METR 2103 - Physical Mechanics
Fall 2000

Study Guide for Comprehensive Final Exam

The Final Exam is Scheduled for 1:30-3:30 pm on Friday, 15 December 2000 in Room P207 of the Sarkeys Energy Center

The information given below is meant to serve as a guide in preparing for the second hour exam. The absence of a topic or point of discussion on this sheet DOES NOT IMPLY a similar absence on the exam. You should carefully study all lecture notes and handouts, and re-work quizzes and problem sets, to prepare adequately for the exam. Your are encouraged to form study groups and work problems on the blackboard until you're convinced you understand both the mechanics and the concepts.

The final examination will be comprehensive and closed-book, and will account for 30% of your final grade. Approximately 50% of the final exam will cover material from the first two exams, with approximately 50% devoted to material that has been covered since the second exam.

Introduction

- Know the definitions of kinematics, dynamics, and statics and be able to give physical examples of each
- Understand units and dimensions and be able to convert from one system to another.

Newton's Laws and Basic Concepts

- Be able to describe physically all three of Newton's laws of motion
- Know the difference between an inertial and a non-inertial reference frame
- Know how mass is defined
- Understand and be able to apply Newton's universal law of gravitation

1-D Motion

- Be able to apply Newton's 2nd law in its fundamental form as well as via the work-energy theorem
- Understand the inclusion of friction in Newton's second law
- Know the momentum theorem
- Be able to define a conservative force
- Know the definition of potential energy and be able to relate it to force and work
- Understand the conservation of energy and be able to use it to solve problems
- Understand stable, unstable, and neutrally stable equilibria and be able to give physical examples of each

- Understand concepts of static stability of an air parcel moving adiabatically
- Know the equations and solutions for the simple harmonic oscillator and be able to use this concept in understanding air parcel static stability. Note that you do not need to be able to derive the SHO solution, but should be able to show that a solution satisfies the equations and be able to determine the constants from initial conditions.
- Know the parcel method for determining atmospheric stability
- Be able to use the vertical equation of motion with only buoyancy and the PGF acting
- Know the definition and physical basis of CAPE
- Understand the total derivative as well as the Eulerian, Lagrangian, and advective terms
- Be able to solve advection problems
- Understand the concept of finite differences and be able to use them in problems

Vector Calculus and Associated Physical Concepts

- Definition of a vector, magnitude, cross product, dot product, curl, gradient, divergence.
- Understand what is meant by the projection of a vector.
- Be able to compute the total derivative of a vector
- Given the position vector, know how to obtain the velocity and acceleration
- Understand uniform circular motion
- Be able to define absolute, relative, and coordinate system motion in relation to one another and solve problems using them.
- Be able to solve line integrals (e.g., work, circulation)
- Know the relationship between a vector and the gradient of its potential function; be able to find the potential, given the force, and vice versa.
- Be able to compute the vertical velocity using the so-called kinematic method
- Be able to apply Gauss'divergence theorem and Stokes'theorem - also, understand their physical meaning.
- Understand solid body rotation and how to compute the circulation for an object undergoing it.
- Be able to compute the vorticity and divergence given a set of wind observations.

Motion in the 2-D Plane & Rotating Coordinate Systems

- Understand conservative forces and their unique properties insofar as work and potential are concerned.
- Be able to show whether a force is conservative.
- Be able to compute the potential function given its force (see above) and vice versa.
- Understand the concept of central force
- Know the angular momentum theorem
- Know angular momentum conservation and its condition and be able to apply angular momentum conservation to solve problems

MATERIAL COVERED FOLLOWING EXAM #2

- Understand the difference between the total (Lagrangian) derivative in an inertial versus non-inertial reference frame for a rotating coordinate system. Note that you will not be asked to derive the formulas.
- Know the physical meaning behind the term "apparent forces" and understand their origin and significance.
- Be able to write down the vector equation of motion as well as the three component equations contained within it. You must be able to know the difference between total and partial derivatives, and apply vector symbols correctly.
- Understand the difference between centrifugal force and centripetal acceleration.
- Understand the physical meaning of the Coriolis force and be able to solve problems in which it is the only force acting.

- Understand and be able to write down the geostrophic wind equations in Cartesian coordinates. Be sure you know the approximations that lead to the geostrophic wind
- Understand and be able to use natural coordinates for horizontal flow
- Be able to write down the horizontal equations of motion in natural coordinates
- Be able to write down the geostrophic, cyclostrophic, and gradient wind equations in natural coordinates, along with all assumptions made in getting them. Be able to draw pictures of the flow and associated force balances, and also be able to determine which combination of solutions is physically allowed.
- Understand the concept of supergeostrophic and subgeostrophic wind
- Be able to discuss the physical implications of friction on the horizontal flow field and their impact on weather

Important Equations (Equations having to with basic definitions, basic laws, and basic principles and the basic equations of motion should be memorized).

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d^2\vec{r}}{dt^2} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

$$\sum_{i=1}^{n}\vec{F}_i = m\vec{a} = m\frac{d\vec{V}}{dt} = m\frac{d^2\vec{r}}{dt^2}$$

$$\vec{P} \equiv m\vec{V}$$

$$\vec{F}_{net} = \frac{d}{dt}\left(m\vec{V}\right) = \frac{d\vec{P}}{dt}\left(m\vec{V}\right)$$

$$W = \int \vec{F}\cdot d\vec{r}$$

$$K \equiv m\frac{|\vec{V}|^2}{2}$$

$$K_2 - K_1 = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$

$$V(\vec{r}) \equiv \int_{\vec{r}}^{\vec{r}_s} \vec{F}\cdot d\vec{r} \text{ where } \vec{F} \text{ is a conservative force}$$

$$\frac{1}{2}m|\vec{V}|^2 + V(x) = KE + PE = \text{constant} \text{ when total energy is conserved}$$

$$\vec{F} = -\nabla V \quad \text{for conservative force}$$

$$\int_{LFC}^{EL}\frac{d}{dz}\left(\frac{w^2}{2}\right)dz = \int_{LFC}^{EL} B\,dz = CAPE \rightarrow w_{EL}^2 - w_{LFC}^2 = 2\,CAPE$$

$$m\frac{d^2x}{dt^2} = F = -kx \quad \text{where } F = -kx \text{ is a spring restoring force}$$

Solution to the above equation is $x(t) = A\cos(wt) + B\sin(wt) = C\cos(wt + q_0)$ with $w = \sqrt{\frac{k}{m}}$.

For a parcel in the atmosphere without friction,

$$\frac{d^2z}{dt^2} = g\left[\frac{g-\Gamma_d}{T_0}\right]z = -\,K\,z \rightarrow \text{ where } K = -g(g-\Gamma_d)/T_0 \text{. Stability depends on } K.$$

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} + w\frac{\partial F}{\partial z} = \frac{\partial F}{\partial t} + \vec{V}\cdot\nabla F$$

$$\frac{\partial F}{\partial t} = \frac{dF}{dt} - \left(u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} + w\frac{\partial F}{\partial z}\right) = \frac{dF}{dt} - \vec{V}\cdot\nabla F$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \vec{C}\cdot\nabla T$$

$$\vec{V}_{abs} = \vec{V}_{rel} + \vec{V}_{coord}$$

For uniform circular flow,

$$\vec{r} = \hat{i}\,r\cos(wt) + \hat{j}r\sin(wt)\,,\ \vec{V} = \frac{d\vec{r}}{dt} = rw[-\hat{i}\sin(wt) + \hat{j}\cos(wt)]\,,\ \ \vec{V}\perp\vec{r}\,,$$

$$\vec{a} = \frac{d\vec{V}}{dt} = rw^2[-\hat{i}\cos(wt) - \hat{j}\sin(wt)] = w^2(-\vec{r})$$

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos(q)\,,\quad \vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{A}\times\vec{B} = AB\sin(q)\,\hat{u} = \vec{C}\,,$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$$

$$grad(A) = \nabla A \equiv \hat{i}\frac{\partial A}{\partial x} + \hat{j}\frac{\partial A}{\partial y} + \hat{k}\frac{\partial A}{\partial z}$$

$$\mathrm{Div}(\vec{A}) = \nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{z} \equiv \nabla\times\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \hat{i}\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right) - \hat{j}\left(\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}\right) + \hat{k}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$$

When $\vec{A} = \nabla f$, $\oint \vec{A}\cdot d\vec{r} = 0$

$$\nabla\cdot\vec{V} = 0 \rightarrow \frac{\partial w}{\partial z} = -\nabla_h\cdot\vec{V}_h \rightarrow \int_{z_1}^{z_2}\frac{\partial w}{\partial z}dz = -\int_{z_1}^{z_2}\nabla_h\cdot\vec{V}_h dz = -\int_{z_1}^{z_2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dz$$

$$\iiint_{\Omega}\nabla\cdot\vec{V}d\Omega \equiv \iint_{S}\vec{V}\cdot\vec{n}\,ds$$

$$C = \int\vec{V}\cdot d\vec{r}$$

$$\iint(\nabla\times\vec{V})\cdot\vec{n}\,ds = \int\vec{V}\cdot d\vec{r}$$

$\nabla\times\vec{F} = 0$ if $\vec{F}$ is conservative.

$\vec{F} = -\nabla V \rightarrow \frac{\partial V}{\partial x} = -F_x$, $\frac{\partial V}{\partial y} = -F_y$, $\frac{\partial V}{\partial z} = -F_z$, integrate them to obtain *V*.

$\vec{F} = F(r)\hat{r}$ for central force

Torque $\vec{t} = \vec{r}\times\vec{F}$

Angular momentum $\vec{L} = \vec{r}\times m\vec{V}$

In plane polar coordinates (2D)

$$\vec{r} = r\,\hat{r}(q)\,,\quad \vec{V} = \frac{dr}{dt}\hat{r} + \left(r\frac{dq}{dt}\right)\hat{q} = V_r\hat{r} + V_q\hat{q}\,,\ \vec{a} = \left[\frac{d^2r}{dt^2} - r\left(\frac{dq}{dt}\right)^2\right]\hat{r} + \left[r\frac{d^2q}{dt^2} + 2\frac{dr}{dt}\frac{dq}{dt}\right]\hat{q}$$

Eqautions of motion: $m(\ddot{r} - r\dot{q}^2) = f_r$ and $m(r\ddot{q} + 2\dot{r}\dot{q}) = f_q$.

Angular momentum $L = mr^2\dot{q}$ is conserved when $f_q = 0$.

$$\frac{d}{dt}(mr^2\dot{q}) = \frac{d}{dt}(mrV_q) = 0 \rightarrow mr^2\dot{q} = \text{constant}$$ - Angular momentum conservation

$$\frac{d\vec{L}}{dt} = \vec{t}$$ - Angular Momentum Theorem. Torque causes angular momentum to change.

When torque is zero, angular momentum is conserved.

$$\frac{d\vec{V}}{dt} = \frac{\vec{F}_{real}}{m} \qquad \text{(in inertial reference frame)}$$

$$\frac{d\vec{V}}{dt} = \frac{\vec{F}_{real}}{m} + \frac{\vec{F}_{apparent}}{m} \qquad \text{(in non-inertial reference frame)}$$

$$\frac{d\vec{V}}{dt} = -\frac{1}{r}\nabla p + \vec{g} - 2\vec{\Omega}\times\vec{V} + \Omega^2\vec{R} = -\frac{1}{r}\nabla p + \vec{g}_{net} - 2\vec{\Omega}\times\vec{V}$$

$$\vec{F}_{cor} = -2\vec{\Omega}\times\vec{V} = \hat{i}(2\Omega v\sin f - 2\Omega w\cos f) + \hat{j}(-2\Omega u\sin f) + \hat{k}(2\Omega u\cos f)$$

$$\frac{du}{dt} = -\frac{1}{r}\frac{\partial p}{\partial x} + 2\Omega v\sin(f) - 2\Omega w\cos(f) = -\frac{1}{r}\frac{\partial p}{\partial x} + fv - \tilde{f}w$$

$$\frac{dv}{dt} = -\frac{1}{r}\frac{\partial p}{\partial y} - 2\Omega u\sin(f) = -\frac{1}{r}\frac{\partial p}{\partial y} - fu$$

$$\frac{dw}{dt} = -\frac{1}{r}\frac{\partial p}{\partial z} - g + 2\Omega u\cos(f) = -\frac{1}{r}\frac{\partial p}{\partial z} - g + \tilde{f}u$$

$$f = 2\Omega\sin(f) \text{ and } \tilde{f} = 2\Omega\cos(f)$$

For large-scale 2D horizontal flow (*w* is smaller), $\frac{du}{dt} = -\frac{1}{r}\frac{\partial p}{\partial x} + fv$ and $\frac{dv}{dt} = -\frac{1}{r}\frac{\partial p}{\partial y} - fu$.

Geostrophic balance: $-\frac{1}{r}\nabla p - f\hat{k}\times\vec{V}_g = 0$

Geostrophic wind in Cartesian coordinates:

$$\vec{V}_g = \frac{1}{f\,r}\hat{k}\times\nabla p, \quad u_g = -\frac{1}{f\,r}\frac{\partial p}{\partial y} \text{ and } v_g = \frac{1}{f\,r}\frac{\partial p}{\partial x}$$

2D equations of motion in natural coordinates: $\frac{dV}{dt} = -\frac{1}{r}\frac{\partial p}{\partial s}$ and $\frac{V^2}{R} = -\frac{1}{r}\frac{\partial p}{\partial n} - fV$

Geostrophic wind in natural coordinates: $V_g = -\frac{1}{f\,r}\frac{\partial p}{\partial n}$

Cyclostrophic balance: $0 = -\frac{1}{r}\frac{\partial p}{\partial n} - \frac{V^2}{R}$

Cyclostrophic wind in natural coordinates: $V = \sqrt{-\frac{R}{r}\frac{\partial p}{\partial n}}$

Gradient wind balance: $0 = -\frac{1}{r}\frac{\partial p}{\partial n} - fV - \frac{V^2}{R}$

Gradient wind in natural coordinates: $V = -\frac{fR}{2} \pm \left(\frac{f^2R^2}{4} - \frac{R}{r}\frac{\partial p}{\partial n}\right)^{1/2}$

Three force blance including friction: $-\frac{1}{r}\nabla p - f\hat{k}\times\vec{V} + \vec{F}_{friction} = 0$

Important Concepts

Units and Dimensions
Inertial and non-inertial reference frame
Velocity (scalar and vector definitions)
Acceleration
Forces, real and apparent forces
Momentum
Kinetic energy
Potential energy
Newton's laws
Momentum theorem, Momentum conservation
Work-energy theorem
Total energy conservation
Work, Impulse
Conservative force in 1D, 2D and 3D
Stability, Atmospheric stability
CAPE
Simple harmonic motion
Restoring and repelling force
Local (Eulerian) and total (material, substantial and Lagrangian) change in time
Advection
Change following moving object
Vector algebra, dot and cross products
Line integral (e.g., for calculating work and circulation)
Absolute and relative motion
Vorticity and divergence
Gauss divergence theorem relating divergence and fluxes
Stokes' theorem relating vorticity and circulation
Kinematic method for calculating vertical velocity from horizontal divergence
Obtaining potential energy from conservative force
Central force
Angular momentum
Angular momentum theorem
Torque
Angular momentum conservation
Acceleration due to earth rotation
Centripetal acceleration
Centrifugal force
Coriolis force and it components
Equations of motion in rotating earth coordinate
The role of Coriolis force in Foucault pendulum
Geostrophic wind and geostrophic balance
Natural coordinates
Radius of curvature

2D equations of motion in natural coordinates
Cyclostrophic balance and cyclostrophic flow
Gradient wind balance and gradient wind and their role in weather
Frictional effects on geostrophic balance and its impact on weather

GOOD LUCK WITH YOUR FINALS!