

## Chapter 2. Review of Newton's Laws, Units and Dimensions, and Basic Physics

You are all familiar with these 3 important laws. But which are based on experiments and which are matters of definition?

**FIRST LAW** – an object moves uniformly (or remains at rest) provided that there is no net force acting on it.

$$\sum \vec{F} = 0 \rightarrow \text{no change in motion.}$$

One can test this using experiment with an air hockey table. When the table is perfectly level, the puck will not move! You have to be very careful, however, about the coordinate system in which you observe the object. If you are standing on a rotating table without being aware of the rotation, you will see the object move due to centrifugal force and be puzzled!

One is free to choose any coordinate system he/she likes (fixed to table, moving with the puck or standing on the ground) – but you have to know what it is. If a coordinate system is not accelerating, it is called **INERTIAL** and one always can be found.

This is the essence of **FIRST LAW**. This has important implications in meteorology – we study the weather in a reference frame of the rotating earth – it is a non-inertial reference frame, and thus Newton's laws (especially the second) can be applied to the atmosphere only if we take into account the fact that the coordinate system (earth) is accelerating (due to constant change of the direction of motion of objects on the earth surface). This is where Coriolis force comes in, as we will see later. Coriolis force of one of the most important force for large scale atmospheric flows!

**SECOND LAW** – Suppose we attach a rubber band to our air hockey puck. If we stretch it, the puck starts to move..., if we keep it stretched the same amount, what is the resulting motion? The speed (really the velocity) of the puck increases uniformly with time – constant acceleration.

Now, replace the light plastic puck with a bar of gold, now if we stretch the rubber and by the same amount, the acceleration will be much different – far less in fact! So the acceleration depends upon a fundamental property of the object – its mass.

Let's define  $m_p$  = mass of the puck  
 $m_g$  = mass of the gold bar

If we now measure (experimentation is important!) the acceleration of the two masses along the line of acceleration, we will find that

$$\frac{a_p}{a_g} = \frac{m_g}{m_p},$$

We thus have

$$m_g = m_p \left( \frac{a_p}{a_g} \right)$$

which is actually the definition of mass! Note that it is always relative gravity until we assign a value for  $m_p$ . That is, if  $m_p \equiv 1$  gram, then we can find  $m_g$  relative to it. [The standard of mass is a metallic weight kept in France.]

To this point, we have not really defined how the puck or bar is accelerated, but the stretching produces an applied force. If we apply 2 bands on opposite sides of the puck, it doesn't move! If we put two on the same side, it accelerates at twice the rate.

### The force

How do we bring force into the picture?

Let's say that 1 force unit is what is needed to produce 1 unit of acceleration for a unit mass. What is a 'unit mass'? It could be 1 kg or 1 g. Thus,

F units of force produces F units (different unit) of acceleration, i.e.,

$$F \propto a$$

We also know that

$$\frac{1}{m} \propto a$$

if we define

$$F \left( \frac{1}{m} \right) = \text{units of acceleration}$$

then we have

$$a = \frac{F}{m} \text{ or}$$

$F = ma$  ← the fundamental equation of NWP and all of meteorology!

This above is the familiar form of Newton's Second Law. However, that is not all there is to it.

**Point I:** Principle of Superposition – Acceleration is a Vector (magnitude + direction), and mass is a Scalar. So, force has to be a vector too →

$$\vec{F} = m\vec{a} .$$

So, Newton's law is really 3-D! And, the acceleration is a result of the NET FORCE, i.e., the vector sum of all forces. It's just like a tug of war.

$$\vec{F} = \sum_{i=1}^n \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n = m\vec{a}$$

**Point II:** The 2nd law is really

$$\vec{F}_{net} = \frac{d}{dt} (m\vec{V}) = \Delta(\text{momentum})/\Delta t$$

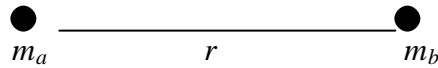
where  $\vec{V}$  = velocity vector. The force introduces a change in momentum.  $\vec{F} = m\vec{a}$  assumes that the mass of the object remains constant, that needn't to be the case ... think of an air parcel that is losing rain water content as it rises. We will study momentum later.

**Point II:** Force is not a matter of definition only. Forces always arise by virtue of interactions among systems of bodies, and the interaction is the significant part. All of our discussion has been for isolated bodies, i.e., no moon, stars, etc. But, such interactions are usually proportional to  $1/r^2$ , so we can reduce them as much as desired. Think of gravity, more on this later.

**THIRD LAW** – Forces always operate in pairs – so to every action there is an equal and opposite reaction. There are no lone forces! If body  $a$  exerts  $\vec{F}_a$  on body  $b$ , then body  $b$  exerts  $\vec{F}_b$  on  $a$ , and  $\vec{F}_a = -\vec{F}_b$ . Think of pushing against a wall!

**Gravitation.** Gravity is the most familiar of the fundamental forces (compare to e.g., electromagnetic force).

Newton was responsible for showing how gravity worked, and he did this in the context of planetary orbits. He, at age 26 (!), showed that the elliptical orbits of planets could be accounted for if every pair of bodies had associated with them a force of attraction proportional to their mass and inversely proportional to the Square of the distance between them.



$$|\vec{F}_{ab}| \propto \frac{m_a m_b}{r^2}$$



By taking measurements, one can determine the constant of proportionality and thus write

$$|\vec{F}| = \frac{Gm_a m_b}{r^2} \quad \leftarrow \text{Newton's Law of Gravitation}$$

where  $G$  = gravitational constant =  $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ .

The gravitational force  $|\vec{F}|$  between 2 particles acts on a line connection the centers of the particles (or the center of mass in case of a group) and is always attractive.

## Gravity

Consider now an object of mass  $m$  at the surface of the earth, whose mass is  $m_e$ . Then

$$|\vec{F}| = \frac{Gmm_e}{r^2}, \quad r = \text{radius of Earth.}$$

Now, define  $\frac{Gm_e}{r^2} \equiv g \rightarrow F = mg$ .

For  $m_e$  = mass of earth and  $r$  its radius,  $g=9.802 \text{ ms}^{-2}$ .

This brings out a mysterious property of the gravitational force – the acceleration of a particle under the influence of gravity is independent of its mass!! (not immediately obvious – does feather falls to ground faster than a stone from the same height?)

$g$  = acceleration due to gravity

Before we get down to solving some sample problems, let's examine some fundamental tools that you will need.

## Units and Dimensions

The 3 fundamental units are mass, length, and time (see handout). The biggest hassle is moving from one system to another, though fortunately, the world is increasingly standardizing on one system.

- a. **CGS** (centimeter-gram-second) – used a lot in early days of science and engineering
- b. **MKS** (meter-kilogram-second) – also called SI (International System of Units, Système International) – the one of choice. Required units of American Meteorological Society (AMS) Journals!
- c. **FPS** (foot-pound-second) – English units. Non-metric system. Unfortunately this county remains one of increasingly fewer countries who still sticks to it. Not even the English.

See article on Mars Climate Orbiter – it says Units are very important!! Measurement numbers without unit are meaningless – and can lead to disaster!

## Consistency of Dimensions

The biggest point to be made here is CONSISTENCY of dimensions on both sides of an equation. By checking for this, you can frequently uncover errors in your equations and/or solutions. Make it a habit to check the dimensions on both sides of equations!

Example: Let's verify the gravitational equation

$$F = \frac{G m_1 m_2}{r^2}$$

Left hand side:

$$F \sim ma \sim \frac{kg \ m}{s^2}$$

Right hand side:

$$\frac{G m_1 m_2}{r^2} = \left( \frac{m^3}{s^2 \ kg} \right) \left( \frac{kg \ kg}{m^2} \right) = \frac{kg \ m}{s^2} \rightarrow \text{consistent!}$$

In this class, we will not use Newtons as the unit for force, but rather  $kg \ m / s^2$ .

Note that  $\frac{kg\ m}{s^2} = kg\ m\ s^{-2}$ , either form is okay, although the latter is a recommended form of notation of certain Journals, such those of AMS.

### **Unit Conversions**

Example: Convert  $\frac{ft \cdot (lbs)^3}{hour}$  to MKS.

$$x \frac{ft\ (lbs)^3}{hour} = x \frac{ft\ (lbs)^3}{hour} \left( \frac{\_m}{1ft} \right) \left( \frac{\_kg}{1lbs} \right)^3 \left( \frac{1hour}{\_s} \right)$$

All you have to do is to fill the blanks for the conversion factors.

Let's look at  $13 \frac{ft \cdot (lbs)^3}{hour}$ :

$$13 \frac{ft \cdot (lbs)^3}{hour} = 13 \frac{ft\ (lbs)^3}{hour} \left( \frac{1m}{3.281ft} \right) \left( \frac{0.45kg}{1lbs} \right)^3 \left( \frac{1hour}{3600s} \right) = 1.04 \times 10^{-4} mkg^3 s^{-1}$$