

METR 5303 – Lecture #9, #10

The Barnes Objective Analysis Procedure

Reading: Koch *et al*, (1983) paper (handed out); Daley, p. 90-93.

We will study the Barnes scheme in detail (2-3 lectures) because it is commonly used in research and software packages (e.g., GEMPAK; McIDAS) for diagnostic analysis of meteorological data.

A Barnes objective analysis refers to the use of a Gaussian filter as the distance-dependent weight function:

$$w_m = \exp [- r_m^2 / \kappa] \quad (1)$$

where r_m is the distance between the m^{th} observation and the grid point, and κ is a parameter governing the shape of the resulting response function (see Fig. 3.3 in Daley).

κ has units of r^2 (e.g., m^2 if doing an analysis in x - y space), and should be related to the average spacing of the observations (and also the data distribution if it is non-uniform).

E.g., if have dense observations, κ should become smaller so that there is a smaller weight w_m for a given r_m (*i.e.* – only data closest to grid point are weighted highly)

The Barnes scheme can be designed to include (exclude) the scales you want (don't want) to retain by the choice of κ and number of scans.

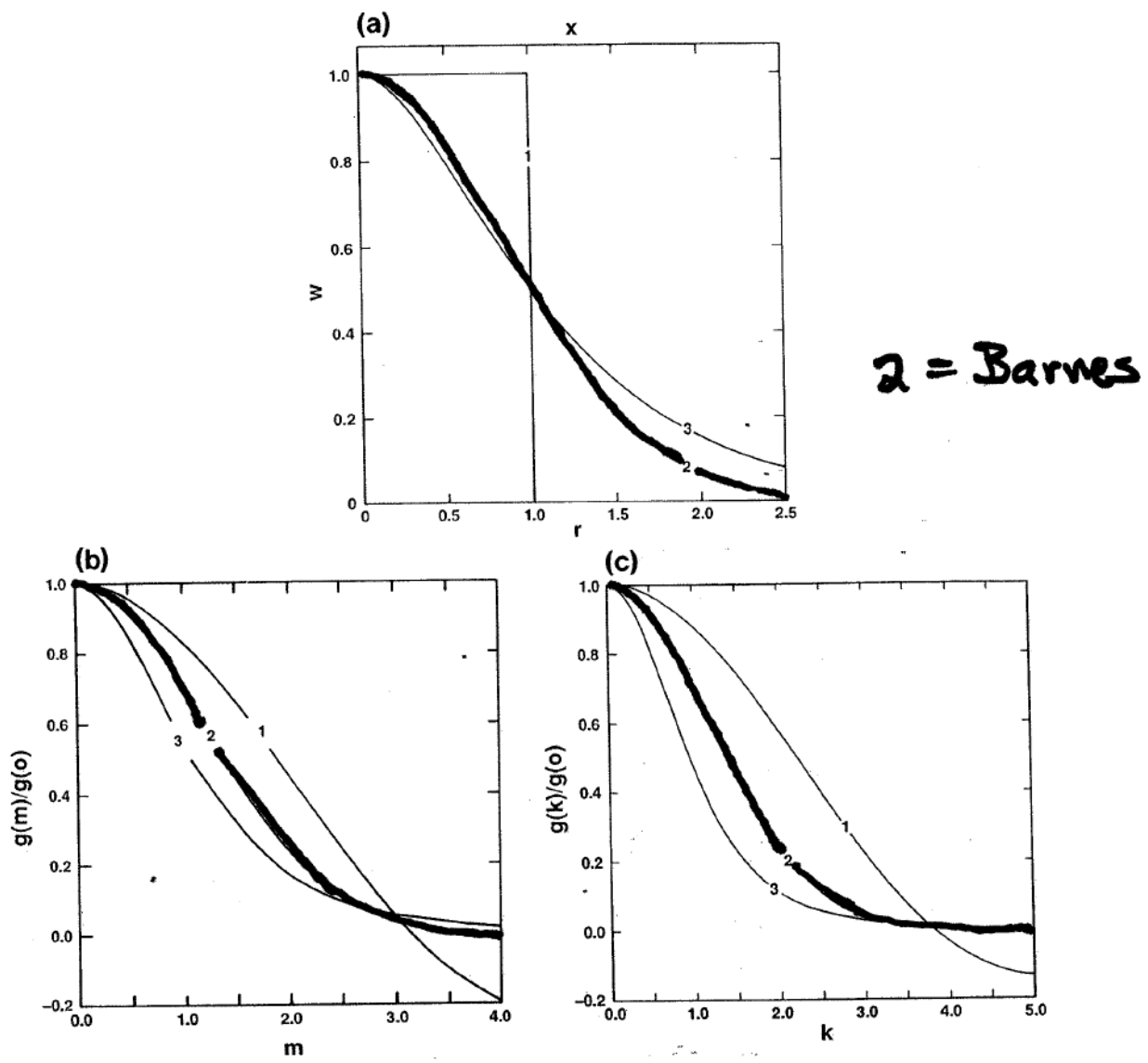


Figure 3.3 (a) Weight functions and (b,c) transfer functions. (b) is the one-dimensional case; (c) is the axisymmetric case.

A response function for eq. (1) can be obtained analytically by assuming sinusoidal fields and continuous data (Barnes, 1973; Pauley and Wu, 1990). The response function for one pass is

$$D_0 = \exp [- \kappa_0 (\pi / \lambda)^2] \quad (2)$$

where λ is wavelength.

Thus the analyzed field $g_0(i, j)$ can be related to the original data field $f(x, y)$ via

$$g_0(i, j) = D_0 f(x, y) \quad (3)$$

Figure 1 in Koch *et al* is a plot of D_0 vs a normalized wavelength $\lambda^* = \lambda/L$, where $L = 2\Delta n$, where Δn could be interpreted as the mean station separation. The curves are for various values of the normalized weight parameter $\kappa_0^* = \kappa_0 / L^2$. This figure shows how different wavelengths are filtered during the first pass as a function of the weight parameter. The plot shows that we can get a fairly smooth analysis (short wavelengths are filtered out) for $\kappa_0^* = 1$ or greater, which means that $\kappa_0 \geq (2\Delta n)^2$.

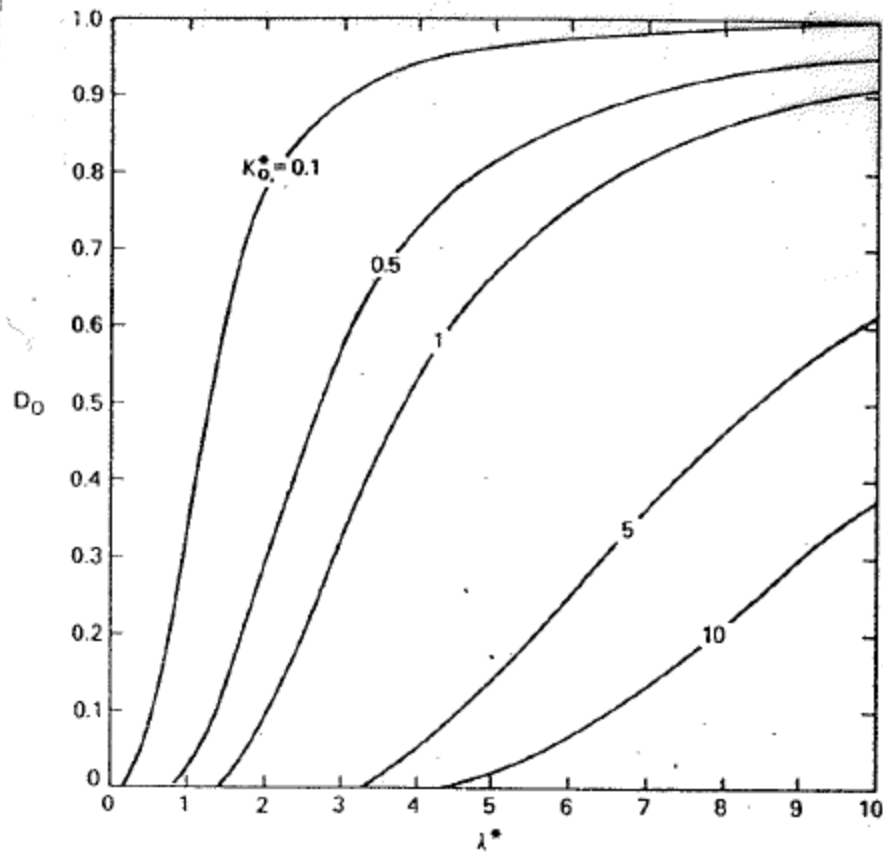


FIG. 1. Relationship of first pass response D_o to dimensionless wavelength $\lambda^* = \lambda/L$ for various values of dimensionless weight parameter $\kappa_o^* = \kappa_o/L^2$, in which L is an arbitrary scale length based upon the data distribution (e.g., $L = 2\Delta n$).

Barnes (1964) applied eq. (1) several times to allow the scheme to “recover” (retain) more amplitude in the medium scale waves that are overly smoothed in the first pass. However, we still want the amplitude of the $2\Delta n$ wavelength to be near zero.

Barnes (1973) and Koch *et al* (1983) show that we can obtain our “desired analysis” in 2 scans by modifying κ_0 on the second pass via $\kappa_1 = \gamma\kappa_0$, where γ is a “convergence factor”, $0 < \gamma < 1$. The smaller γ is, the closer one fits the observations.

The analysis equation for this second pass is :

$$g_1(i, j) = g_0(i, j) + D_1 [f(x, y) - g_0(x, y)] \quad (4)$$

where the $g_0(x, y)$ is obtained by interpolating the $g_0(i, j)$ field from eq. (3) to the stations, and where

$$D_1 = \exp [- \gamma\kappa_0 (\pi/\lambda)^2] = D_0^\gamma \quad (5)$$

is the response function for the modified weight function

$$w'_m = \exp [- r_m^2 / (\gamma\kappa_0)] \quad (6)$$

Since $\gamma\kappa_0 < \kappa_0$, w'_m is smaller for any r_m , and less weight is given to more distant observations than before, thus forcing the analysis closer to the nearby obs.

Note, however, that D_1 is not the total response to both passes. To get the total response D_1^* , we substitute eqs. (3) and (5) into (4) to get:

$$g_1(i, j) = D_0 f(x, y) + D_0^\gamma [f(x, y) - D_0 f(x, y)]$$

where we have assumed for the last term that the response on the (x,y) data field is the same as on the (i, j) gridded field. This can be rearranged to get

$$g_1(i, j) = D_0 [1 + D_0^{\gamma-1} - D_0^\gamma] f(x,y)$$

Therefore,

$$D_1^* = D_0 [1 + D_0^{\gamma-1} - D_0^\gamma] \quad (7)$$

The change in the response from the first to the 2nd pass for two different values of γ is shown in Fig. 3 in Koch *et al.* Note that $\gamma = 1$ means that the 2nd pass was made with no accelerated convergence. The greatest increase in response occurs at the shorter wavelengths. However, we want to make sure that we don't significantly increase the response at the $2\Delta n$ wavelength, which is usually noisy. Here, the increase is about 40%, which is worrisome; - ok if have accurate, well-distributed data; not ok if not the case.

Since D_1^* depends on γ , which we specify, and D_0 , which is a function of κ_0 , the main task w.r.t. designing the desired response is to devise a procedure to determine κ_0 .

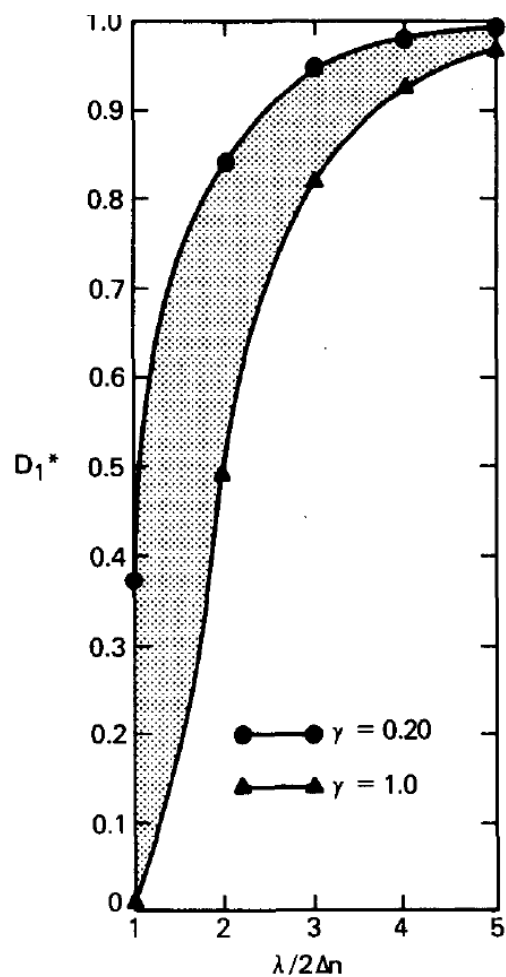


FIG. 3. Permissible second (correction) pass filter responses as functions of multiples of the $2\Delta n$ wavelength in the interactive Barnes scheme.

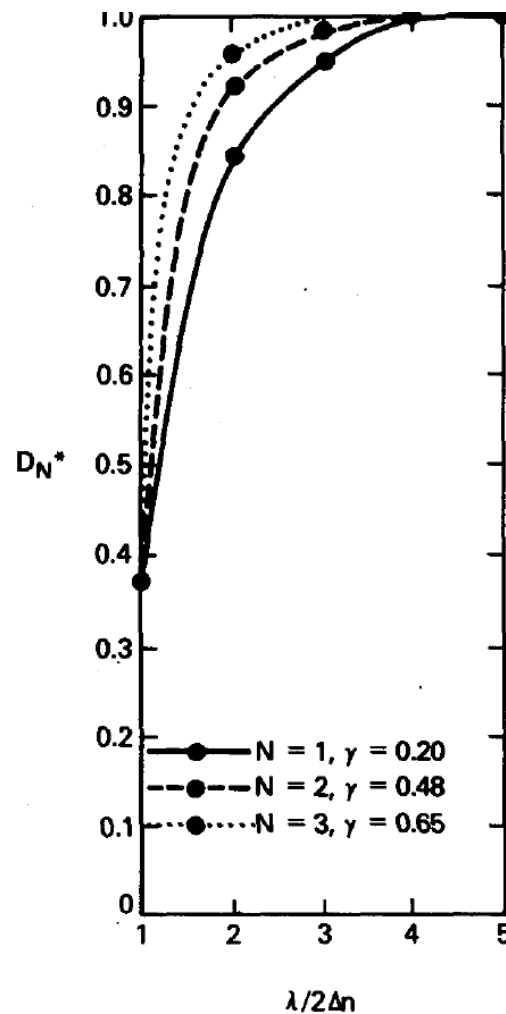


FIG. A2. Effective enhancement of filter response D_N^* produced by making multiple passes, as function of resolvable wavelengths $\lambda > 2\Delta n$.

First, though, we summarize the Barnes analysis scheme as presented by Koch *et al*:

1. Assuming we have no background field, the first pass is

$$g_0(i, j) = \frac{\sum_{m=1}^M w_m f(x_m, y_m)}{\sum_{m=1}^M w_m} \quad (8)$$

where the weight function w_m is given by eq. (1), and M is the number of observations within the cutoff radius R_c .

2. Interpolate $g_0(i, j)$ to stations to get $g_0(x_m, y_m)$
3. Compute the observation increment $f'(x_m, y_m) = f_m(x_m, y_m) - g_0(x_m, y_m)$
4. The second pass is then obtained from

$$g_1(i, j) = g_0(i, j) + \frac{\sum_{m=1}^M w'_m f'(x_m, y_m)}{\sum_{m=1}^M w'_m} \quad (9)$$

where the weight function is

$$w'_m = \exp[-r_m^2 / \gamma \kappa_0] \quad (10)$$

with $0 < \gamma \leq 1$.

Thus, control over what you want to accomplish with the Barnes analysis is obtained by *a priori* choices of κ_0 , γ , Δx , R_c , and the number of passes. Here R_c is a cut off radius which can be infinity.

An important difference between the Barnes scheme and the Cressman scheme is that Cressman scheme has a finite cutoff radius while that of the Barnes scheme does not have to be finite. For the finite cutoff radius, problem can arise when very few data are found within the cutoff radius. One solution is to increase the cutoff radius locally to ensure a minimum number of observations available, although this can introduce additional noise. See Koch paper for additional discussions in page 1490.

Tailoring the Barnes Scheme to Your Needs (according to Koch *et al*, 1983)

(a) Choice of domain

See text for discussion. One important point is to choose your objective analysis domain to be smaller than the data domain.

(b) Calculation of Δn

The mean distance between stations Δn plays an important role in the tailoring of the Barnes scheme. In Koch *et al*, Δn_c is the value that one computes from your station data. The quantity Δn_r is the spacing one obtains by assuming an even distribution of the M observations over the domain area A , which is given by

$$\Delta n_r = A^{1/2} \frac{1+M^{1/2}}{M-1}, \text{ which goes to } \left(\frac{A}{M}\right)^{1/2} \text{ for large } M.$$

For example, if $M = 100$, $\Delta n_r = 1/9 A^{1/2}$, whereas the simple formula gives $1/10 A^{1/2}$.

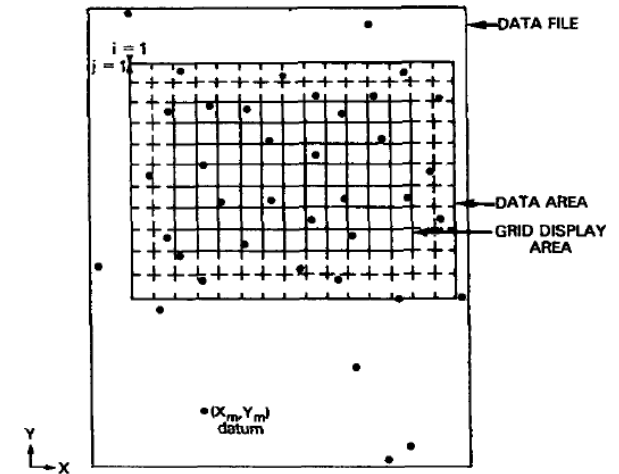
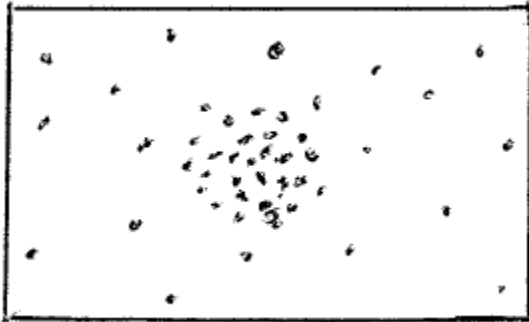


FIG. 2. Schematic illustration of data and mesh domains used within the interactive Barnes scheme.

If you have poor data distribution, then $\Delta n_c \ll \Delta n_r$, and a value of Δn larger than Δn_c should be selected. E.g., if the distribution looks like:



we have $\Delta n_r > \Delta n_c$, and we would need to use a value closer to Δn_r to avoid excessive noise in the analysis if we were using only one value of κ_0 . However, we could do either of the following:

- (i) Produce two analyses, one with $\Delta n = \Delta n_r$, and a second one just in the data dense region only with $\Delta n = \Delta n_c$ in that region.
- (ii) Compute Δn at every grid point from nearby stations, thus changing κ_0 at every grid point (as well as the response function).

(c) Control of detail in the analysis (via Koch *et al*, 1983)

This means that we will determine κ_0 by requiring the response function to have certain properties. First, we will specify $\gamma = 0.2$, which means we believe the data are accurate and well-distributed. (GEMPAK default value used to be 0.3). Recall that the total response of the resulting 2-pass Barnes scheme is eq. (7) from the previous lecture:

$$D_1^* = D_0 [1 + D_0^{\gamma-1} - D_0^\gamma] \quad (7)$$

where D_0 is given by eq. (2)

$$D_0 = \exp [- \kappa_0 (\pi/\lambda)^2] \quad (2)$$

Solving eq. (2) for λ , we have

$$\lambda = \pi \left[\frac{-\kappa_0}{\ln D_0(\lambda)} \right]^{1/2}$$

and if $\lambda = 2\Delta n$, we have

$$2\Delta n = \pi \left[\frac{-\kappa_0}{\ln D_0(2\Delta n)} \right]^{1/2}$$

If we take the ratio of these last two equations,

$$\frac{\lambda}{2\Delta n} = \left[\frac{\ln D_0(\lambda)}{\ln D_0(2\Delta n)} \right]^{1/2}$$

Thus we can solve for $D_0(\lambda)$ as a function of λ and Δn only (basically, it gives a formula for $D_0(\lambda)$ in terms of $D_0(2\Delta n)$, class exercise)

$$D_0(\lambda) = [D_0(2\Delta n)]^{\left(\frac{2\Delta n}{\lambda}\right)^2} \quad (11)$$

Now that we know $D_0(\lambda)$, we can use this in eq. (7) to determine the total response D_1^* . But we first need to determine a value for $D_0(2\Delta n)$. Koch et al determined this value by specifying that the total response D_1^* for the wavelength $2\Delta n$ should be e^{-1} ; that is, after 2 scans, the response at $\lambda = 2\Delta n$ should be $\sim .37$. (Note: This may be too high for many purposes.)

Working backwards via eq. (7), we can verify that this requires $D_0(2\Delta n) = 0.0064$.

We can now use eq. (2) with $\lambda = 2\Delta n$ to compute κ_0 :

$$\kappa_0 = - (2\Delta n/\pi)^2 \ln D_0(2\Delta n)$$

or
$$\kappa_0 = 5.052 (2\Delta n/\pi)^2 \quad (12)$$

Recall that (12) applies only for 2 passes with $\gamma = 0.2$, and $D_1^*(2\Delta n) = e^{-1}$.

Example:

Assume $\Delta n = 50$ km. Then κ_0 from (12) = 5119 km². This compares with $(\Delta n)^2 = 2500$ km². Thus using the above κ_0 value yields a smaller response for a given λ - more smoothing. [The opposite is true if you use $\kappa_0 \sim (\Delta n)^2 = 10,000$ km²]

Verify the following: In the first pass of this example, with $w_m = \exp[-r_m^2/\kappa]$,

we have $w_m = e^{-1}$ for $r_m = \sqrt{5119}$ km ~ 71 km, and $D_0 \leq e^{-1}$ for $\lambda \leq 225$ km.

For the 2nd pass, (with $\gamma = 0.2$), we have $w'_m = e^{-1}$ for $r_m \sim 32$ km, and $D_1^* \leq e^{-1}$ for $\lambda \leq 100$ km (which was what was specified above).

Thus we are significantly weighting only stations very close to the grid point on the 2nd pass. If this proves too noisy, could choose a larger γ or a smaller value for $D_1^*(2\Delta n)$.

(d) Choice of grid distance Δx

This choice should also be related to Δn , noting that the minimally resolved wave in the data is $2\Delta n$. However, we would like 5-6 grid points to resolve this wave and its first derivative. This suggests that

$$\Delta x < \Delta n/2.$$

In GEMPAK, Δx is recommended to be $(0.3 - 0.5) \Delta n$. A smaller Δx could be used, but it is not justified by the observations.

NOTE: A smaller Δx could be justified for an analysis used in NWP: If the large-scale forcing is accurate, and the physics is correct, a small-scale grid permits mesoscale (or convective-scale) features to develop in the forecast on scales much less than Δn .

(e) Quality control

[Note: this term usually means a procedure to eliminate bad or unrepresentative data, but this is not addressed; we are just using the Koch *et al* sub-category.]

(i) Choice of influence or cutoff radius R_c

R_c governs the largest value of r_m that is used in eq. (1). We could use all the data without harm, since the weight w_m just goes to zero for large r_m , so R_c is used simply to save computer time. However, it has to be large enough so that the weight is nearly = 0 when $r_m = R_c$ (i.e. – the analysis is not affected).

Two ways to select R_c include:

1. Choose R_c large enough such that all grid points are affected by at least, e.g., 3 data points – so that you get a reasonable analysis in the most data sparse region. One could go further and require that R_c is large enough such that there are data in at least 3 quadrants surrounding the grid point.
2. In Koch *et al*, they select $R_c = (20 \kappa_0)^{1/2}$. This makes $w_m = 2 \times 10^{-9}$ when $r_m = R_c$. We can put this in terms of grid size if we assume that $\Delta x = \Delta n/2$. Then, using eq (12), we have

$$R_c = (4\Delta x)/\pi (20 * 5.052)^{1/2} = 12.8 \Delta x.$$

This result yields the two ratios:

$$R_c / \Delta x = 12.8 ; \quad R_c / \Delta n = 6.4$$

(ii) Evaluation of the analysis

A common procedure is to compute the root mean square difference (rmsd) between the observations and the analysis interpolated to the stations (i.e. – how well does the analysis fit the data?). The rmsd will usually converge toward the data (i.e., approach 0) with increasing number of passes, but we don't want it to become zero if there are errors in the data; recall this is called overfitting.

What is an appropriate value of rmsd? One guideline is that it shouldn't be less than the rms error of observation for the data you are using. Therefore, you should know this error value in order to make good decisions in your analysis procedure. Note that it is not a wise goal to get the rmsd as close to zero as possible. Why?

Figure 6 in Koch *et al* (and Fig. 3.9 in Daley) shows the effects of different values of γ on an analysis of surface data in the Great Plains. Difference fields taken between the analyses would better show how smaller scales are “built back in” as γ becomes smaller.

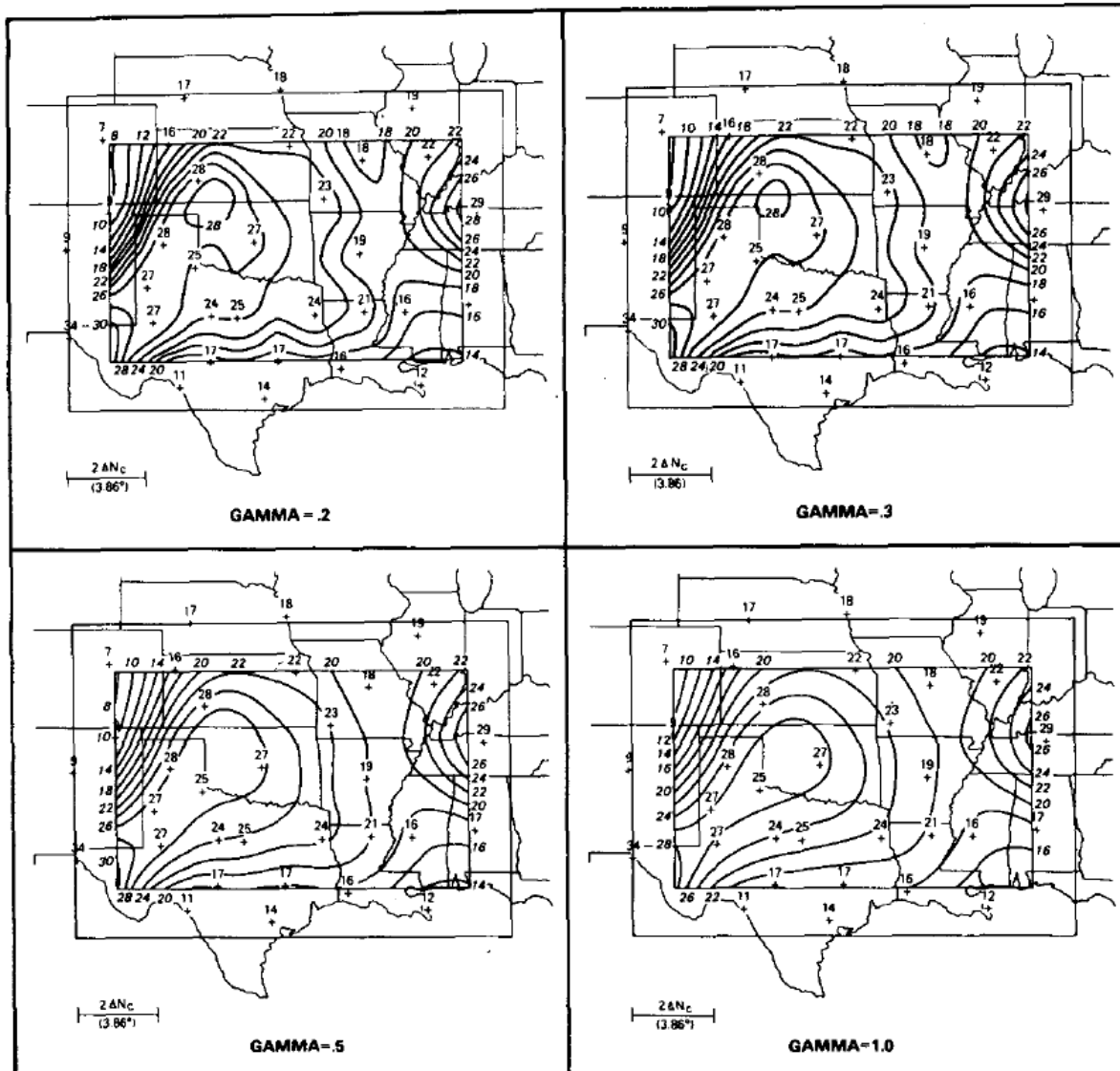


FIG. 6. Changes in the objective analysis of the SESAME RAWIN data set brought about by variations in the value input for γ (analysis with $\gamma = 0.3$ is identical to that in Fig. 5a):

Miscellaneous comments on the Barnes scheme

A. The Gaussian weight function can be used for any or all 4 dimensions

Examples include:

(a) Suppose one has many x - y analyses at successive times. We can introduce “time continuity” via

$$w_{m,n} = \exp [- r_m^2 / \kappa - t_n^2 / \tau]$$

over $n = 1 \sim N$ time periods. We choose τ using similar procedures to the choice of κ .

(b) Use on wind profiler time-height data:

Here we use $w_{m,n} = \exp [- z_m^2 / \kappa_z - t_n^2 / \tau]$

(c) Could be used on 3-D x - y - z analyses; x - z cross-sections; x - y - z - t , etc.

B. Alternative strategy to determine “best” analysis procedure

In 1994, Barnes wrote a series of three papers in *J. Atmos. Oceanic Tech.* to clarify some misconceptions users had about the Barnes scheme. The primary purpose was to clarify the four “selectable parameters” κ , γ , R_c and the number of passes n for different wavelengths and sampling distributions.

Barnes considered “well-sampled waves” to have 9 observations per wavelength if only that original field was desired, but if derivatives were calculated from the analysis, 13-15 observations per wavelength were required to prevent them from being too noisy.

Barnes defined “marginal under-sampling” as 5 obs. per wavelength, and true under-sampling to be ≤ 4 observations. It is possible to produce an acceptable analysis under these conditions if κ is chosen to be large, and you can accept noisy derivatives.

He shows how different station distributions affect the analysis. In general, a purely random distribution of stations will have 100-300% more error than the same analysis procedure applied to an evenly-spaced network.

Barnes’ strategy to produce the “best analysis” is:

1. Choose a domain that is as regularly sampled as possible. Have observations outside the domain. Create synthetic or “bogus” observations in data sparse regions to avoid uneven data distributions. (Could create these from a first pass with very large R_c and κ that produces a smooth field.)
2. Compute the uniformity ratio = $(\Delta n_r - \Delta n_c) / \Delta n_c$, where Δn_c and Δn_r are defined earlier.

The uniformity ratio is ≥ 0 since $\Delta n_r \geq \Delta n_c$. [Barnes used mean distance to 6 nearest stations to compute Δn_c]

The station distribution is “quasi-random” if the uniformity ratio is ~ 1.1 .

Note: Obs. that are *very* close together will adversely affect the Δn_c calculation. Often such obs. are averaged together to create a “super-ob”.

3. Determine the “bandwidth” one has in your data domain. For example, if you have only 5-6 stations across the domain width, you essentially have no bandwidth, since the shortest possible wavelength for

which you can get derivative information is also the longest. Recall that the longest or “fundamental wave” is the domain dimension; the shortest resolvable wave is $2\Delta n_r$, with $\lambda \geq 2\Delta n_r$ needed to get derivative information.

4. Perform tests with an analytic function that contains the permissible waves. Use the observed station distribution and choose a large R_c . Explore a reasonable range of κ , γ , and n values.
5. Compute the mean absolute error at grid points (m.a.e.g) for the different analyses and their derivatives. We can do this rather than the rmsd at stations because we know the truth at the grid points via the analytic fields. Thus it is permissible to get the m.a.e.g as small as possible. Determine what combination of κ , γ , n values minimizes the m.a.e.g.

Barnes (and others) have found that 3-4 passes with $\gamma = 1$ (no accelerated convergence) yields the best analysis for the purpose of computing derivatives. This leaves only κ to determine, and it is selected to minimize m.a.e.g.

References:

Barnes, S. L., 1994: Applications of the Barnes Objective Analysis Scheme. Part I: Effects of Undersampling, Wave Position, and Station Randomness. *J. Atmos. Oceanic Tech.*, **11**, 1433-1448.

Barnes, S. L., 1994: Applications of the Barnes Objective Analysis Scheme. Part II: Improving Derivative Estimates. *J. Atmos. Oceanic Tech.*, **11**, 1449-1458.

Barnes, S. L., 1994: Applications of the Barnes Objective Analysis Scheme. Part III: Tuning for Minimum Error. *J. Atmos. Oceanic Tech.*, **11**, 1459-1479.