Hour Exam #1
Physical Mechanics
Monday 25, 2000

Answers

1. (15%) In which reference framework is the Newton's second law valid - inertial or non-inertial system? Is a coordinate system fixed to the earth inertial or non-inertial and why?

   It's valid only in inertial system.
   It's a non-inertial system because the earth is rotating. The coordinate system fixed to the earth has a constant radial acceleration.

2. (25%) A 1500 kg car is picked up by an F5 tornado from the ground level and tossed upward at a vertical speed of 50m/s at 200 m height level – to achieve this speed (at 200m level), what is the lifting force of the tornado assuming the force is constant?

   Given m=1500kg, v_0 = 0 m/s, v_1=50m/s, z_0=0, z_1=200m, want F.

   The work done by net force = change in kinetic energy, and the forces are constant →

$$(F - mg)(z_1 - 0) = \frac{1}{2} m v_1^2 - 0$$

$$\therefore F = mg + \frac{m v_1^2}{2 z_1} = 1500 \times 9.8 + \frac{1500 \times 50^2}{2 \times 200} = 24075\, kg\, m\, s^{-2}$$

3. (60%) When an air parcel of mass *m* is displaced vertically by *z* meters, the *net* buoyancy force it experiences is

$$F = -mN^2 z\,,$$

   assuming the parcel is initially located at z=0. Here N is a non-zero real number and has a unit of s^{-1}.

   a) (15%) Does this force represent a restoring or a repelling force? Is the atmosphere stable, unstable or neutral?

      Because for positive z, F < 0, and negative z, F>0, the force is always in the opposite direction as the displacement, it is therefore a *restoring* force.

      Because the displaced parcel is subject to a restoring force, the atmosphere is stable.

   b) (15%) The equation describing the motion of this parcel can be obtained from the Newton's second law:

$$\frac{d^2 z}{dt^2} = -N^2 z\,. \qquad (1)$$

      Show that

$z(t) = A \sin(\omega t - \theta_0)$ (2)

is a (general) solution to Eq.(1) when ω=N. Here A is the amplitude and θ_0 is the initial phase.

To show that (2) is indeed a solution to (1), we plug (2) into (1):

$$\text{LHS}=\frac{d^2z}{dt^2}=-Aw^2\sin(wt-q_0)=-AN^2\sin(wt-q_0)=-N^2z=RHS$$

Therefore (2) is a solution to (1) when ω=N.

c) (15%) Solution (2) represents a simple harmonic oscillation of the air parcel around its equilibrium level z = 0. Given that the parcel's location is at z =0 when t=0, and the maximum displacement of the parcel from its initial location is 100 m, determine θ_0 and A, and write out the final solution for z(t).

According to definition, the amplitude for a SHO is the maximum displacement given, therefore A=100m. From z(t=0) =0, we get

$0 = z(t=0) = A \sin(\omega 0 - \theta_0) = A \sin(\theta_0) \rightarrow \theta_0 = 0$ ($= n\pi$ is okay too).
The final solution is $z(t) = 100 \sin(\omega t)$

d) (15%) If the reference height of zero potential energy is chosen at z=0, what is the potential energy associated with the net buoyancy force when it is at z= 100 m? Here m=1 kg, N = 0.01 s^{-1}. [Hint: potential energy associated with a (conservative) force at a given position is defined as the work done by this force to bring an object from this position to a standard reference location].

$$V(z)=\int_{100}^{0}(-mN^2z)dz=-mN^2\left.\frac{z^2}{2}\right|_{100}^{0}=1\times(0.01)^2\,\frac{(100)^2}{2}=0.5\,kgm^2s^{-1}=0.5J$$

e) (10 bonus points). What is the velocity of the parcel when it passes through z=0 level?

At z=100, the location of maximum displacement, v (velocity)=0, therefore kinetic energy K=0.
At z=0, potential energy=0. We can find K therefore v at z=0 from total energy conservation:

$$V(100)+0=0+\frac{1}{2}mv^2 \rightarrow$$

$$v=\sqrt{\frac{2\times V(100)}{m}}=\sqrt{\frac{2\times0.5}{1}}=1m/s$$