperature introduces additional terms into the force-
restore equations.

Suppose this seasonal mean temperature profile of the soil is given by \( \tilde{T}_s = \tilde{T}_s + \gamma z \), where 
\( \tilde{T}_s \) is the mean temperature at the surface, and \( \gamma \) is the “lapse rate” of the mean temperature
(positive as temperature decreases with depth). Thus, under sinusoidal surface forcing with
single dominant period, as in the form of Eq. (6), the soil temperature as a solution to Eq.(5) can
be found to be

\[
T(z,t) = \tilde{T}_s + \gamma z + Ae^{\frac{z}{d}} \sin(\omega t + \phi_0 + \frac{z}{d}),
\]

(10)

where \( \phi_0 = \omega t_0 \) is the initial phase and \( t_0 \) is the time at which the amplitude of surface
temperature oscillation is zero. \( d = \sqrt{K_T \tau / \pi} = \sqrt{2\lambda / C\omega} \) is the e-folding damping depth
at which the amplitude of surface temperature oscillations is reduced by a factor of \( e^{-1} \) (about
0.37). Obviously, it is a function of soil thermal diffusivity and the period of forcing. E-folding
damping depth is much larger for annual forcing than for diurnal forcing. Higher thermal
diffusivity also results in larger damping depth.

Soil water content plays a role through its double effects on soil heat capacity and soil
thermal conductivity. It increases \( C \), but at the same time increases \( \lambda \). This makes it hard to
determine the net effects of soil moisture on \( K_T \). In general, for all types of common soil, the
largest thermal diffusivity is achieved when soil volumetric water content reaches about 25%. \( K_T \)
was assumed to be constant with depth in this study as well as in B75.

The general solution for soil temperature (10) tells us that the amplitude of soil
temperature fluctuations decreases exponentially with depth and the phase delay increases
linearly with depth; the amplitude is dampened faster with depth for high frequency modes, which justifies our using of only one mode (daily cycle) in the analysis.

The background soil temperature profile also implies an extra soil heat flux of constant magnitude $-\lambda \gamma$ for all depths. Thus, at the surface $(z=0)$, the soil heat flux equals $G(0,t) = G_0 - \lambda \gamma$, or, a modification to the definition of Eq. (3). Plug the solution of $T$ in Eq. (10) into the definition of ground heat flux given in (2), we obtain

$$G(z,t) = -\lambda \left\{ \gamma + \frac{1}{d} A e^{d z} \left[ \sin(\omega t + \phi_0 + \frac{z}{d}) + \cos(\omega t + \phi_0 + \frac{z}{d}) \right] \right\}. \quad (11)$$

From Eq.(10) and its time derivative we have

$$A e^{d z} \sin(\omega t + \phi_0 + \frac{z}{d}) = T(z,t) - \tilde{T}_s - \gamma z, \quad (12)$$

and

$$A e^{d z} \cos(\omega t + \phi_0 + \frac{z}{d}) = \frac{1}{\omega} \frac{\partial T(z,t)}{\partial t}. \quad (13)$$

In obtaining Eq.(13), we have assumed that the time rate of change in $\gamma$ is a very slow-varying function of time, therefore its time derivative can be neglected. Plug Eqs. (12) and (13) into Eq. (11), to obtain

$$G(z,t) = -\lambda \left\{ \gamma + \frac{1}{d} \left[ T(z,t) - \tilde{T}_s - \gamma z + \frac{1}{\omega} \frac{\partial T(z,t)}{\partial t} \right] \right\}. \quad (14)$$

Applying Eq.(14) to $z=0$, and making use of surface energy balance equation (3), we obtain, after some reorganization,

$$\frac{\partial T(0,t)}{\partial t} = \frac{2\pi}{\tau} [\tilde{T}_s - T(0,t)] + \frac{d \omega}{\lambda} \left[ R_{net} - LE - H \right] \quad (16)$$

where $(d \omega)/\lambda \approx 1/c$. If we designate $C_G \equiv 1/c \approx 2/(d C)$, Equation (16) can be rewritten as

$$\frac{\partial T(0,t)}{\partial t} = C_G [R_{net} - LE - H] + \frac{2\pi}{\tau} [\tilde{T}_s - T(0,t)]. \quad (17)$$
Equation (17) is essentially the force-restore equation for surface temperature derived by B75 and used in models of D78 and NP89. The key issue here is the definition of the temperature towards which the surface temperature is restored. In Eq. (17), this temperature is $\tilde{T}_s$, the temporal mean surface temperature.

In Eq. (17), $\tilde{T}_s$, defined as the time mean surface temperature, is not known. It is a slow varying quantity and we choose to relate it to a temperature of the deep layer soil of some sort. Here we set out to derive an equation for the mean (vertically averaged) temperature of the deep layer. The mean temperature of the layer extending from the ground level to a depth at $z$ is defined by $\overline{T}_z = \frac{1}{z} \int_0^z T(z', t)dz'$. Plugging in $T(z)$ given by Eq. (10) gives

$$\overline{T}_z = T_s + \frac{\gamma z}{2} + \frac{dA}{2z} e^{\frac{\gamma}{d}} \left[ \sin(\omega t + \phi_0 + \frac{z}{d}) - \cos(\omega t + \phi_0 + \frac{z}{d}) \right] - \frac{dA}{2z} \left[ \sin(\omega t + \phi_0) - \cos(\omega t + \phi_0) \right].$$

Taking a time derivative of Eq. (18) and making use of Eq. (12), we have

$$\frac{\partial \overline{T}_z}{\partial t} = \frac{dA}{2z} \left[ A e^{\frac{\gamma}{d}} \left[ \sin(\omega t + \phi_0 + \frac{z}{d}) - \sin(\omega t + \phi_0) \right] + [T(z, t) - T(0, t) - \gamma z] \right].$$

(19)

Letting $z = -\pi d$, i.e., choosing the depth of the layer for which we look for the vertical mean to be $\pi d$, Eq. (19) becomes

$$\frac{\partial \overline{T}_{(0-\pi d)}}{\partial t} = \frac{A \omega}{2\pi} \left[ e^{-\frac{\pi}{d}} + 1 \right] \cos(\omega t + \phi_0) - \frac{1}{\tau} [T(-\pi d, t) - T(0, t) + \gamma \pi d].$$

(20)

Let $z = -\pi d$ again, we obtain the mean temperature in the $\pi d$ deep layer as

$$\overline{T}_{(0-\pi d)} = \tilde{T}_s - \frac{\gamma \pi d}{2} + \frac{A(1+e^{-\pi})}{2\pi} \left[ \sin(\omega t + \phi_0) - \cos(\omega t + \phi_0) \right].$$

(21)

Applying Eq. (12) at $z = -\pi d$, rearranging, we have
\[
\tilde{T}_s = T(-\pi d, t) + A e^{-\pi} \sin(\omega t + \phi_0) + \gamma \pi dz. \tag{22}
\]

Plug the above equation into Eq. (21), we obtain
\[
\overline{T_{(0-\pi d)}} = T(-\pi d, t) + \frac{\gamma \pi d}{2} + A \left[ e^{-\pi} + \frac{1 + e^{-\pi}}{2\pi} \right] \sin(\omega t + \phi_0) - \frac{A(1 + e^{-\pi})}{2\pi} \cos(\omega t + \phi_0). \tag{23}
\]

Let
\[
\overline{T_{(0-\pi d)}} = T^{(1)} + \frac{\pi d}{2} \gamma + \frac{A}{2\pi} (e^{-\pi} + 1) \sin(\omega t + \phi_0), \tag{24}
\]

where
\[
T^{(1)} = T(-\pi d, t) - \frac{A(1 + e^{-\pi})}{2\pi} \cos(\omega t + \phi_0) + A e^{-\pi} \sin(\omega t + \phi_0).
\]

and \( \alpha = \tan^{-1} \left[ \frac{-1 + e^{-\pi}}{2\pi e^{-\pi}} \right] \approx -0.42\pi, \) and \( B = \left[ \frac{1 + e^{-\pi}}{2\pi} \right]^2 + e^{-2\pi} \right]^{0.5} \approx 0.17. \)

Plug \( \overline{T_{(0-\pi d)}} \) given in (24) into (20), making use of Eq.(13) applied at \( z=0, \) we obtain
\[
\frac{\partial T^{(1)}}{\partial t} = -\frac{1}{\tau} \left[ T(-\pi d, t) - T(0, t) + \gamma \pi d \right]
= -\frac{1}{\tau} \left[ T^{(1)} - T(0, t) + \gamma \pi d \right] + \frac{AB}{\tau} \sin(\omega t + \phi_0 + \alpha). \tag{26}
\]

Equation (26) is similar to the 'deep layer' temperature equation used in traditional force-restore models (see, e.g., NP89), except for the sine term on the \( rhs \) and \( \gamma \pi d \) term. The sine term considerably complicates the equation. We want to see if the equation can simplified through further variable transform. We define \( T^{(1)} = T^{(2)} - \frac{AB}{2\pi} \cos(\omega t + \phi_0 + \alpha) \) and plug it into (26) to obtain
\[
\frac{\partial T^{(2)}}{\partial t} = -\frac{1}{\tau} \left[ T^{(2)} - T(0, t) + \gamma \pi d \right] + \frac{AB}{2\pi \tau} \cos(\omega t + \phi_0 + \alpha). \tag{27}
\]
Eq. (27) is more attractive than Eq. (26) because the last term on the rhs is a factor of $2\pi$ smaller than that in Eq. (26). The next transform would be $T^{(2)} = T^{(3)} + \frac{AB}{(2\pi)^2} \sin(\omega t + \phi_0 + \alpha)$, and Eq. (27) can be then rewritten as

\[
\frac{\partial T^{(3)}}{\partial t} = -\frac{1}{\tau} \left[ T^{(3)} - T(0,t) + \gamma \pi d \right] - \frac{AB}{(2\pi)^2} \sin(\omega t + \phi_0 + \alpha),
\]

whose last term on the rhs is yet another factor of $2\pi$ smaller! The next transformation

\[
T^{(3)} = T^{(4)} + \frac{AB}{(2\pi)^3} \cos(\omega t + \phi_0 + \alpha)
\]

yields equation

\[
\frac{\partial T^{(4)}}{\partial t} = -\frac{1}{\tau} \left[ T^{(3)} - T(0,t) + \gamma \pi d \right] - \frac{AB}{(2\pi)^3} \cos(\omega t + \phi_0 + \alpha).
\]

We notice that a pattern has emerged with both the transformation and the second term on the rhs of the prognostic equation. The following general transformation can be applied,

\[
T^{(2n)} = T^{(2n+1)} - (-1)^n \frac{AB}{(2\pi)^{2n}} \sin(\omega t + \phi_0 + \alpha),
\]

\[
T^{(2n+1)} = T^{(2n+2)} - (-1)^n \frac{AB}{(2\pi)^{2n+1}} \cos(\omega t + \phi_0 + \alpha), \quad n \geq 0.
\]

where integer $n$ is the transformation order, with $T^{(0)} = T(-\pi d, t)$.

Applying this transformation indefinitely results in the magnitude of the sinusoidal term on the rhs of the equation eventually approaching the limit of zero. At this limit, the variable that is predicted by the equation becomes

\[
T_2 = T^{(\infty)} = T(-\pi d, t) + AB \left[ \sum_{n=0}^{\infty} \left( \frac{-1}{4\pi^2} \right)^n \right] \left[ \sin(\omega t + \phi_0 + \alpha) + \frac{1}{2\pi} \cos(\omega t + \phi_0 + \alpha) \right]
\]

\[
= T(-\pi d, t) + AB \frac{4\pi^2}{4\pi^2 + 1} \left[ \sin(\omega t + \phi_0 + \alpha) + \frac{1}{2\pi} \cos(\omega t + \phi_0 + \alpha) \right]
\]

\[
= T(-\pi d, t) + AB' \sin(\omega t + \phi_0 + \alpha')
\]

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where $B' = \frac{2\pi}{\sqrt{4\pi^2 + 1}} B \approx 0.17$, $\alpha' = \alpha + \tan^{-1}(\frac{1}{2\pi}) \approx -0.37 \pi$ and we give variable $T^{(\infty)}$ the name $T_2$ to match the name used for the ‘deep layer’ temperature as in Eq. (9). The prognostic equation for $T_2$ is therefore

$$\frac{\partial T_2}{\partial t} = -\frac{1}{\tau} [T_2 - T_s + \gamma \pi d],$$  \hspace{1cm} (31)$$

the same as Eq. (9) except for the term related to mean lapse rate $\gamma$. Here, we renamed $T(0, t)$ as $T_s$ to be consistent with earlier notations in Eqs. (8) and (9).

Now, we go back to the prediction equation (17) for surface temperature, $T_s$, and replace $T_2$ in the equation with the following

$$\tilde{T}_s = T(-\pi d, t) + \pi d \gamma + A e^{-\pi} \sin(\omega t + \phi_0)$$

$$= T_2 + \pi d \gamma - AB' \sin(\omega t + \phi_0 + \alpha') + A e^{-\pi} \sin(\omega t + \phi_0)$$

$$= T_2 + \pi d \gamma + AB'' \sin(\omega t + \phi_0 + \alpha''),$$  \hspace{1cm} (32)$$

where $\alpha'' = \tan^{-1} \frac{AB' \sin \alpha'}{AB' \cos \alpha' - A e^{-\pi}} \approx 0.45 \pi$; $B'' = \sqrt{(e^{-\pi} - B' \cos \alpha')^2 + (B' \sin \alpha')^2} \approx 0.158$, so that

$$\frac{\partial T_s}{\partial t} = C_G (R_{net} - LE - H) - \frac{2\pi}{\tau} (T_s - T_2 - \pi d \gamma) - \frac{2\pi}{\tau} AB'' \sin(\omega t + \phi_0 + \alpha'').$$  \hspace{1cm} (33)$$

Equations (31) and (33) are the new force-restore equations we obtained for predicting the ‘deep layer’ and surface soil temperature. One of the advantages over the original force-restore equations is the clear definition of $T_2$ and a rigorously derived prediction equation for it. The main differences of these equations from those of NP89 include the extra $\pi d \gamma$ terms in both equations, and which we will show through verification experiments against the OASIS data are the most significant aspect of improvement with this new set of equations. The last term in (33)
is a troublesome term, because the amplitude of diurnal cycle, $A$, is not known a priori. Fortunately, experiments show that the neglect of this term has little impact on the solution; in practice it can be neglected or estimated based on the mean amplitude of the previous days.

3. OASIS data at Norman site and the use of data

This study make use of the OASIS data of Norman, Oklahoma site which was also used by Brotzge and Weber (2002) for soil model calibrations. The OASIS sites are part of the Oklahoma Mesonet (Brock et al. 1995) that have routine measurements of the surface energy budget. Due to their high initial and maintenance cost, only 10 of the 90 OASIS sites are equipped with sonic anemometers. These 10 sites are called super sites and Norman, Oklahoma (station named NORM) is one of them. At super sites, all components of the surface energy budget are directly measured by instruments while at the standard sites, the latent heat flux is the residual term required to close the surface energy budget. The Norman site is flat and its immediate surroundings can be considered as uniform on a scale of several thousands meters at an elevation of 360 m. The parameters used for characterizing the land surface are summarized in Table 1.

At the Norman site, the routine meteorological measurements include surface temperature, mixing ratio, precipitation rate, wind, and surface pressure. An infrared sensor records surface skin temperature. The raw data of surface atmospheric meteorological variables and soil skin temperature are recorded at 5 minutes intervals. The measurements of soil moisture and soil temperature were made using the 229-L sensors, every half an hour at 0.05, 0.25, 0.60 and 0.75 m from the surface downward. The soil moisture measurements at 0.05 m are directly used to specify the near-surface layer soil moisture and soil moisture measurements at 0.25, 0.6 and 0.75 m are weighted by their representing soil depths (i.e., 10, 30, 40, and 70 cm) to come up
with a mean value for the deep soil moisture content. Since our focus here is the soil temperature prediction, time interpolated observed values of soil moisture (or depth mean values) are used to specify both near-surface and deep-layer soil moisture in our validation experiments, therefore only soil temperature is predicted by the model. The soil temperature measurements at 0.05 and 0.25 m are processed according to Eq. (30) to arrive at values of $T_2$, which are then used to initialize as well as to verify the force-restore model.

Details of soil moisture measurements using 229-L probes, including theory, sensor calibration and data manipulation are as described in Basara (2001). All four components of the surface energy balance, i.e., net radiation, sensible heat flux, latent heat flux, and ground heat flux are directly measured every five minutes and are available for the whole study period. The instrument used for net radiation measurement is NR-Lite radiometer. Additional quality control is applied during periods of precipitation and when dew is suspected. Observations are gathered every three seconds and averaged to yield 5-minute observations. A combination method (separate estimation of ground flux and storage terms) is used to estimate the total ground heat flux. The sensible heat flux is estimated using eddy correlation method. Values of latent heat flux are estimated using a co-located Krypton hygrometer. When any one of the four components of the energy budget was missing, the residual was used. Detailed discussion can be found in Brotzge (2000).

The vegetation parameters recorded by OASIS include vegetation type, leaf area index (LAI), vegetation cover, and NDVI index. This study involves four forecast periods selected from each season of year 2000. Each forecast period is 6 days long. Because vegetation properties are slowly varying functions of time, they were kept constant within each forecasting period.
Since $T_2$ is now defined as a composite value, by Eq. (30), its initialization and verification should also use the same definition. Pre-processing of the deep soil temperature is performed first. In the equation, the most important is the determination of damping depth of the soil temperature.

The generalized amplitude-phase method (Sellers 1965, p134-139) is used to determine the damping depth (See Appendix) for each testing period because of its soil moisture dependency. This method uses the soil temperature information of two different depths at four equally separated times of a clear day. Given the data availability, amplitude-phase method can be applied to the soil temperature measurement pairs at 5 and 25cm, 5 and 60 cm or 25 and 60 cm, to obtain three different values of the damping depth. The final optimally estimated damping depth is obtained by taking a weighted average of the three, with the first measurement being given the largest weight of 0.6 and the remaining two given the same weight of 0.2. Because the assumption of sinusoidal diurnal oscillations can be violated in the real soil temperature data at times, the calculated damping depth can be enormously large at these times, causing some spikes in the curves as shown in Figure 2. Values larger than 30 cm are considered bad values and are discarded. The remaining values are then averaged over each period to obtain the mean values. It was found that the mean damping depth could vary between 12 and 17 cm at the Norman site, mainly depending on the moisture contents. For our four selected periods, the mean values are 0.155, 0.148, 0.164 and 0.146 m for spring, summer, fall and winter periods, respectively, as given in the last column of Table 2.

Amplitude $A$ in Eq. (10), by definition, is the temperature amplitude at the land surface. When the infrared-sensor-measured skin temperature is generally good, skin temperature amplitude is calculated daily, and the average amplitude over the six day period is used.
However, when the infrared sensor measurements are contaminated by dense, sharp and irregular spikes caused by temporary instrument failure (e.g., during the much of the winter season of 2000 in OASIS data and true for the selected winter test period to be described later), the surface soil temperature are obtained by extrapolation from the temperature measured at 5 cm depth using the exponential decaying relation described by Eq. (10).

\[ T_2 \] in Eq. (30) includes soil temperature at depth \(-\pi d\) and a sinusoidal part. For every damping depth we determined, the depth of \(-\pi d\), in general, does not happen to be at one of the four fixed measurement depths of 5, 25, 60, and 75 cm. It therefore has to be derived from the measurements. Using the method for determining the damping depth, soil temperature at \(z = -\pi d\) can be derived from measurements at any available depth, i.e., at 5, 25 (in this study) or 60 cm. Aside from the afore-mentioned amplitude \(A\), the sinusoidal term in Eq. (30) includes an initial phase \(\phi_0\). This parameter signifies physically the phase delay of surface soil temperature to surface forcing and is obtained in this study by a comparison of the maximum surface soil temperature occurrence time and the time of maximum net radiation.

4. Numerical model and experimental design

The implementation of the two-layer soil-vegetation model in the ARPS basically follows the ISBA model (NP89) with some of its later enhancements. In the model, the surface layer depth is set as 0.1 m and the deep layer is assumed to be at 1 m. The deep soil layer acts as a reservoir for heat as well as for soil water.

The amplitude of daily soil temperature cycle depends highly on the volumetric heat capacity of the combined ground-vegetation surface layer, volumetric heat capacity for ground in ISBA depends on both soil texture and the wetness of the soil at the time. The heat capacity of vegetation is set as \(2 \times 10^{-5} \text{ K m}^2 \text{ J}^{-1}\) at NORM. The volumetric heat capacity also determines the
relative importance of surface forcing (energy balance) term and the restore term in surface temperature equation. According to NP89, for Norman site (with slope of logarithmic water retention curve $b=10.4$), the soil heat capacity can vary by a factor of 7 between that when soil moisture content is near saturation ($w_{\text{sat}}=0.45 \text{ m}^3 \text{ m}^{-3}$) and that at wilting point (heat capacity is $3.729 \times 10^{-6} \text{ K m}^2 \text{ J}^{-1}$ when $w_{\text{wilt}}=0.19 \text{ m}^3 \text{ m}^{-3}$).

Because our primary goal is to evaluate the performance of the soil model, in our experiments, we run the soil model in a stand-alone, forced mode to avoid uncertainties due to atmospheric processes. Short wave (solar) radiation reaching the ground (which is needed in the parameterization of evapotranspiration process), net radiation, wind speed (at 2 m AGL), surface pressure, air temperature (2 m AGL), and specific humidity (2 m AGL) are all specified using OASIS measurements which are linearly interpolated to the model time steps where they are needed. The time step size used for the land surface model is 1 minute (much larger step size can be used without stability problem. The smaller value is used for accuracy here). The surface latent heat and sensible heat fluxes are calculated using the stability-dependent surface flux model in the ARPS, instead of using those from OASIS measurements. Doing so permits the feedback of surface soil temperature and moisture prediction to the surface energy balance through surface flux calculations.

The ARPS allows the use of vertically stretched grids. To match the height (2m AGL) at which the meteorological measurements are taken, the minimum vertical grid spacing near ground is set to 4 m so that the first scalar level (where temperature, moisture and horizontal wind are defined) is 2 m AGL. The remaining parameters, including the leaf area index, soil type, surface roughness, minimum stomatal resistance and vegetation cover are specified according to the properties of Norman site (See Tables 1 and 2 for some of the parameter
values). The vegetation cover and surface roughness data provided by OASIS is not seasonally varying, and values of 0.75 and 0.03 m, respectively, are used following Brotzge and Weber (2002). Jacquemin and Noilhan (1990) found that latent heat flux estimation in ISBA is not sensitive to roughness. Vegetation coverage is however a more important parameter in surface energy partitioning. In this study, the soil texture-related parameters are specified for silty clay soil type according to Table 2 of NP89. The minimum stomatal resistance is set as 200 s m$^{-1}$ and the threshold solar radiation strength for shutdown transpiration at dust is 50 W m$^{-2}$.

Four different groups of experiments are performed for each season, and they are termed ‘original’, ‘lapse rate only’, ‘sine-term only’ and ‘revised’, respectively, based on the formulation of the equations used. By ‘lapse rate only’ we mean that the only modification to the original formulation is to take into account the difference between the average temperatures of the top and deep layers, i.e., to include the $\pi d\gamma$ terms in the restoring terms in the rhs of Eqs. (31) and (33). By ‘sine-term only’ we mean that the modification is only limited to the sinusoidal term introduced in Eq. (33). By ‘revised’ we mean that we fully implemented the terms in Eqs. (31) and (33). All results are compared with OASIS properly processed measurements (denoted ‘observation’ in the figures). For T$_2$, this means that Eq.(30) (with and without the transient term depending the run) is used to determine its observed value. Such values are also used to initialize T$_2$ at initial time. Since the sine term is not easy to determine in advance in practical applications, it is also this term that complicates the skin temperature prediction equation (Eq. 33), we want to see if its inclusion in the equations is significant. Finally, as mentioned earlier, in all experiments to be presented, the time-dependent soil moisture content is specified according to OASIS observations, with some smoothing applied to the time series; therefore soil
temperatures at the two layers are the only two prognostic variables in these experiments. Comparison runs with predicted soil moisture show similar results, however.

c) Study periods selected

To examine the effects of the modifications to the force-restore equations, in particular, the inclusion of the seasonal mean temperature lapse rate terms, four periods of 6 days each from four different seasons in year 2000 were selected. They are: March 25-31 from spring, August 12-18 from summer, September 17-23 from early fall and January 5-11 from winter. In selecting the study periods, the top priority is given to the coherency among the soil and atmospheric measurements. Quiescent, high pressure dominated clear sky conditions are preferred to have more periodical surface forcing. Figure 3 shows, using summer season as an example, regular periodic behaviors in both fluxes at the surface and in the temperature of air and soil. Figure 3a exhibits the periodic daily cycles of various measured energy fluxes. Figure 3b shows that, under such periodic forcing, both surface skin temperature and the soil temperatures at 5 cm show periodic diurnal cycles of evolution.

The selected winter period is different from the other three periods in two aspects: this period is wetter (superficial soil moisture content $w_g = 0.379$ m$^3$ m$^{-3}$ and deep soil moisture content $w_2 = 0.384$ m$^3$ m$^{-3}$) than the other periods ($w_g = 0.25$ m$^3$ m$^{-3}$ and $w_2 = 0.270$ m$^3$ m$^{-3}$ for summer; $w_g = 0.24$ m$^3$ m$^{-3}$ and $w_2 = 0.269$ m$^3$ m$^{-3}$ for fall; $w_g = 0.36$ m$^3$ m$^{-3}$ and $w_2 = 0.38$ m$^3$ m$^{-3}$ for spring period). This fact is also reflected in the damping depths (Table 2), which is shallower for the winter. As mentioned in Section 3, during much of the winter season, the infrared sensor measurements are contaminated. For January 5-11, 2000, the surface soil temperatures are obtained by extrapolation from the temperature measured at 5 cm depth using the exponential decaying relation described by Eq. (10).
5. Results of Experiments

We present in this section results of the numerical experiments outlined earlier. We first look the results from the six-day summer period starting from August 12, 2000.

Figure 4 shows that model predicted and observed skin and deep soil temperature, $T_s$ and $T_2$, for this period using different formulations. The surface temperature forecasts, by either the original or modified formulations, are most accurate for the first two days and those for the rest of the period are pretty good too (Fig. 4a). The root mean squared error is 1.5 K for the entire period. No apparent phase error exists and the amplitude difference is generally less than 3 K. The maximum amplitude errors occur mainly at the time of maximum daytime heating, and a maximum difference of 3.86 K occurred at around 17 UTC of 14 August 2000. There exists a general, though small, cold bias in the skin temperature forecast of about -0.58 K, as indicated by both minimum and maximum daily temperatures.

The deep soil temperature predicted by the original formulation has large errors (Fig. 4b), the rms error is 3.32 K for the six-day period (Table 3). The predicted values are consistently 4 to 5 K higher than the observed ones only half a day from the initial time although the diurnal oscillations are generally in phase with the observations. A careful look indicates that there is a tendency for the daily mean value of $T_2$ to approach the daily mean of $T_s$, a result, we believe, caused primarily by the neglect of mean lapse rate of the soil temperature, which is 4.7 K over a 46 cm depth (see Table 2).

The $T_2$ predictions using 'revised' and 'lapse rate only' formulations are much improved over the original formulation (Fig. 4b). The rms error is reduced to about 0.7 K in both runs, and the reduction is mainly due to the removal of the upward drift observed in the original case. The figure also shows that the inclusion of sine-term only made very little difference from the original solution, and the solutions of revised and lapse rate only cases are also very close. This indicate that the inclusion of the sine-term in both cases has very little impact, and because of the difficulty with knowing the amplitude of surface oscillations a priori, the sine or transient terms in our derived equations can be safely neglected in practice. As shown in Table 3, the 'revised' formulation consistently improves the $T_2$ forecast in all four seasons, and the improvement is most dramatic in summer and winter. The lapse rate related terms are primarily responsible for such improvement.

The inclusion of the lapse-rate related term in the skin temperature prediction equation, i.e., Eq. (33), did not affect the already rather accurate prediction of skin temperature much in this case. The slight cold bias in the skin temperature still exists (Fig. 4a). This must be because in the skin temperature equation, the forcing from the net radiation and sensible heat fluxes plays a much larger role than the extra restore terms we added. The net radiative flux is, especially during the daytime heating period, the dominant term. We noted earlier that there is little phase delay in skin temperature prediction, a problem reported by Brotzge and Weber (2002) in tests.
with a day in May and two days in August of the same year using the same model and data set. This improvement can be shown to be due to better behavior of the soil moisture, which in our case is specified according to observations. Improvements to the ARPS soil moisture prediction equations have been made since the work of Brotzge and Weber (2002) by the current authors and the predicted soil moisture content is now much closer to the observed values; in fact, a test using predicted soil moisture values with improved soil moisture equations with the current case produced very similar temperature forecast.

The results from the six-day winter period starting from 00UTC 5 January 2000 are presented in Fig. 5. As pointed out earlier, the observed skin temperature in this case is extrapolated from the soil temperature measured at 5 cm using Eq.(10). In general, the revised model, or the version that includes the lapse-rate related terms, provides an improved deep soil temperature forecast (Fig. 5b). In this case, the $T_2$ predicted by the original formulation has a tendency to drift below the temperature of observed values, opposite in direction to the summer case. This can again be explained by the fact that the mean lapse rate is neglected in the original formulation and in winter the mean surface temperature is lower that the mean temperature at the deeper layer. The original model tends to pull the deep layer temperature towards that of the surface. The improvement in $T_2$ forecast is not as consistent through the period as in the summer, however. This can be attributed to the fact that during this period, the surface temperature is not very periodic from day 4 (8 January 2000). The first three days represent is a clear calm period after a cold front passage, whereas the last two days is a warming up period. For day four, the observed shortwave solar radiation fluxes reaching the ground clearly indicate cloudy sky conditions and the daily maximum downward longwave radiation shows a 30% increase that prevented surface temperature from decreasing as much at night. The aperiodic behavior in the
skin temperature leads to poorer prediction of both skin and deep layer temperatures from day four to day five.

In contrast to the selected summer period, the daytime forcing is milder in the winter. Also, soil moisture content is larger due to several antecedent rainfall events (with the most recent one occurring on 03 January). Estimated using the soil moisture content at 5 cm, the volumetric soil heat capacity of this winter period is 2.6 times that of the summer period. This effect works in accord with the reduced energy balance term and significantly reduces the relative importance of $C_g (R_{net} - LE - H)$ term in Eq. (33). Using the same error statistics, Table 4 illustrates the effect of our modifications on the surface temperature prediction. The $rms$ error for surface temperature with revised version is reduced to 0.73 K from 1.06 K. The mean bias error is also significantly reduced from 0.85 K to 0.34 K. Similar results were found when we applied the same modifications to another wet period starting from 00UTC 6 April, 2000, suggesting that our revised formulation also improves the forecast of surface temperature and the effect is more evident when the primary force term in the equation is weaker.

The results from the six-day early fall period starting from September 17, 2000 are shown in Fig. 6. The daily minimum skin temperature is well predicted, with difference from the observation being less than 2 K for all the days. The model fails to produce as high day-time maximum temperatures as observed for all of the days although the difference is generally less than 5 K except for day four when the maximum skin temperature is significantly lower than the other days. This abrupt change in the atmospheric forcing must have been contributed to this larger error since interruption of sinusoidal behavior of surface conditions is expected to increase errors in force-restore model prediction. The predictions of the deep layer soil temperature using the revised or lapse-rate-only version of the model is generally better than the original or the
sine-term-only case, except for day four (20 September), when surface temperature exhibited non-sinusoidal behavior (Fig. 6b). The original deep soil temperature curve drifts downward then oscillates around a level that is about 0.6 K colder than the observed mean deep soil temperature. After including the lapse-rate related terms, the $T_2$ curve oscillates around a value closer to the true mean value of the deep soil temperature, giving a much closer fit to the observations. A more careful look shows that in the first half day, the difference between the modified and the original version is small, but the difference grows larger with time and reaches a steady level after a couple of days. This is so because the restore term in the original formulation acts to drag the deep layer temperature towards the mean surface temperature, which is about 298.6 K, instead of the seasonally mean temperature of 299.2 K in this case. Again, the inclusion of the lapse rate terms is most effective in improving the deep soil temperature prediction and the effect of including the sine-terms is negligible, results consistent with those of earlier cases.

For the spring of 2000, it was hard to find a weeklong period with totally clear sky conditions at the Norman site. For the 25-31 March period, the first four days generally satisfy the periodic atmospheric forcing conditions at the surface. At day five, there was a cold front passage that caused significant daytime temperature drop (Fig. 7a). Our calculation obtains the average soil temperatures using data from all six days, resulting in a smaller average temperature difference between the surface and deep layer. This explains at least partly why the deep soil temperature trend is not totally removed for the first four days of simulation (Fig. 7b) when the atmospheric forcing is rather periodic. Still, with the inclusion of the lapse rate term, the deep layer temperature error is reduced by about half (Fig. 7b and Table 3), or 2-3 degrees most of the time. The improvement in the skin temperature forecast is evident (see Table 3), though not as
large. It should be noted that the skin temperature forecast is not bad ($rms \approx 1.5$ K and maximum absolute error $\approx 3$ K for the revised model) despite the non-periodic behavior around day four.

To avoid the interference of the cold front passage in the later part of the period, we repeated the test using data from the first three days only, for which the mean temperature difference between the surface and the deep layer is 3.5 K instead of the 2 K. In this case, the revised scheme gives a much improved deep soil temperature forecast (figures not shown). The difference between the forecast and observations are within 1 K and no apparent phase error is found. The original formulation has a maximum error of more than 2 K. Note that because the difference in the e-folding scaling depth in the two cases, $T_2$ is not defined at exactly the same depth.

The numerical experiments for all four seasons share the commonality that for deep soil temperature, the most effective factor for improving the original force-restore model is to take into account of the seasonal-mean temperature lapse rate in the equations. The sine terms are of minimal significance and can therefore be neglected. The improvement to the skin temperature by the revised formulation is evident though not as dramatic.

6. Summary and Conclusions

In an attempt to clarify the definition and to improve the forecast of the 'deep layer' temperature in the soil-temperature force-restore model, we re-derived the equations starting from the heat transfer equation. Our derivation led to a 'deep layer' temperature, commonly denoted as $T_2$, that is defined as the soil temperature at depth $\pi d$ plus a transient term where $d$ is the e-folding damping depth of soil temperature diurnal oscillations (c.f. Eq. (30)). Corresponding to this new definition, the prediction equation for $T_2$, Eq. (31), has the same form as the commonly used one (e.g., NP79), except for an additional term involving the lapse rate of
the 'seasonal-mean' soil temperature and the damping depth $d$. A term involving the same also appears in the skin temperature prediction equation, Eq. (33), which also includes a transient term. The impact on the soil temperature prediction by these additional terms are tested against OASIS observations for four week long periods selected from out of four different seasons in 2000.

The results from these experiments show clear improvement in the prediction by our revised formulation of both skin and deep layer temperature, with the improvement in the latter being much more dramatic. The inclusion of the transient (sine) terms and the lapse-rate related terms are tested separately. It is found that the most effective modification that improves the deep soil temperature forecast is the one that takes into account of the seasonal-mean soil temperature lapse rate. The inclusion of the transient (sine) terms is of minimal impact therefore the transient terms in both the definition of $T_2$ and the skin temperature equation can be neglected without much impact, resulting in much cleaner equations. The recommended equations to use are therefore,

$$\frac{\partial T_s}{\partial t} = -\frac{1}{\tau}[T_2 - T_s + \pi d \gamma], \quad (34)$$

$$\frac{\partial T}{\partial t} = C_o (R_{net} - LE - H) - \frac{2\pi}{\tau} (T_s - T - \pi d \gamma). \quad (35)$$

where

$$T_2 = T(-\pi d, t). \quad (36)$$

It was found that without the inclusion of the $\pi d \gamma$ terms, the predicted deep-layer temperature would drift from observed initial value towards the mean of skin temperature, and such drift was found to be on the order of 5 K for winter and summer for the Norman site and the drift is of opposite sign in winter and summer. The inclusion of $\pi d \gamma$ terms virtually removes such a drift.
We note here that as hard as we have searched the literature, we did not find a rigorous derivation of equation like (34) for the prediction of $T_2$. Presenting a clean definition of $T_2$ and its prediction equation is one of the main contributions of this paper.

For dry conditions and periods with relatively strong daytime heating, our revision does not impact the skin temperature forecast as much but the improvement becomes more significant for wetter periods. The value of our revision is further supported by the results of our recent work in which the improved force-restore model, together with equations for soil moisture, is used to build an adjoint-based 4DVAR system for retrieving initial conditions of the soil model. The retrieval of the initial soil temperature and moisture is much better with the revised formulation.

We note here that it may be argued that if we kept the original formulation of the force-restore model and definite and initialize $T_2$ as the mean of skin temperature, the drift we observed should not happen. We have verified that this is true, which is not surprising because of the (correct) original definition by e.g., D75, of $\bar{T}$ towards which $T_s$ is restored. The key problem is that the $T_2$ used here is commonly considered the deep layer temperature (see review in Introduction) and its value is used in parameterizing vegetation processes that involve deep roots. For this reason, there is clear value in having available the deep layer temperature. Consensus of the modeling community is that “the root zone temperature must be included in any skillful model parameterization" (Roger Pielke Sr., personal communications). This will be especially relevant when the land surface scheme is used in coupled mode with the atmospheric components because there will be more feedback from the calculations of heat and moisture fluxes, including that from evapotranspiration.
To use the revised system given by Eqs. (34)-(36), one does need to determine $\pi d$ first. $\pi d$ is the difference between the 'seasonal mean' skin and deep layer temperature, with the latter is defined at depth $\pi d$. For NWP applications that range from few hours to a couple of weeks, these two values can be estimated from data of the proceeding days, given that deep layer soil moisture, a quantity that affects $d$ most, is slowly varying. The data can be either observed or model forecast values, with the former being preferred. For longer term applications, climatological values are suggested. Since the needed values are 'seasonal mean' ones, the use of climatological values is not as bad as it may sound. Parameter $d$ can be determined by the amplitude-phase method, as done in this paper.

Finally, we note that even though our numerical tests were performed for Norman site only, the results should be valid for other sites too because the underlining physics are the same. Furthermore, the land surface processes should be tested within the integrated land-atmospheric system. In fact, experiments have been performed within the ARPS in a coupled mode and our modifications are found to improve the forecast of the overall system as well.
Appendix. The amplitude-phase method for determining e-folding damping depth of soil temperature

Suppose four observations are taken regularly during each day, at 6 hour intervals and at two different depths, it is straightforward to get the following estimate for soil thermal diffusivity (Sellers 1965):

\[ K_T = \frac{4\pi(d_2 - d_1)^2}{\tau} \left\{ \ln \left[ \frac{[T_1(d_1) - T_2(d_1)]^2 + [T_3(d_2) - T_4(d_2)]^2}{[T_1(d_2) - T_2(d_2)]^2 + [T_3(d_2) - T_4(d_2)]^2} \right] \right\}^{-2} \]

Where \( d_1 \) and \( d_2 \) are two different depths, \( \tau \) is period of daily cycle, i.e., 86400 seconds, \( T_1, T_2, T_3 \) and \( T_4 \) are temperature measurements at 00, 06, 12, and 18UTC, respectively. The e-folding damping depth can then be calculated according to \( d = \sqrt{K_T \tau / \pi} \).

This method works better for clear sky conditions. Also, to have a better result, \( d_1 \) and \( d_2 \) should be separated as far as possible but all within the damping depth \( d \). Since daily sinusoidal cannot penetrate beyond 60 cm for normal soils, it is suggested that the two selected depths should all be limited to within 60 cm depth.

For example, on March 25, 2000, from OASIS measurements at NORM, 5 cm soil temperatures at 00, 06, 12 and 18 UTC are 292.52, 290.28, 288.61 and 289.90 K, respectively, and 25 cm soil temperatures at are 288.83, 289.30, 289.08 and 288.40K, respectively. Using the formula, one obtains \( K_T = 7 \times 10^{-7} \) m² s⁻¹. Thus \( d = 13.87 \) cm, and \( \pi d = 43.6 \) cm. Soil temperature damping depths for the four seasons are obtained by applying this method to each day and then taking then average.
Acknowledgement

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Figure 1. OASIS observed soil temperatures for 7 day periods starting from August 12 (a) and
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Figure 2. Example damping depths (SD) for soil temperature calculated using amplitude-phase
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Figure 3. OASIS measurements for the period of August 13 through August 17, 2000. In panel
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Figure 4. Model predicted and observed temperatures for the 6-day summer period, starting from
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thin solid line is for the original force-restore model (original case), thin solid line with
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without marker refers to the ‘sine term only’ case, and the long dashed line refers the case
with a complete implementation of our modifications to the force-restore model, the
‘revised’ case. The long-short dash line is for OASIS measurements. Panels (a) and (b) represent surface and deep soil temperature, respectively.

Figure 5. As Fig. 4 but for the six-day winter period starting from 00UTC, January 05, 2000. The observed surface temperature series are estimated using OASIS soil temperature measurements at 05 and 25 cm depths and the amplitude-phase method.

Figure 6. As Fig. 4 but for the six-day early fall period starting from 00UTC, September 17, 2000.

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Table 4. Error statistics of $T_s$ predictions with different formulations for the winter period
Table 1. General information of Norman super OASIS site

<table>
<thead>
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<th>Site ID</th>
<th>Location (deg)</th>
<th>Elevation (m)</th>
<th>Slope (deg)</th>
<th>Land use</th>
<th>Soil type</th>
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<td>360.0</td>
<td>0.0</td>
<td>Scrub</td>
<td>Silty clay</td>
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</table>
Table 2. Leaf area index (LAI), difference between seasonal mean surface and deep layer temperature ($\pi d\gamma$), observed period-average amplitude of skin temperature ($A_0$) and of deep-layer temperature ($A_{\text{deep}}$), and damping scale depth ($d$) in the four study periods.

<table>
<thead>
<tr>
<th>Period</th>
<th>LAI</th>
<th>$\pi d\gamma$ (K)</th>
<th>$A_0$ (K)</th>
<th>$A_{\text{deep}}$ (K)</th>
<th>$d$ (m)</th>
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<td>9.62</td>
<td>0.74</td>
<td>0.155</td>
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<tr>
<td>Summer (8/12-8/18)</td>
<td>0.60</td>
<td>4.70</td>
<td>15.52</td>
<td>1.34</td>
<td>0.148</td>
</tr>
<tr>
<td>Fall (9/17-9/23)</td>
<td>0.50</td>
<td>-0.51</td>
<td>12.50</td>
<td>1.23</td>
<td>0.164</td>
</tr>
<tr>
<td>Winter (1/5-1/11)</td>
<td>0.06</td>
<td>-2.17</td>
<td>2.87</td>
<td>0.61</td>
<td>0.146</td>
</tr>
</tbody>
</table>
Table 3. Error statistics of $T_2$ predictions for different formulations and periods

<table>
<thead>
<tr>
<th>Period and mean $T_2$</th>
<th>Formulation</th>
<th>$rms$ (K)</th>
<th>Mean Bias Error(K)</th>
<th>Max. Absolute Error (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Summer</strong> $\bar{T}_2 = 301.98$</td>
<td>Original</td>
<td>3.3246</td>
<td>3.1323</td>
<td>5.00</td>
</tr>
<tr>
<td></td>
<td>Revised</td>
<td>0.6839</td>
<td>-0.3237</td>
<td>1.89</td>
</tr>
<tr>
<td></td>
<td>Lapse rate only</td>
<td>0.7019</td>
<td>-0.3699</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>Sine term only</td>
<td>3.3772</td>
<td>3.1866</td>
<td>5.12</td>
</tr>
<tr>
<td><strong>Winter</strong> $\bar{T}_2 = 282.52$</td>
<td>Original</td>
<td>1.439</td>
<td>-1.282</td>
<td>2.69</td>
</tr>
<tr>
<td></td>
<td>Revised</td>
<td>0.920</td>
<td>0.10</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>Lapse rate only</td>
<td>0.895</td>
<td>0.14</td>
<td>1.57</td>
</tr>
<tr>
<td></td>
<td>Sine term only</td>
<td>1.434</td>
<td>-1.268</td>
<td>2.73</td>
</tr>
<tr>
<td><strong>Fall</strong> $\bar{T}_2 = 298.59$</td>
<td>Original</td>
<td>1.086</td>
<td>-0.64</td>
<td>3.50</td>
</tr>
<tr>
<td></td>
<td>Revised</td>
<td>1.043</td>
<td>0.43</td>
<td>2.60</td>
</tr>
<tr>
<td></td>
<td>Lapse rate only</td>
<td>1.029</td>
<td>0.39</td>
<td>2.59</td>
</tr>
<tr>
<td></td>
<td>Sine term only</td>
<td>1.074</td>
<td>-0.60</td>
<td>3.50</td>
</tr>
<tr>
<td><strong>Spring</strong> $\bar{T}_2 = 286.98$</td>
<td>Original</td>
<td>2.20</td>
<td>1.739</td>
<td>3.87</td>
</tr>
<tr>
<td></td>
<td>Revised</td>
<td>1.54</td>
<td>0.062</td>
<td>3.09</td>
</tr>
<tr>
<td></td>
<td>Lapse rate only</td>
<td>1.55</td>
<td>0.021</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td>Sine term only</td>
<td>2.24</td>
<td>1.783</td>
<td>3.93</td>
</tr>
</tbody>
</table>
Table 4. Error statistics of $T_s$ predictions with different formulations for the winter period

<table>
<thead>
<tr>
<th>Winter $\bar{T}_i = 280.352$</th>
<th>$rms$ (K)</th>
<th>Mean Bias Error (K)</th>
<th>Max. Absolute Error (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>1.06</td>
<td>0.85</td>
<td>2.53</td>
</tr>
<tr>
<td>Revised</td>
<td>0.73</td>
<td>0.34</td>
<td>2.33</td>
</tr>
<tr>
<td>Lapse rate only</td>
<td>0.77</td>
<td>0.36</td>
<td>2.40</td>
</tr>
<tr>
<td>Sine term only</td>
<td>1.04</td>
<td>0.84</td>
<td>2.63</td>
</tr>
</tbody>
</table>
Figure 1. OASIS observed soil temperatures for 7 day periods starting from August 12 (a) and January 5, 2000 (b), respectively. Solid lines designate soil temperature skin temperature for August case and that at 5 cm depth for January case, dot dash lines are those of 25 cm, and dot dot dash lines are the soil temperature at 60 cm. The weekly averages are shown as horizontal lines of the same styles. The values of the period-mean temperatures are labeled.
Figure 2. Example damping depths (SD) for soil temperature calculated using amplitude-phase method. The damping depths are calculated from soil temperatures at 5 and 25 cm depths. The starting point are March 24 (cross), August 12 (thick solid), September 17 (dot dash) and January 9 (star) for selected period in spring, summer, fall and winter seasons, respectively. Amplitude-phase method worked the best for the August 12, 2000 period.
Figure 3. OASIS measurements for the period of August 13 through August 17, 2000. In panel (a), SWin is incoming shortwave flux, SWout is reflected shortwave flux, LWout means outgoing longwave radiation, LWin means downward longwave radiation, and Rnet is the net radiation. In panel (b), Tair is air temperature at 2 m, Tskin is skin temperature measured by the infrared instrument, and T05 is soil temperature at 5 cm depth.
Figure 4. Model predicted and observed temperatures for the 6-day summer period, starting from 00UTC, August 12, 2000, using several different versions of the force-restore model. The thin solid line is for the original force-restore model (original case), thin solid line with cross markers is for the ‘lapse-rate only’ case (see text for definition), the dotted line without marker refers to the ‘sine term only’ case, and the long dashed line refers the case with a complete implementation of our modifications to the force-restore model, the ‘revised’ case. The long-short dash line is for OASIS measurements. Panels (a) and (b) represent surface and deep soil temperature, respectively.
Figure 5. As Fig. 4 but for the six-day winter period starting from 00UTC, January 05, 2000. The observed surface temperature series are estimated using OASIS soil temperature measurements at 05 and 25 cm depths and the amplitude-phase method.
Figure 6. As Fig. 4 but for the six-day early fall period starting from 00UTC, September 17, 2000.
Figure 7. As Fig. 4 but for the six-day spring period starting from 00UTC, 25 March, 2000.