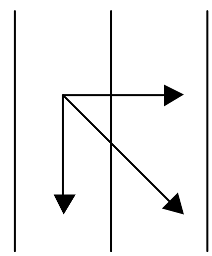
Hour Exam #2
Physical Mechanics
Monday, November 6, 2000

Total 100 Points
Time to complete the test: 60 minutes

Answers

1. (20%) A N-S oriented cold front is passing through Norman. The surface weather shows that the temperature contours at 1 K intervals are 20 km apart with air being colder towards west. At 12pm noon, the northwesterly wind at Norman is measured at 20 m/s, and the temperature is 20 C°.

   a) (10%) Assume there is no diabatic heating to the air parcels, what will be the temperature at Norman by 2 pm?



Since there is no diabatic, $\frac{dT}{dt} = 0$.

$\frac{\partial T}{\partial t} = \frac{dA}{dt} - \vec{V} \cdot \nabla T = -\vec{V} \cdot \nabla T$. We need to know $\vec{V}$ and ∇T. They are found below.

$$\vec{V} = [20\cos(45°)\hat{i} - 20\sin(45°)\hat{j}] \quad (m/s)$$

$$\nabla T = \frac{\partial T}{\partial x}\hat{i} + 0\,\hat{j} = \frac{1K}{20000m}\hat{i}$$

$$\frac{\partial T}{\partial t} = 0 - \vec{V} \cdot \nabla T = -[20\cos(45°)\hat{i} - 20\sin(45°)\hat{j}] \cdot \frac{1K}{20000m}\hat{i}$$

$$= -\frac{20\cos(45°)m/s \times 1K}{20000m} = -7.07 \times 10^{-4} K/s$$

Therefore T(2pm) = T(12pm) + $\frac{\partial T}{\partial t} = 20 - 7.07 \times 10^{-4} \times 2 \times 3600 = 14.91 C°$

b) (10%) If the diabatic solar heating is causing the temperature of air parcels to rise at 1 C°/hour, what will be temperature at Norman at 2 pm then?

Due to diabatic heating, the temperature following air parcel is rising →

$\frac{dT}{dt} = 1K / hour$. In two hours, it causes additional temperature change of +2K = 2C°. Therefore the final temperature at 2 pm = 14.91 + 2 = 16.91 C°.

(The rate of change in T is now $\frac{\partial T}{\partial t} = \frac{dT}{dt} - \vec{V} \cdot \nabla T = -7.07 \times 10^{-4} K / s + 1K / hour$).

2. (30%) A thunderstorm is moving eastward at 15 m/s while the wind measured ahead of the thunderstorm is from the southwest at 20 m/s.

a) (10%) What is the speed and direction of the *storm-relative* velocity?

Suppose you are standing on an open deck training that is moving eastward at 15 m/s. A person on the ground throws a ball at you in the westward direction at 20 m/s. What is the speed of the ball seen by you, i.e., what is the you-relative speed of the ball?

Choosing a coordinate system whose x-axis is in the direction of the train, the ground-relative absolute speed of the ball is

V_{rel} = - 20 m/s (minus because it is in the opposite direction of x-axis)

The speed of the moving coordinate following the train is V_{coord} = 15 m/s. You need to find V_{rel}, the speed of the ball relative to the moving coordinate/you/train.

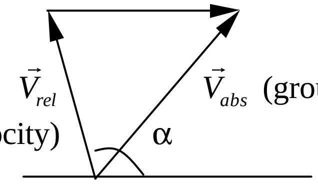
Since $V_{abs} = V_{rel} + V_{coord}$, therefore

$V_{rel} = V_{abs} - V_{coord} = -20 - 15 = -35$ m/s!

Now, what if the ball is thrown at an angle to the direction of the train? Now, we are dealing with a 2D problem, we need to use vector velocities:

$$\vec{V}_{rel} = \vec{V}_{abs} - \vec{V}_{coord} .$$

Going back to our problem. The thunderstorm is our training, and you are to ride with the thunderstorm to observe the storm-relative wind velocity. The air parcel is the ball, and the velocity of the air parcel is the ground relative wind velocity $\vec{V}_{abs}$.



According to the problem

$$\vec{V}_{coord} = 15\hat{i} \quad (m/s)$$

$$\vec{V}_{abs} = 20\cos(45°)\hat{i} + 20\sin(45°)\hat{j} \ (m/s)$$

therefore

$$\vec{V}_{rel} = \vec{V}_{abs} - \vec{V}_{coord} = [20\cos(45°) - 15]\hat{i} + 20\sin(45°)\hat{j} \ (m/s)$$
$$= -0.858\hat{i} + 14.14\hat{i} \ (m/s)$$

The relative speed

$$|\vec{V}_{rel}| = \sqrt{0.858^2 + 14.14^2} = 14.17 (m/s)$$

and it points in the direction that is

$$a = \tan^{-1}(-0.858/14.14) = 93.47°$$

from the x axis, i.e., pointing to north-northwest.

b) (10%) If in the reference frame moving with the storm, the storm-relative horizontal inflow *speed* is reduced to zero beneath the thunderstorm over a 20 km distance (due to the blocking effect by a low-level outflow boundary moving with the storm), what is the horizontal divergence beneath the thunderstorm? (Hint – you can choose a moving coordinate system whose x-axis is parallel to the storm-relative wind to simplify the divergence calculation, but you do not have to do so though)

Choosing a coordinate system whose axis is parallel to the direction of storm-relative wind and that moves with the thunderstorm, we reduce the problem to a one dimensional one. The x-axis directs away from the storm, therefore the storm-relative wind speed is V_{rel} = - 14.17 m/s.

The divergence $D = \frac{\partial u}{\partial x} = \frac{\Delta u}{\Delta x} = \frac{(-14.17-0)m/s}{20000m} = -7.085\times 10^{-4} s^{-1}$

Since D is negative, the flow is convergent near the thunderstorm, consistent with the fact the speed is reduced to zero beneath the thunderstorm.

c) (10%) Assume this storm-relative inflow (therefore the divergence) is constant in the 2 km deep sub-cloud layer, what will be the vertical velocity at the cloud base, e.g., at 2 km height level, assuming the ground is flat?

Integrate zero-divergence continuity equation from 0 to 2 km:

$$w(2km) - 0 = -\int_0^{2km} D dz = -D\times 2000 = 7.085\times 10^{-4}\times 2000 = 1.42 m/s$$

This is a vertical motion induced by horizontal convergence.

3. (20%) Give the definition of conservative force, assuming the problem is 2-D or 3-D (not 1-D). Is force $\vec{F} = x\hat{i} + y^2\hat{j} + z^3 x\hat{k}$ conservative (need to show it)?

If a work done by a force is independent of the path taken, the force is called conservative. The curl of a conservative force is always zero, and vice versa.

$$\because \nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial u}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y^2 & z^3 x \end{vmatrix} = 0\hat{i} - z^3\hat{j} + 0\hat{k} \neq 0,$$

therefore $\vec{F} = x\hat{i} + y^2\hat{j} + z^3 x\hat{k}$ is not conservative.

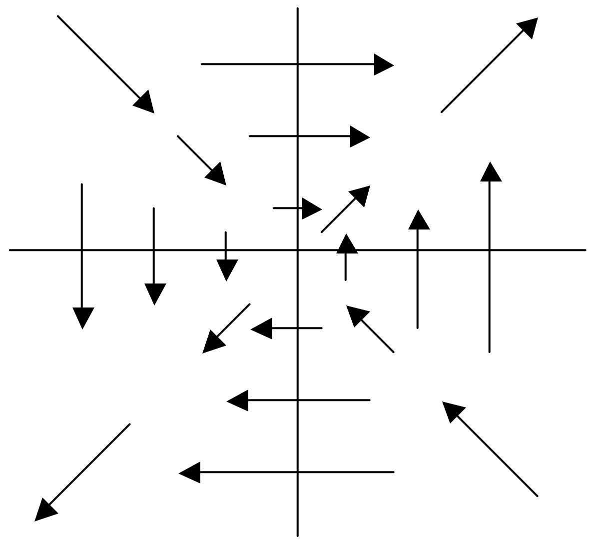
4. (30%) A 2-D flow field is given by

$$u = y,\ v = x$$

where u and v are the velocity components in the x and y directions, respectively.

a) (5%) Sketch out flow pattern

The flow is illustrated as follows: u is positive (negative) for positive (negative) y and increases with |y|. Similarly for v.



b) (10%) What is the vorticity and divergence of this flow?

Divergence $D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial y}{\partial x} + \frac{\partial x}{\partial y} = 0 + 0 = 0$

Vorticity $z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial x}{\partial x} - \frac{\partial y}{\partial y} = 1 - 1 = 0$

c) (7.5%) What is the total circulation ($C = \oint \vec{V} \cdot d\vec{r}$) along a circle with radius 1 and center at origin (0,0)?

Using 2D version of Stokes theorem given, the circulation is

$$\oint_c \vec{V} \cdot \vec{dl} = \iint_\Omega \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) ds = \iint_\Omega z\, ds = 0$$ because ζ = 0.

d) (7.5%) What is the net outward flux going through the circle?

Using Gauss divergence theorem, net flux = $\oint_c \vec{V} \cdot \hat{n}\, dl = \iint_\Omega \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) ds = 0$

because divergence = 0.