Theories on the Optimal Conditions of Long-Lived Squall Lines

• References:


Theories of Intense / Long-lived Squall Lines

- Rotunno, Klemp and Weisman (1987) – RKW Theory

Perspective

- The RKW theory for long-lived squall lines, though widely cited, remains controversial
- We try to look at more careful look at the theory here
Key Findings of Thorpe, Miller and Moncrieff 1982 - TMM82

$P_0$ is quasi-stationary and produced maximum total precipitation

Thorpe, Miller and Moncrieff 1982 - TMM82

- All cases required strong low-level shear to prevent the gust front from propagating rapidly away from the storm;

- TMM concluded that low-level shear is a desirable and necessary feature for convection maintained by downdraught.
RKW Theory

RKW's Vorticity Budget Analysis to Obtain the 'optimally' balanced condition

c. The cold pool with and without low-level shear

To obtain a quantitative criterion for the low-level shear needed to "balance" a cold pool, and to distinguish clearly the physical difference between a cold pool spreading in a no-shear environment from that in an environment with low-level shear, we perform the following analysis. Equation (1) may be written, without further approximation, as

\[
\frac{\partial \sigma}{\partial t} = -\frac{\partial}{\partial x} (w\nu) - \frac{\partial}{\partial z} (w\nu) - \frac{\partial \beta}{\partial x}.
\]  

(4)

We fix ourselves in a frame of reference moving with the edge of the cold air and integrate Eq. (4) from a point to the left, \( x = L_s \), to a point to the right, \( x = R \), at the cold-air edge, and from the ground to some level, \( z = d \), and obtain

\[
\frac{\partial}{\partial x} \int_0^d \sigma dz + \int_0^d (w\nu) dz - \int_0^d (w\nu) dz - \int_0^d \beta dz.
\]

(5)

\[
\frac{\partial}{\partial x} \left( \int_0^d \sigma dz \right) + \int_0^d (w\nu) dz + \int_0^d B d dz.
\]  

Since we are looking for a steady balance, we set the tendency to zero. Also, in the circumstances investigated herein, there is negligible buoyancy of the air approaching the cold pool, thus we take \( \beta = 0 \). Finally, we recognize that \( \sigma = \sigma_z \) away from the edge of the cold air. Under these conditions, Eq. (5) becomes

\[
0 = \left( \frac{w_{1,d} - w_{2,d}}{2} \right) \left( \frac{w_{1,d} - w_{2,d}}{2} \right) - \int_0^d (w\nu) dz + \int_0^d B dz.
\]  

(6)

We consider the situation where the cold air is stagnant (relative to the cold-air edge), so that \( w_{1,d} = 0 \), and restricted to a height, \( z = H \), where \( H < d \). With these simplifications, Eq. (6) becomes

\[
0 = \frac{w_{2,d}^2}{2} - \left( \frac{w_{1,d}^2}{2} \right) + \int_0^d (w\nu) dz + \int_0^d B dz.
\]  

(7)
RKW’s Vorticity Budget Analysis to Obtain the ‘optimally’ balanced condition

Now consider a situation where there is low-level shear as illustrated in Fig. 18d. We look for the optimal state where the low-level flow is turned by the cold pool in such a way as to exit as a vertically oriented jet. Thus we set $u_{L,D}$, $u_{R,D}$ and $\int_0^L (w_0) d\chi = 0$ in Eq. (7) to obtain

$$\Delta u = c,$$

where $\Delta u = u_{R,D} - u_{R,B} = -u_{L,D}$. (10)

Although this formula is almost identical to Eq. (8), the physical interpretation is fundamentally different. In light of Eq. (5), Eq. (10) states that the import of the positive vorticity associated with the low-level shear just balances the net buoyant generation of negative vorticity by the cold pool in the volume.

In the above, $c$ is defined by

$$c^2 = 2 \int_0^L (-B_c) d\chi = 2 g \frac{\Delta \rho}{\rho_0} H = 2 g \frac{\Delta \rho}{\rho_0} H$$

which is exactly the density current propagation speed we derived earlier! Therefore the optimal condition obtained based on RKW’s vorticity budget analysis says that the shear magnitude in the low-level inflow should be equal to the cold pool propagation speed.

RKW Optimal Shear Condition
Based On Vorticity Budget Analysis

\[
\Delta u = -u_{R,0} = c
\]

\[
\frac{\partial(u\eta)}{\partial x} + \frac{\partial (w\eta)}{\partial z} = -\frac{\partial B}{\partial x}
\]
RKW Optimal Shear Condition

Vorticity Budget Analysis of RKW

\[ \frac{\partial (u\eta)}{\partial x} + \frac{\partial (w\eta)}{\partial z} = -\frac{\partial B}{\partial x} \]

\[ \int_0^L (w\eta)_d d\eta = 0 \]
\[ \int_0^L (u\eta)_d d\eta = 0 \]
\[ \int_0^L B_d d\eta \approx -g\Delta \theta / \theta, H = -c^2 / 2 \]
\[ \int_0^L (u\eta)_d d\eta = -\frac{u_{x,0}^2}{2} \]

\[ \Delta u = -u_{R,0} = c \]

RKW Numerical Experiment of a Spreading Cold Pool

Area To be Shown
RKW Density Current Simulation Results

Circulation are induced by cold pool propagation, NOT vorticity or shear

Questions

- Does the Low-level Inflow have to Contain Shear or Vorticity?
- Can long-lived squall lines be supported even without shear in the lowest few kms of the troposphere?
First, Theoretical Models of Density Currents

Inviscid Steady-state Density Current Models in Variable Vertical Shear
(Xue, Xu, Droegemeier, 1997 JAS; Xue 2000 QJ)

Two shear layers allow for more flexibility with inflow configuration, e.g.,
Summary of Theoretical Model Results

- Positive inflow shear, either at low-levels or at upper-level, supports a deep cold pool, steep frontal interface, and therefore a deep updraft.

- A deep updraft can be supported even without low-level inflow shear

- The RKW Theory, however, considers the low-level shear essential for deep updraft to form

Results from a Time-dependent Numerical Model
Zero Upper-Level Shear, Different Low-Level Shear

\[ \alpha = -1 \]
\[ \beta = 0 \]

No Cold Pool Induced Internal Circulation

Figures Plotted to Scale

Zero Low-level Shear with Opposite-Sign Upper-Level Shears

\[ \alpha = 0 \]
\[ \beta = +2 \]

\[ \beta = -2 \]
\[ \alpha = 0 \]

Cold pool structure strongly influenced by upper-level shear too; not considered by RKW
Conclusions from Numerical Experiments

• The overall flow is dictated by the overall vorticity distribution in the domain.

• Low-level shear is not necessary to establish a deep cold pool, contrary to what RKW theory suggests.

Numerical Simulations of Squall Lines in Support of Our Last Argument
2-D Squall Line Simulations of Xue (1989, 1991)

Fig.5.14. Schematic diagram of the inflow profiles for squall line experiments. (a) shows step type inflow profiles which have uniform speed therefore zero vorticity below 1.5 km, (b) shows shear inflow profiles which have a constant vorticity.

Step Profile Cases

Fig.5.11. The surface observed case (arrows) for experiments: (a) SLEIA, (b) SLEIB, (c) SLEIC, and (d) SLEID respectively.
Low-level Linear Shear Inflow Cases (0-4 hours)

(a) Linear Low-level Shear

(b) 12 m/s

(c) 15 m/s

(d) 20 m/s

(e) 28 m/s

Line is Quasi-Stationary
Conceptual Model of Xue (1991)

\[ c = \text{cloud-relative cold pool speed} \]

RKW Simulation Results

Optimal Condition

- No need for ‘Cold Pool Circulation’ or ‘Inflow Shear’
Conclusions of Xue et al

- The shear between the ground level and the steering level is a more important factor in determining the propagation of cold pool relative the cloud system above.

- The updraft orientation is a function of vorticity distribution throughout the entire domain, and a global solution should be obtained by solving the vorticity equation with proper boundary conditions.

- To determine the behavior of the updraft branch of inflow over the cold pool, we need to know the vorticity distribution in the entire domain and the boundary conditions. Vorticity in an air parcel alone cannot tell us its trajectory.

Conclusions Xue et al – continued…

- In general, a cold pool that propagates at the speed of, or slightly faster than, the steering level wind (or the propagation speed of a cloud) creates an optimal condition for intense, long-lasting squall lines.

- The role of the low-level system relative inflow is to prevent the cold pool from propagating away from the overhead cloud. The surface system-relative wind speed, rather than the shear, is most important.

- Our optimal condition based on front propagation speed and surface and steering level winds makes few assumptions and is more generally valid.