Review before first test
Physical Mechanics Fall 2000

Newton's Laws – You should be able to state these laws using both words and equations.

The 2nd law most important for meteorology.

Second law:

$$\boxed{\text{net force = mass} \times \text{acceleration}}$$

Note that it is the **net** force – all forces acting on the object have to be taken into account – remember the gravity in one of your homework problem?

Inertial and non-inertial reference frame – in which reference frame are the Newton's Law valid? Can Newton's Law be used in a non-inertial reference frame? Anything needs to be done to do so?

In equation form, the second law is

$$\boxed{F = m\,a}$$

where a is the acceleration rate.

$$a = \frac{dV}{dt}$$

where V is the velocity defined as

$$V = \frac{dx}{dt}.$$

x is the coordinate of the objection, and is a function of time t.

$F = ma$ can have the following forms:

$$\boxed{m\frac{dV}{dt} = F \text{ and } m\frac{d^2x}{dt^2} = F.}$$

Momentum, momentum theorem, impulse

For an object with non-constant mass, the more accurate way of expressing the second law is

$$\boxed{\frac{dP}{dt} \equiv \frac{d(mV)}{dt} = F \quad \text{and} \quad \frac{d}{dt}\left(m\frac{dx}{dt}\right) = F}$$

which says that

$$\boxed{\text{net force = rate of change of momentum}}$$

where $P \equiv mV$ is the definition of momentum. The above is the **differential form of momentum theorem.**

The integral form of momentum theorem is

$$\boxed{P_2 - P_1 = \int_{t_1}^{t_2} F dt \equiv \text{Impulse}} \qquad (6)$$

which says

$$\boxed{\textbf{Impulse from the net force (F × Dt) = change in momentum}}$$

Note that the change in momentum does not require displacement. The impulse can be applied to an object before an appreciable displacement occurs. Think of two colliding balls of equal mass – the momentum is transferred from one ball to the other at the instance of collision.

Kinetic energy, work, work-energy theorem

Work done by a force is equal to the spatial displacement times the force in the direction of this displacement.

When the force is not constant, the work should be expressed in an integral form:

$$W = \int_{x_1}^{x_2} F dx$$

where F is the force acting in the x direction, over an interval between x_1 and x_2. It can also be written in the form of

$$W = \int_{t_1}^{t_2} (Fv) dt \; .$$

Here Fv is the **power**, representing the amount of work done by force F in unit time.

The net work done to an object causes the kinetic energy to change, the amount of change is equal to the total work done, this is the **WORK-ENERGY THEOREM:**

Work done by net force = change in kinetic energy

It can be in the following form:

$$\boxed{K_2 - K_1 = \int_{x_1}^{x_2} F dx = Work}$$

where $K \equiv m\frac{v^2}{2}$ is the **kinetic energy**.

You should be able to derive the above theorem from Newton's second law.

Note that the *net* force acting on an object changes the kinetic energy only, not potential energy (will be discussed later). E.g., when a crane lifts an object vertically at a *uniform* speed, it does work and changes the potential energy of the object. But since this lifting force equals the gravity, the net force acting on the object is zero, therefore there is no change in the kinetic energy.

Potential Energy and Total Energy Conservation

For 1-D problems, if a force is independent of t and velocity and is dependent on the space location only, the force is said to be conservative.

Examples of such conservative forces include elastic force from a stretching spring F(x) = - kx, the gravity F(z) = -mg. The gravity is a special case of spatially dependent force – its not dependent none of t, v or z.

The frictional force F(z) = - αw is not a conservative force.

The work done by a conservative force between two spatial locations can be described by a spatial function called the potential energy.

For the next 2 years, and certainly if you go to grad school, you will hear a lot about <u>conservation</u>, <u>conservation laws</u>, and <u>conservative quantities</u>. This will be our first look at them. Later, we will see that conservative forces lead to work that is independent of path.

Consider the <u>special</u> case where F = F(x) <u>only</u>.

Then,

$$m\frac{dv}{dt} = F(x) \qquad (28)$$

and by (9) ($K_2 - K_1 = \int_{x_1}^{x_2} Fdx$), we have

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \int_{x_0}^{x} F(x)dx$$

The **POTENTIAL ENERGY** [V(x)] is defined as the work done by a force F(x) when a particle is brought from an arbitrary point x to some standard point x_s:

$$\boxed{V(x) \equiv \int_{x}^{x_s} F(x)dx = -\int_{x_s}^{x} F(x)dx}$$

If all forces acting on an object are conservative, and the potentially energy associated with the net force is V(x), the from the Newton's second law, we can show that

$$\boxed{\frac{1}{2}mv^2 + V(x) = V(x_0) + \frac{1}{2}mv_0^2}$$

where the right hand side terms are the potential and kinetic energy of the object at position x_0 and the LHS are the corresponding energy at any position x.

We define the **TOTAL (mechanical) ENERGY** $\equiv \frac{1}{2}mv^2 + V(x)$, which **remains the same for all x**. Therefore, we have a **Conservation Law of Total Mechanical Energy:**

$$\boxed{\textbf{E = KE + PE = constant following the motion}}$$

From the definition of V, we see that

$$\boxed{F = -\,dV/dx}$$

Applications of total energy conservation

Examples:

Geopotential or gravitational potential energy – the potential energy associated with gravity,

$$F = - mg.$$

For geopotential, we usually define the sea level z = 0 as the reference (standard) height where the potential energy is zero, therefore

$$V(z) = \int_z^0 (-mg)dz = mgz$$

The total energy conservation gives

$$\frac{1}{2}mw^2 + mgz = \frac{1}{2}mw_0{}^2 + mgz_0 \ .$$

This formula is often used to solve problems on free falling and vertically ejected/thrown object.

E.g., you home work problem – what height can a ball rise to with an initial vertical velocity of 30 m/s?

$$\frac{1}{2}\times 30^3 + 0 \ = 0 + 9.8H \rightarrow \text{H} = 45.92 \text{ m!}$$

Elastic potential energy

The restoring force of an elastic spring is F(x) = - kx , a conservative force. The potential energy of an objected attached to its end is

$$V(z) = \int_x^0 (-Kx)\,dx = \frac{1}{2}kx^2$$

again we usually choose the x =0 as the reference position where V = 0. Using the total energy conservation, you can determine the velocity of the objection at any particular x position. This method is used to obtain the solution of an oscillating spring attached objection in Symon's book on pages 31-32 – see your handout.

Stability

Stable Equilibrium - a state of potential energy minimum. If a particle is displaced slightly in whichever direction, it will experience a restoring force that tends to return it to its starting location. When the total energy is conserved (e.g., when friction is neglected), because the state is associated with a potential energy minimum, any small displacement will result in an increase in potential energy, at the expense of kinetic energy. This state is stable.

Unstable Equilibrium - a state of potential energy maximum. If a particle is displaced slightly in whichever direction, it will experience a repelling force that tends to drive the object away from its initial location further. When the total energy is conserved (e.g., when friction is neglected), because the state is associated with a potential energy maximum, any small displacement will result in a decrease in potential energy, giving rise to more kinetic energy. This state is unstable.

Neutral Equilibrium – In this case, if a particle or air parcel is displaced slightly, it will experience neither restoring force nor repelling force. This state is associate with the inflection point in the potential energy curve.

You should be able to determine the state of stability knowing the potential energy function V(x) or the force acting on the object as a function of spatial coordinate, i.e., F(x).

You should be able to apply these stability concepts to simple problems of vertical displacement of air parcels.

You should know the expected behavior of an objected when displaced from a stable/unstable/neutral equilibrium state, and be able to explain the behavior in terms of both force-acceleration and energy conservation.

CAPE, the **Convective Available Potential Energy**

CAPE, the Convective Available Potential Energy in meteorology is defined as the **work that can be (potentially) done by the buoyancy force to accelerate an air parcel of unit mass vertically to its maximum speed.**

By integrating the vertical equation of motion, we obtain

$$\int_{LFC}^{EL} \frac{d}{dz}\left(\frac{w^2}{2}\right) dz = \int_{LFC}^{EL} B dz = CAPE$$

which can be used to calculate the maximum vertical velocity given the CAPE, or the CAPE from the buoyancy function. Here LFC is for level of free convection (LFC) and

EL for the equilibrium level near the top of convective cloud where the parcel temperature is reduced to the environmental value.

Simple Harmonic Oscillations

When the atmosphere is stable, the vertical displacement will introduce restoring force that draws the air parcel to its original equilibrium level. The restoring force, as will be shown later, is proportional to the amount of vertical displacement, in the same way a the restoring force from a stretched or compressed spring. The air parcel and an object attached to the end of a spring will undergo a periodic **simple harmonic oscillation**.

When friction is present, the amplitude of the oscillation will decrease with time, resulting in **damped harmonic motion.** When the fiction/damping is strong enough, the motion may not be able to complete a single period of oscillation, resulting in **over-damped oscillation.**

Oscillatory motion can only occur when there is restoring force – which only occurs under stable condition. Under unstable condition, the displaced parcel will never return!

Gravity waves in the atmosphere are manifestations of the stable oscillations of air parcel. The convective storms are results of unstable displacement!

Assuming the restoring force is given by

$$\boxed{F = -k\,x}$$

from Newton's second law, the equation describing the motion is

$$\boxed{m\frac{d^2x}{dt^2} + kx = 0}$$

which is a second-order <u>ordinary differential equation</u> (ODE).

One form of the general equation to this equation is

$$\boxed{x(t) = C\cos(\theta_0 - \omega t)}$$

where $w = \sqrt{\frac{k}{m}}$. C is the <u>amplitude</u>, ω the <u>frequency</u>, and $T = (2\pi)/\omega$ the <u>period</u> of oscillation. $\theta_0 - \omega t$ is called the <u>phase</u> of oscillation and θ_0 the initial phase.

The frequency is dependent on the physical property of the oscillator, and the amplitude C and initial phase θ_0 are specific to each problem. Given two proper initial conditions, we will be able to determine their values. In most general cases, you will need to solve two simultaneous equations to obtain the values of the two arbitrary constants (C and θ_0 here).

Other general requirements:

You should know how to perform **unit conversions** and be able to check the **consistency of dimensionality** in equations.

You should understand the fundamental physical concepts and remember the basic equations, and be able to use them to solve problems.

Make sure you do can do all your homework problems.

Make sure you understand all examples we discussed in class and found in the Notes.

Review the steps of problem solving, and apply them to your exam problems. For the actual exam, you will not be asked to put down detailed description of these steps, but it is important to understand what are given and what are sought after, which will help you decide on the best equation(s) to use. A sketch is often very helpful.

You need to be able to give simple physical interpretation of the mathematical solutions you obtain.

You will need a basic calculator for your test.