P11.5 USING A LOW-ORDER MODEL TO CHARACTERIZE TORNADOES IN MULTIPLE-DOPPLER RADAR DATA

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1. INTRODUCTION

Several methods have been used to fit low-order models of velocity fields to radial velocity data from a single Doppler radar. The Velocity-Azimuth Display (VAD; Lhermite and Atlas 1962) and Volume Velocity Processing (VVP; Waldteufel and Corbin 1979) techniques use spatially-linear wind models, and are often successful as long as the conditions for their validity are approximately met. A relatively new class of low-order wind models is being used to analyze hurricanes and tornadoes: the Velocity Track Display (VTD, Lee et al. 1994), Extended Velocity Track Display (EVTD; Roux and Marks 1996) and the Ground-Based Velocity Track Display (GBVTD, Lee at al. 1999, Lee and Marks 2000). These methods seek to obtain the key axisymmetric and low-order asymmetric components of the tangential wind from a harmonic analysis of radial velocity data from a single Doppler radar.

A new multiple-Doppler radar analysis technique is being developed for the objective detection and characterization of tornadoes. Observed radial wind data is fit to an analytical low-order model of a tornado in order to retrieve key characteristics of the wind field, particularly the location, movement and strength of the tornado. This technique is being designed for use within the network of densely-spaced, adaptively scanning radars envisioned by the CASA (Collaborative Adaptive Sensing of the Atmosphere) Engineering Research Center (McLaughlin et al. 2005).

2. LOW-ORDER MODEL

The tornado model is a combination of four idealized flow fields: a uniform flow, linear shear flow, linear divergence flow, and modified combined Rankine vortex (representing the tornado). The latter three fields are allowed to translate with different speeds and directions. The model is described using 19 parameters, including vortex center, translation, and radius, as well as maxima and decay exponents for the tangential and radial wind components (Table 1). Model winds are projected in the direction of the radar(s) to obtain the model radial wind, V_r:

$$V_r =$$

$$\frac{(x_n - x)*[a + b(y - v_{ts}t) + c(x - u_{td}t) + \frac{V_R}{R}(x - x_0 - u_{tr}t) - \frac{V_T}{R}(y - y_0 - v_{tr}t)] + (y_n - y)*[d + e(x - u_{ts}t) + f(y - v_{td}t) + \frac{V_R}{R}(y - y_0 - v_{tr}t) + \frac{V_T}{R}(x - x_0 - u_{tr}t)]}{\sqrt{(x_n - x)^2 + (y_n - y)^2}}$$

$$\text{for } \sqrt{(x - x_0 - u_{tr}t)^2 + (y - y_0 - v_{tr}t)^2} < R$$

$$V_r =$$

$$\frac{(x_n - x)*\left\{a + b(y - v_{ts}t) + c(x - u_{td}t) + \frac{RV_R(x - x_0 - u_{tr}t)}{\left[(x - x_0 - u_{tr}t)^2 + (y - y_0 - v_{tr}t)^2\right]^{\beta}} - \frac{RV_T(y - y_0 - v_{tr}t)}{\left[(x - x_0 - u_{tr}t)^2 + (y - y_0 - v_{tr}t)^2\right]^{\alpha}}\right\} + (y_n - y)*\left\{d + e(x - u_{ts}t) + f(y - v_{td}t) + \frac{RV_R(y - y_0 - v_{tr}t)}{[(x - x_0 - u_{tr}t)^2 + (y - y_0 - v_{tr}t)^2]^{\beta}} + \frac{RV_T(x - x_0 - u_{tr}t)}{[(x - x_0 - u_{tr}t)^2 + (y - y_0 - v_{tr}t)^2]^{\alpha}}\right\}}{\sqrt{(x_n - x)^2 + (y_n - y)^2}}$$

$$\text{for } \sqrt{(x - x_0 - u_{tr}t)^2 + (y - y_0 - v_{tr}t)^2} \geq R$$

where (x_n, y_n) are the radar locations. Figure 1 shows an example of a flow field generated by the model.

Parameter	Description
a,d	uniform flow
b,e	shear field strength
c,f	divergence field strength
u_{ts}, v_{ts}	shear field translation
u_{td}, v_{td}	divergence field translation
x_0, y_0	Rankine-combined vortex center
R	radius of maximum wind (vortex)
V_R, V_T	maximum tangential, radial wind
u_{tr}, v_{tr}	vortex translation
α, β	tangential, radial wind decay exponents

Table 1. Description of parameters in low-order model.

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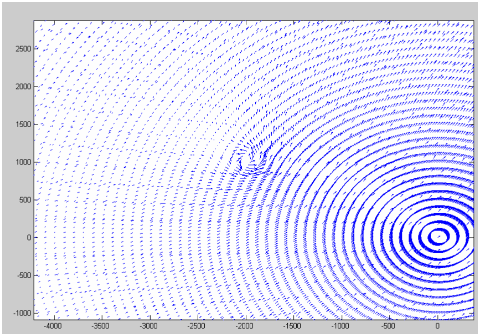

Fig. 1. Example wind field generated by model. Axes indicate distance (in meters) from radar at (0,0).

3. OPTIMIZATION

A cost-functional J accounts for the discrepancy between model and observed radial wind:

$$J \equiv \sum_{n=1}^{N} \int_{A_n} [V_{r_n}^{obs} - V_{r_n}]^2 \, dA_n$$

where $V_{r_n}^{obs}$ is the radial wind data from the n'th radar, V_{r_n} is the model radial wind, and A_n is the four-dimensional domain of integration for each radar. J is evaluated over both space and time so that observations can be used at the time they were acquired, thus bypassing the need for time interpolation, moving reference frames or other ad-hoc procedures. Since the model radial wind is calculated from a set of nonlinear equations in the unknown parameters, J generally cannot be minimized by linear minimization methods. Instead, the Polak-Ribiere conjugate-gradient algorithm is used to minimize J and so determine the low-order model parameters (Polak 1971). The model parameters are rescaled before the minimization procedure in order to make the cost function less elliptical.

4. PRELIMINARY EXPERIMENTS

In our experiments, two emulated radars are separated by 10 km, and have adjustable range and azimuthal coverage. It has been found that placing the true or guessed vortex center near the edge of the dual-radar domain can produce spurious local minima in J. However, these minima have much larger magnitudes than the true minimum. Such minima can be detected by viewing 2-D cross-sections of J. Figure 2b shows J as a function of the first guess vortex center, where the true center is located at $(x_0,y_0)=(-2000,1000)$, just outside the domain of one of the radars (Figure 2a). All other model parameters are fixed at truth. The true wind field is the same as that in Figure 1. Even in this idealized scenario, the minimization algorithm would not converge to the global minimum were the first guess of

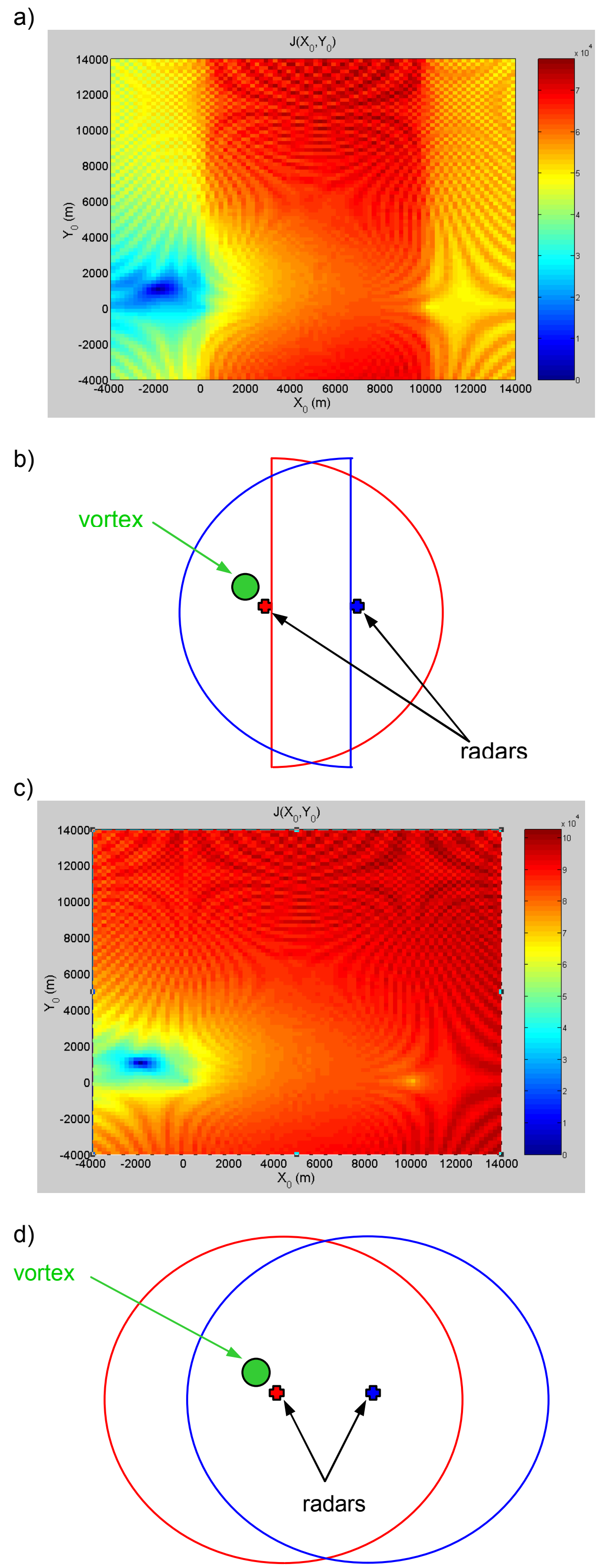

Fig. 2. a) J as a function of first guess vortex center (x_0,y_0) for radar domain in (b); c) same as (a) except for radar domain in (d).

the vortex center too far from truth. Once the dual-radar domain is extended over the vortex region, however, the additional minimum is mitigated, increasing the likelihood of the true vortex center being retrieved (Figures 2c,2d). In reality, the data distribution problem will always be a threat due to both limited radar coverage and missing data. Additional local minima may result from non-tornadic vortices. The existence of these minima necessitate the use of many first guesses of the vortex center and possibly other parameters in order to ensure any tornadic vortices are detected and characterized.

5. FUTURE WORK

The robustness of this technique is being explored by systematically varying the first guess of the model parameters. The algorithm will next be tested against an ARPS (Advanced Regional Predicting System) dataset of a tornado. This will help assess how widespread local minima in J may be in nature. Level I data (power spectra) will later be used to address the degradation of azimuthal resolution with distance from the radar.

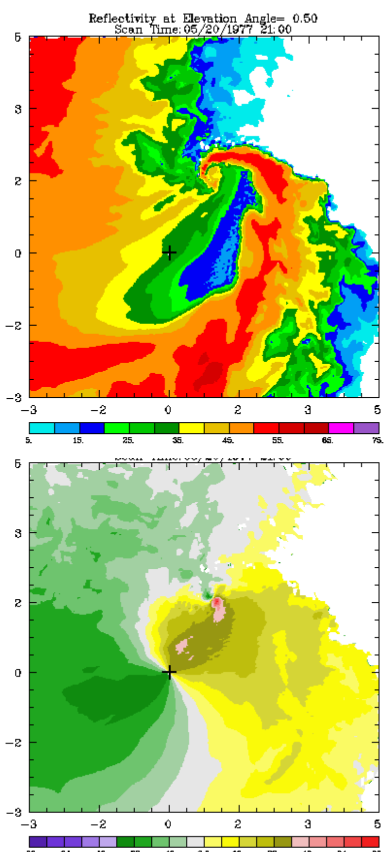

Fig. 3. Emulated radar observation of reflectivity (top panel) and radial velocity (bottom panel) of model simulated tornado to be fit to low-order model. The radar location is marked by a cross.

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