P11.3 EFFECTS OF NESTING FREQUENCY AND LATERAL BOUNDARY PERTURBATIONS ON THE DISPERSION OF LIMITED-AREA ENSEMBLE FORECASTS

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1. Introduction

It is known that "one-way" lateral boundary conditions constrain the growth of initial perturbations in limited-area ensemble forecasts (Paegle et al., 1997, references therein). External boundary conditions (LBCs) typically lack fine-scale features, and in the case of ensemble forecasts, also lack consistent perturbations. Perturbations growing on the nested domain become displaced by the coarsely resolved LBCs (Errico and Baumhefner, 1987; Vukicevic and Paegle, 1989) while the domain size itself determines the maximum wavelength attainable by the perturbations (Vukicevic and Errico, 1990).

Another aspect of the boundary condition problem that has previously received little attention is the impact of LBC update interval (Warner et al., 1997). Commonly used linear interpolation between relatively infrequent LBC updates acts as a filter that exacerbates the scale deficiency problem.

Short-range ensemble forecast experiments have shown that the ensembles often are under dispersive. That is, the verifying analysis does not fall within the range of possibilities forecast by the ensemble. Du and Tracton (1999) found that a regional ensemble with a larger domain produces greater spread than does an ensemble with a smaller domain. Furthermore, they found that the contribution of different LBCs to ensemble spread increases with time while that of initial condition perturbations decreases with time. These and other similar results (Hamill and Colucci, 1997; Hou et al., 2001; Stensrud et al., 2000) demonstrate that, with time, the spread of the LAM ensemble forecast becomes increasingly determined by the spread in the global ensemble as high frequency and small scale components are "swept" from the LAM domain.

To help restore the error variance lost at small scales, we propose to apply coherent perturbations to the LBCs at scales unresolved by the external model. The amplitude of such perturbations will be chosen to mimic the exponential error growth curves generated from simulations in unbounded domains. The use of more frequently updated boundary conditions and the inclusion of small-scale boundary perturbations will be shown to enhance the dispersion for limited-area ensemble forecasts.

In this paper, necessary analysis tools are introduced along with the parameterized potential vorticity model used in this study. We will then present results on the effect of LBC update interval on the nested-grid ensemble dispersion. Other results will be presented at the conference.

2. Ensemble MSE and Dispersion

Before introducing the underlying hypothesis for this work in section 3., it is useful to review standard ensemble statistical methods. Consider N members of an ensemble forecast $\mathbf{f}(t)$ and corresponding analyses $\mathbf{a}(t)$ given as vectors on a p-element grid. Although identical in a regular ensemble configuration, let the analyses corresponding to each forecast be unique for generality. The ensemble mean-square error (MSE) is

$$V^2 = \frac{1}{N}\sum_{i=1}^{N} \|\mathbf{f}_i - \mathbf{a}_i\|^2 \qquad (1)$$

where $\|\cdot\|^2$ is the average sum of squares (dot product) over the grid (Stephenson and Doblas-Reyes, 2000). To gain further insight, add and subtract the ensemble mean forecast $\bar{\mathbf{f}}$ and analysis $\bar{\mathbf{a}}$ inside (1), then expand and manipulate so that

$$V^2 = \frac{1}{N}\sum_{i=1}^{N} \|\mathbf{f}_i - \bar{\mathbf{f}}\|^2 \;+\; \frac{1}{N}\sum_{i=1}^{N} \|\mathbf{a}_i - \bar{\mathbf{a}}\|^2 \qquad (2)$$
$$-\;\frac{2}{N}\sum_{i=1}^{N} \frac{1}{p}(\mathbf{f}_i - \bar{\mathbf{f}})\cdot(\mathbf{a}_i - \bar{\mathbf{a}}) \;+\; \|\bar{\mathbf{f}} - \bar{\mathbf{a}}\|^2.$$

Note that an analogous expression may be found by adding and subtracting the grid mean (scalar)

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forecast and analysis for each ensemble member, multiplied by the identity vector. The variance and covariance terms in (2) may be combined to write the *total biased error variance*

$$\sigma^2 \equiv V^2 - \|\bar{\mathbf{f}} - \bar{\mathbf{a}}\|^2 = \frac{1}{N}\sum_{i=1}^{N}\left\|(\mathbf{f}_i - \mathbf{a}_i) - \overline{(\mathbf{f} - \mathbf{a})}\right\|^2. \tag{3}$$

Ensemble forecasts usually are verified against one analysis. In that case, $\mathbf{a}_i = \bar{\mathbf{a}} = \mathbf{a}$, and both (2) and (3) reduce to

$$\begin{aligned} V^2 &= \frac{1}{N}\sum_{i=1}^{N}\|\mathbf{f}_i - \bar{\mathbf{f}}\|^2 \; + \; \|\bar{\mathbf{f}} - \mathbf{a}\|^2 \\ &= D^2 + \|\bar{\mathbf{f}} - \mathbf{a}\|^2, \end{aligned} \tag{4}$$

where D^2 is the ensemble dispersion, or spread. This result shows that the squared error of the ensemble mean is less than the ensemble MSE V^2 because ensemble dispersion allows unpredictable components of flow to be averaged out in the ensemble mean (Leith, 1974; Stephenson and Doblas-Reyes, 2000). Note that when comparing all ensemble members to one analysis as in (4), $D^2 = \sigma^2$.

As initial forecast errors grow with time, ensemble forecast members become uncorrelated with analyses and their covariance is zero. If forecasts are unbiased and have the same variance as the analyses, then the expected value of (2) converges as $V^2 = 2D^2$. This is the classic result obtained by Leith (1974), and motivates the common practice that total error variances be normalized by the climate variance of analyses.

3. Impact of LBCs on Total Error Variance

Equation (4) reveals a linear, additive relation between V^2 and D^2, thereby providing a direct link between ensemble MSE and ensemble dispersion. The following argument highlights changes in ensemble MSE (hence, ensemble dispersion) resulting from coarsely resolved lateral boundary forcing.

Suppose we decompose the forecast and analysis fields on the nested domain into longwave and shortwave components so that $\mathbf{f} = \mathbf{f}_l + \mathbf{f}_s$ and $\mathbf{a} = \mathbf{a}_l + \mathbf{a}_s$. The shortwave components are those scales not resolved by the external model, while longwave components are those scales commonly resolved on both external and internal grids.

Let the shortwave components of the forecast be decomposed further to include losses by advective "sweeping" and regeneration by nonlinear dynamic downscaling so that $\mathbf{f}_s = \mathbf{f}_\alpha + \mathbf{f}_\eta$. With time, $\mathbf{f}_\alpha$ components are displaced by the coarsely resolved LBCs, suggesting the assumption that their amplitudes tend toward zero. Note that temporal interpolation of LBCs acts as a filter that may lengthen the scale $\mathbf{f}_\alpha$ components. Following from (1) while dropping i subscripts, the resulting impact of LBCs on the ensemble MSE is

$$\begin{aligned} V^2 &= \frac{1}{N}\sum_{i=1}^{N}\|\mathbf{f}_l + \cancelto{0}{\mathbf{f}_\alpha} + \mathbf{f}_\eta - \mathbf{a}_l - \mathbf{a}_s\|^2 \\ &= \frac{1}{N}\sum_{i=1}^{N}\|(\mathbf{f}_l - \mathbf{a}_l)\|^2 + \frac{1}{N}\sum_{i=1}^{N}\|\mathbf{f}_\eta - \mathbf{a}_s\|^2 \\ &\quad + \frac{2}{N}\sum_{i=1}^{N}\frac{1}{p}(\mathbf{f}_l - \mathbf{a}_l)\cdot(\mathbf{f}_\eta - \mathbf{a}_s). \end{aligned} \tag{5}$$

This equation provides insight to the hypothesis that limited-area models are able to produce fine-scale information not present in the coarse-grid external model. The extent to which small-scale wave components are skillfully regenerated will depend on the size and location of the nested domain and on the LBC update interval. Thus, an important aspect of this work is to determine the relative contribution of the regenerated variance compared to the total variance.

To quantify the contributions of each term in (5), variances are computed spectrally using the method outlined by Errico (1985). Specifically, if $F(k)$ is the discrete Fourier transform of the error field, then (3) may be obtained as

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{K-1} 2\left|F_i(k)\right|^2, \tag{6}$$

where $k = 1, \ldots, K-1$ are the set of Nyquist resolved wavenumbers.

Skill is measured by normalizing variances within different wavelength bands by the climatological variances obtained spectrally. When normalized in this manner, results may be compared to other studies such as Laprise et al. (2000).

4. Parameterized Potential Vorticity Model

A simplified model is used for this work to help isolate *only those errors associated with LBCs* in a controlled and efficient manner. Emphasis is directed towards large scale mid-tropospheric flow since these are the patterns that are important for accurate placement of developing mesoscale

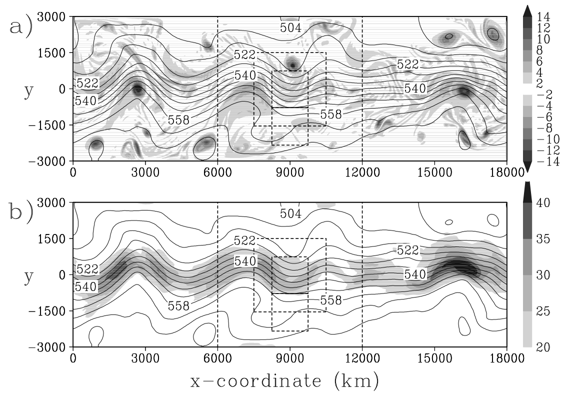

Figure 1: PPV model simulation on a 25 km grid, 15 days after initializing with a perturbed shear flow. Streamfunction contours are shown in both panels ($\psi \times 10^6$ m^{2}s^{-1}). Shading in (a) indicates relative vorticity greater than $\pm 2 \times 10^{-5}$ s^{-1}. Shading in (b) shows wind speeds greater than 20 m s^{-1}. Boxes show outlines of four different sub-grids used for singly-nested model configurations.

and smaller features (Paegle et al., 1997). Furthermore, we wish to exclude from consideration in this work the hypothesis that predictability is enhanced under the influence of surface forcing (Van Tuyl and Errico, 1989; Warner et al., 1989).

The single-level grid point model runs on a midlatitude beta channel and is based on an approximation of the quasi-geostrophic potential vorticity equation. Let $\xi \equiv \zeta - \lambda^2\psi$ define a parameterized relative potential vorticity, where ζ is the relative vorticity, ψ is the streamfunction, and $\lambda = 7.071 \times 10^{-7}$ m^{-1} is an inverse length scale based on the Rossby radius of deformation (Holton, 1979). The parameterization represents the *first-order* effects of vertical motions in a baroclinic atmosphere (vortex stretching). The parameterized potential vorticity (PPV) model is

$$\frac{\partial \xi}{\partial t} = -\frac{\partial \psi}{\partial x}\frac{\partial \xi}{\partial y} - \frac{\partial \psi}{\partial y}\frac{\partial \xi}{\partial x} - \beta\frac{\partial \psi}{\partial x} - \nu\nabla^4\xi. \qquad (7)$$

If $\lambda = 0$ the PPV model reduces to the standard barotropic vorticity model, except for the 4th order numerical diffusion term. An example of the vorticity and streamfunction fields produced by the PPV model is shown in Fig. (1).

5. Impact of Nesting Interval

The impact of nesting interval is explored in a perfect model configuration following the method described by Laprise et al. (2000). Perfect initial conditions and LBCs are provided by an external control simulation that may be low-pass filtered to remove power at small scales. Initial conditions are not perturbed, so the only source of error is that due to interpolation between available LBC updates and the wave absorbing zone used for "one-way" nesting. Normalized variances for the resulting fields were computed spectrally as discussed above and averaged over 100 cases. Dispersion statistics from planned ensemble experiments should appear similar to results shown here.

A frequency analysis of a time series of ξ at a single grid point reveals that 99.6% of the variance is explained by waves having periods longer than 3 hours. The perfect model simulations were consistent, showing that boundary-induced errors were minimal when LBCs are updated at intervals of 3 hours or less. Errors become much larger when LBCs are updated every 6 hours as shown in Fig. 2.

In an otherwise perfect model simulation, LBC interpolation causes inconsistencies between the external and internal fields. Attempts to smooth the discontinuity across the boundary zone generates a small scale wave that enters the nested domain. From there, the disturbance may either dissipate or amplify, depending on its amplitude and stability of the ambient flow. Compared to the large domain, boundary-induced errors on the medium and small (center) domains grow faster with time and stabilize at different saturation points (Fig. 2). On the small (south) domain, located outside the main jet profile, the flow is weak and errors grow more slowly with time.

In other experiments, LBCs were low-pass filtered to examine the ability of the nested grid to regenerate short wavelengths, even in the presence of continued coarse-resolution boundary forcing. Figure (3) shows some recovery of skill at small scales due to dynamic downscaling. However, after four days the errors have about the same level of error as the previous case, indicating that LBC errors have swept through the domain. In all cases, longwave components are minimally affected since the LBC is well-sampled at these scales.

6. Summary

The goal of this work is to quantify the extent to which coarsely resolved LBCs modify error variances (hence, ensemble dispersion) for limited-domain solutions. If it can be shown that ensemble dispersion is deficient due, in part, to coarsely

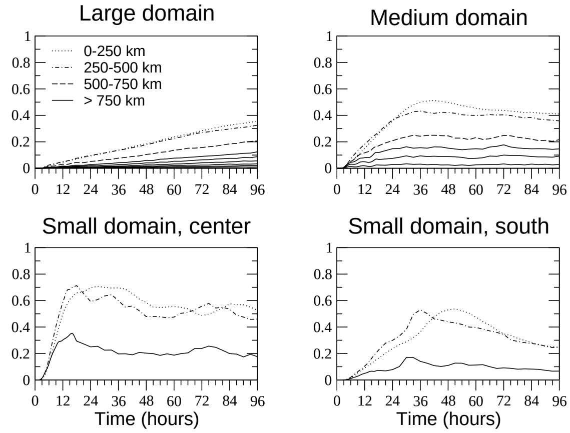

Figure 2: Normalized vorticity variances at 250 km wavelength intervals for perfect model simulations run on a 50-km grid with LBCs low-pass filtered to exclude wavelengths $<$400-km. LBCs are updated every 6 hours.

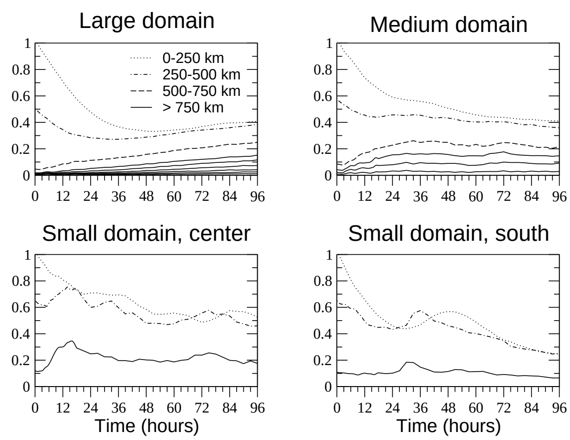

Figure 3: As in Fig. 2, except initial conditions are also low-pass filtered.

resolved LBCs, then a procedure will be developed to add small scale LBC perturbations using parametric exponential error growth curves.

Error growth curves obtained from ensemble simulations on the external periodic channel domain were not yet complete at the time of writing. These results, along with a more developed method for perturbing LBCs at unresolved scales using error growth curves will be presented at the conference.

Acknowledgement The first author is supported by a Williams Graduate Research Fellowship through the Center for Analysis and Prediction of Storms at the University of Oklahoma. Computational support was provided, in part, by NSSL.

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