Quiz #4. Physical Mechanics, 2000
Total 9 points.

Answers

1. (5 points) In the class, we showed that for a particle undergoing a 2D plane motion, its velocity in terms of the plane polar coordinates is given by

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + \left(r\frac{dq}{dt}\right)\hat{q}$$

where $\vec{r} = \vec{r}(q)$ is the position vector and θ is the polar angle.

1) Given that $\frac{d\hat{r}}{dq} = \hat{q}$ and $\frac{d\hat{q}}{dq} = -\hat{r}$, please show that the acceleration is

$$\vec{a} = \frac{d\vec{V}}{dt} = \left[\frac{d^2r}{dt^2} - r\left(\frac{dq}{dt}\right)^2\right]\hat{r} + \left[r\frac{d^2q}{dt^2} + 2\frac{dr}{dt}\frac{dq}{dt}\right]\hat{q}\ .$$

Can you do derivatives

$$\frac{d(xy)}{dt} \text{ and } \frac{d(xyz)}{dt}\ ?$$

That's essentially ALL that's asked of you!!! And you were ASKED to prove this in the class, and I hinted that I might have it in the quiz!!! The proof is:

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}\left[\frac{dr}{dt}\hat{r} + \left(r\frac{dq}{dt}\right)\hat{q}\right]$$
$$= \frac{d^2r}{dt^2}\hat{r} + \frac{dr}{dt}\frac{d\hat{r}}{dt} + \frac{d}{dt}\left(r\frac{dq}{dt}\right)\hat{q} + r\frac{dq}{dt}\frac{d\hat{q}}{dt}$$
$$= \frac{d^2r}{dt^2}\hat{r} + \frac{dr}{dt}\frac{d\hat{r}}{dq}\frac{dq}{dt} + \left(\frac{dr}{dt}\frac{dq}{dt} + r\frac{d^2q}{dt^2}\right)\hat{q} + r\frac{dq}{dt}\frac{d\hat{q}}{dq}\frac{dq}{dt}$$
$$= \left[\frac{d^2r}{dt^2} - r\left(\frac{dq}{dt}\right)^2\right]\hat{r} + \left[r\frac{d^2q}{dt^2} + 2\frac{dr}{dt}\frac{dq}{dt}\right]\hat{q}$$

2) If the force acting on this particle of mass *m* is $\vec{F} = F_r\hat{r} + F_q\hat{q}$, write down the equations of motion for the radial and azimuthal (i.e., tangential) components.

What is the equation of motion????

$$\vec{F} = m\vec{a} \text{ or } \vec{F} = m\frac{d\vec{V}}{dt} \text{ or } \vec{F} = m\frac{d^2\vec{r}}{dt^2}$$

What are the equations of motion for the components???

$$F_r = ma_r = \frac{d^2 r}{dt^2} - r\left(\frac{dq}{dt}\right)^2$$

$$F_q = ma_q = r\frac{d^2 q}{dt^2} + 2\frac{dr}{dt}\frac{dq}{dt}$$

2. (4 points) State the angular momentum theorem.

The time rate of change of angular momentum is equal to the torque or $\frac{d\vec{L}}{dt} = \vec{t}$.

Remember of regular momentum theorem? The time rate of change in momentum is equal to the force it's subject to!

In what situation is angular momentum conserved?

When torque is zero.

Example Answer for angular momentum theorem:

When the torque is 0, angular momentum is 0. When d/dt = 0 → angular momentum is zero.