# Numerical Weather Map Analysis<sup>1</sup>

## By PALL BERGTHORSSON<sup>2</sup> and BO R. DÖÖS, University of Stockholm (Manuscript received June 30, 1955)

### Abstract

A method to analyze upper air charts numerically is presented. The analysis is expressed by the height values of the pressure surface in gridpoints. The computed height in a gridpoint is obtained as a weighted mean of height values derived from the surrounding height and wind observations, the forecast height in the point and the corresponding normal height. Nine 500 mb maps are analyzed with the aid of the Swedish computor BESK, six of these consecutive. The size of the grid was  $32 \times 41$  points.

The analyses have been compared with two independent conventional analyses. The mean values of the root mean square of the differences between the numerical and the conventional analyses were 26 m and 24 m respectively and 26 m between the two conventional ones.

The root mean square of the differences between the observed and analyzed heights was 22 m in the mean. This is roughly what should be expected judging from the existing knowledge about observation errors.

Three barotropic forecasts have been computed from the numerical analyses. They are compared with the corresponding numerical forecasts from conventionally analyzed maps. It was not possible to find any significant difference between the goodness of the forecasts based on the numerical analyses and the conventional analyses.

#### **1.** Introduction

The first attempts at numerical weather forecasting on a routine basis have been characterized by a combination of tedious manual work on one hand and electronic computations with extremely high speed on the other. The weather observations are plotted on maps, examined and analyzed. From this manual analysis values are interpolated at a great number of gridpoints and punched on a paper tape, which afterwards is checked and recopied. Finally the electronic computer can start the forecasting procedure.

The manual part of these operations consumes time that is out of proportion to the time required for the machine computation. This, however, is not the only disadvantage.

Tellus VII (1955), 3

The manual analyst cannot be expected to use systematic and quantitative methods in his interpolations and extrapolations. His work is rather a complicated curve-fitting by the eye based on a number of more or less wellestablished rules. The analysis will, in other words, be subjective and depending on the skill of the meteorologist. It is furthermore very difficult to avoid wiggles and irregularities of small scale which are neither desirable nor justified by observations. These may frequently amplify in the forecast computation and thus reduce the value of the final forecast. Errors in the reading and punching of values in gridpoints are also highly probable.

This leads to the conclusion that it would be desirable to device a method to perform the analysis with the computer. Already in the early stage of experiments with numerical forecasting, PANOFSKY (1949) presented a method of objective analysis. Later SMAGO-RINSKY (unpublished), ELIASSEN (1954), GIL-CHRIST and CRESSMAN (1954), and VÄISÄLÄ (unpublished) have studied this subject. These authors have presented methods which are

<sup>&</sup>lt;sup>1</sup> Part of the research reported in this document has been sponsered by the Geophysics Research Directorate of the Air Force Cambrigde Research Center, Air Research and Development Command, United States Air Force, under contract No. 61 (514)--648-C, through the European Office ARDC.

<sup>&</sup>lt;sup>2</sup> Now at the Icelandic Weather Service, Reykjavik, Iceland.

applicable for interpolation between observed values, to obtain the most probable values in gridpoints.

In our investigation of this problem at the University of Stockholm, we reached the conclusion that quite often it is not possible to get a reasonable analysis only by means of interpolation between synoptic observations. It is quite clear that the distance between the observations must be small compared with the size of the systems to be analyzed. This is certainly not the case in many areas as over the oceans. In such cases any interpolation method will fail, independent of whether it is linear, quadratic or cubic. If, however, some observations were available in the area 12 hours ago, a twelve hour forecast is probably a better approximation than the interpolated analysis. The application of the forecast for the analysis has indeed been suggested by Smagorinsky and Gilchrist and Cressman, and it is a rather natural way of keeping time continuity in the maps.

If the forecast in the case discussed above is of little value, no observations being available when it was made, it is quite possible that even climatological informations will give the most reasonable solution. This is particularly true in low latitudes, where the deviations from normal patterns are surprisingly low. The suggestion of using the normals is due to MR. CHARASCH. At this point it may be adequate to point out, that we can not hope to get a *true* analysis. What we can expect is to obtain the *most probable* analysis. This may serve to justify the use of the climatological informations together with the forecast and the observations.

#### 2. Method of analysis

The following investigation has been restricted to the analysis of the 500 mb flow pattern. However, the same method could in principle be applied to other levels.

As informations we have used the observations of wind and height at the 500 mb surface, the 12 or 24 hour barotropic forecast valid for the same time as the analysis, and the normal height of the 500 mb level for the particular month when the analysis is made.

The principle of the analysis is the following:

We start out with the best available approximation of the 500 mb map. This preliminary field is then modified as far as possible with available observations. The analysis we obtain in this manner can then be used as a preliminary field which again is modified by the observations. We found, however, that in



Fig. 1. Percentage contribution of the forecast in constructing the preliminary field. Tellus VII (1955). 3

most cases fairly satisfactory results were obtained in the first try.

The first preliminary 500 mb heights  $(Z_p)$  are constructed as a weighted mean of the forecast heights  $(Z_f)$  and the normal heights  $(Z_N)$ . For each gridpoint we thus obtain

$$Z_p = \frac{\mu_f Z_f + \mu_N Z_N}{\mu_f + \mu_N} \tag{1}$$

 $\mu_f$  and  $\mu_N$  being the weights of the forecast and normal heights respectively. We assume that  $\mu_f$  is only a function of geographical position and season. Assuming furthermore that the deviations from normals are not correlated with the deviations from the forecast, we can put

$$\mu_f = \frac{\text{const.}}{\sigma_f} \tag{2}$$

in other words, the weight of the forecast is inversely proportional to the root mean square of the differences between observed and forecast heights for a great number of forecasts. In the same manner the weight of the normal in each point is determined as:

$$\mu_N = \frac{\text{const.}}{\sigma_N} \tag{3}$$

 $\sigma_N$  being the root mean square of the deviation of the daily values Z from  $Z_N$ . Fig. 1 shows the geographical distribution of the ratio  $\mu_f/(\mu_f + \mu_N)$  expressed in per cent. The computed height in a gridpoint will now be expressed as a weighted mean of height values derived in three different ways from the surrounding observations and the preliminary Z-field given by (1). From each station observing the 500 mb height and the wind and not farther away from the gridpoint than about 900 km, these three approximate height values for the gridpoint are derived in the following way:

1) Assuming that the difference between the observed height  $(Z_{os})$  and the preliminary height at the station  $(Z_{ps})$  is the same as the difference between the derived height  $(Z_1)$  and the preliminary height  $(Z_{pg})$  in the gridpoint (fig. 2 a), we find:

$$Z_1 = Z_{pg} + (Z_{os} - Z_{ps}) \tag{4}$$

2) Assuming that the observed wind is geostrophic and representative for the area Tellus VII (1955), 3



Fig. 2. Schematic illustration of the derivations of the heights  $Z_1$ ,  $Z_2$  and  $Z_3$ .

between the gridpoint and the station, one can compute the corresponding gradient of Zand finds another approximate height value  $Z_2$  in the gridpoint (fig. 2 b)

$$Z_2 = Z_{os} + \left(\frac{\partial Z}{\partial n}\right)_{os} \cdot l \qquad (s)$$

*l* being the distance between the station and the gridpoint.



Fig. 3. A graphical representation of the weighting functions.

3) Assuming the gradient of the preliminary field *in the gridpoint* to be representative for the area between the station and the gridpoint, the height in the point will be (fig. 2 c)

$$Z_{3} = Z_{os} + \left(\frac{\partial Z}{\partial n}\right)_{pg} \cdot l \tag{6}$$

We have next assumed that the weights of these three heights,  $Z_1$ ,  $Z_2$  and  $Z_3$ , are functions only of the distance between the gridpoint and the station. These weighting functions can be determined statistically with the following method.

We form the regression equation

$$Z_g = \frac{\mu_1 Z_1 + \mu_2 Z_2 + \mu_3 Z_3 + \mu_f Z_f}{\mu_1 + \mu_2 + \mu_3 + \mu_f} \qquad (7)$$

For a given distance from the station we now require that the mean square difference

between  $Z_g$  and the conventionally analyzed height shall be a minimum for a great number of cases. This will determine the ratio between the coefficients  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$  and  $\mu_f$ . Since  $\mu_f$  is assumed to be known from the special investigation of forecast errors this will give  $\mu_1$ ,  $\mu_2$  and  $\mu_3$ . This computation can then be repeated for different distances from the station and thus determines the weighting functions  $\mu_1$ ,  $\mu_2$  and  $\mu_3$ . This investigation gave the result that these weighting factors can be approximated by the following expressions:

$$\mu_1 = \frac{30}{r^4 + 150} - 0.04 \tag{8}$$

$$\mu_2 = \mu_3 = \frac{27}{r^8 + 70} \tag{9}$$

where r is given in gridunits. Having modified the preliminary height in one point by one such station we have

$$Z_{g} = \frac{\mu_{f}Z_{f} + \mu_{N}Z_{N} + \mu_{1}Z_{1} + \mu_{2}Z_{2} + \mu_{3}Z_{3}}{\mu_{f} + \mu_{N} + \mu_{1} + \mu_{2} + \mu_{3}} \quad (10)$$

In the case where the station reports height only or if it is so far from the gridpoint, that the wind is of no value as an information (beyond ca 900 km) we only use assumption 1) which was discussed above in connection with stations observing both height and wind. We obtain

$$Z_h = Z_{pg} + (Z_{os} - Z_{ps}) \tag{II}$$

The weighting function  $\mu_h$  was found to be quite different from  $\mu_1$ . This is because  $Z_h$ is the only contribution from the station.  $\mu_h$ will therefore be higher than  $\mu_1$  near the station (cf. fig. 3). It was found to be of little use to apply the height  $Z_h$  from stations beyond 1,500 km, as the weight  $\mu_h$  then becomes very small. The function  $\mu_h$  can be approximated by

$$\mu_h = \frac{2.25}{r^8 + 5} + \frac{10}{r^4 + 20} - 0.01 \qquad (12)$$

Having modified the preliminary height in one point by one such station using the observed height only, one obtains

$$Z_{g} = \frac{\mu_{f} Z_{f} + \mu_{N} Z_{N} + \mu_{h} Z_{h}}{\mu_{f} + \mu_{N} + \mu_{h}}$$
(13)  
Tellus VII (1955). 3

In order to test how sensitive the analysis is to the shape of the weighting functions (8), (9) and (12) we changed these functions so that  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$  and  $\mu_h$  were twice as large for r = 0. In recomputing one analysis we found that the difference between the analyses was insignificant, at the most 10 m in single points.

In our previous discussion we have only considered one station contributing to the height in a gridpoint. In the case of several stations influencing the height at the point, another factor has to be taken into account. Some of the stations may be very close to each other and therefore give contributions that are correlated, while other stations are isolated and give contributions which are more or less uncorrelated to those of other stations. The weight of the contributions from a station should therefore be reduced. For this reduction we have used a factor inversely proportional to the density of stations surrounding the particular station, i.e.  $\frac{\mathbf{I}}{\varrho}$ . We have evaluated  $\varrho$ as the number of stations within a radius of ca 375 km, the station itself also being counted. The ultimate height at the gridpoint will thus be

tribution  $Z_h$  having the weight  $\mu_h$ . We know the height  $Z_g$  before the modification and the sum of weights  $\Sigma \mu$  used to compute it. The modified height  $Z'_g$  will then be:

$$Z_{g}' = \frac{\sum \mu Z_{g} + \frac{1}{\varrho_{s}} \mu_{h} Z_{h}}{\sum \mu + \frac{\mu_{h}}{\rho_{s}}}$$
(15)

where  $\rho_s$  is the station density at this particular station. After computing  $Z'_g$  we store the quantities  $Z'_g$  and  $\Sigma\mu' = \Sigma\mu\frac{\mu_h}{\rho_s}$  for this point, and the height can later be modified by another station in a similar way.

### 3. The computational program

The method of analysis discussed in the previous section has been carried through with the aid of the Swedish computor BESK.

The computational program can be outlined in the following manner (cf. fig. 4).

I. The entire code is read and stored on the magnetic drum.

II. The data tape, which contains the coordinates of the stations, the heights of the pres-

$$Z_{g} = \frac{\mu_{f} Z_{f} + \mu_{N} Z_{N} + \sum_{i=1}^{n} \frac{1}{\varrho_{i}} [\mu_{1} Z_{1} + \mu_{2} Z_{2} + \mu_{3} Z_{3}]_{i} + \sum_{j=1}^{m} \frac{1}{\varrho_{j}} [\mu_{h} Z_{h}]_{j}}{\mu_{f} + \mu_{N} + \sum_{i=1}^{n} \frac{1}{\varrho_{i}} [\mu_{1} + \mu_{2} + \mu_{3}]_{i} + \sum_{j=1}^{m} \frac{1}{\varrho_{j}} [\mu_{h}]_{j}}$$
(14)

Here *n* denotes the number of stations influencing the height at the point and giving the contributions  $Z_1$ ,  $Z_2$  and  $Z_3$ , and *m* is the number of stations which give contributions from the observed heights only.

This formula is very suitable for numerical computations. One can for example compute the heights in the gridpoints one by one, in every case using all stations that influence the point. We have, however, chosen the method to compute at a time the contributions from one station to the height at all gridpoints affected by the station. Thus it is possible to feed the observations into the machine in an arbitrary order and thus also use stations that arrive during the computations.

Suppose for example that we are going to modify the height at a gridpoint by the con-Tellus VII (1955), 3 sure surface and the wind direction and speed, is read for the first time. At this moment only the density of stations for each station is computed. These values are stored on the magnetic drum for later use.

III. The twelve-hour forecast, based on the previous map, is read, converted from the decimal to the binary system and stored on the drum. It is possible to use the output tape from the barotropic forecast.

IV. The normal heights are read from tape and converted from decimal to binary. Thereafter the weights  $\mu_f$  and  $\mu_N$  for each gridpoint are read. The weighted mean of the normal and the forecast (the preliminary field) is computed and stored.

V. The data tape is read for the second time. In order to save space in the storage only a



Fig. 4. Flowdiagram of the computations. N denotes the total amount of stations on the data-tape.

few (5) stations are read from the tape at a time. For each one of these stations the height of the pressure level is converted from decimal to binary, the geostrophic gradient of the two components of the wind (if it is a windstation) is computed. The densities, which were computed in part II, are extracted for these stations.

VI. Treating one station at a time, the height and the two components of the wind are compared with the corresponding values of the preliminary field. If the difference between the observed height and the interpolated height of the preliminary field exceeds the tolerance (100 m), the coordinates of the station and the both heights are printed to be available for visual inspection. If the corresponding differences for the wind exceeds the tolerance (30 kts), the coordinates and the observed and interpolated gradient are printed. It is possible at this point to disregard the observation if one can conclude that it is erroneous. Thereafter the observation (if it is accepted) is used to modify the heights in the surrounding grid points according to (15). After these five stations have been treated five new stations are read from the tape using part V. This is repeated until all stations have been considered.

VII. The analysis is now completed. Using part V to read the data tape the observations are now tested in the same manner as in part VI. However, the observations are now tested against the analysis instead of the preliminary field. The tolerances are now more strict (50 m for the height and approximately 25 kts for the geostrophic gradient). If an observation now is suspected to be erroneous it is possible by a correction-tape either to remove the observation or to correct it. VIII. The result of the analysis is now printed. The analysis is stored in the machine so that it is possible to use it for a barotropic forecast immediately.

#### 4. Results

The computations were made over a rectangular grid consisting of  $32 \times 41$  points and the gridsize was 300 km at 50°N (fig. 5). The time required to compute the analysis was 22 minutes using about 500 pieces of informations, counting a height as one and a wind as two. For a station which reports both height and wind it takes about two and a half second to modify the heights in the surrounding grid-points and for a station which only reports height about two seconds. (At the BESK the time for addition is 56 microsec and the time for multiplication is 364 microsec).

In all nine analyses have been computed: 26 September 1500 GMT—29 September 0300 GMT 1954, a series consisting of six analyses with 12 hour intervals, 22 January 0300 GMT, 23 January 0300 GMT 1955 and 23 May 1500 GMT 1955. In fig. 6, 8 a and 9 three of these analyses are shown.

The last analysis was carried out on a routine basis in cooperation with the Weather Service of the Royal Swedish Air Force. This was done in order to investigate how much it is possible to gain in time by using this method of analysis. The observations were checked and punched as soon as they arrived. At 2300 GMT (8 hours after the observation time) most of the observations were received. The data tape was then recopied and checked. A few observations, which had arrived in the meantime, were added. At about 24 GMT the data tape was ready for the machine.



Fig. 5. Location of the grid used in analyses and forecasts. The two circles indicate the areas from which observations can influence the height in a gridpoint. The inner circle represents the influence area of a wind observation, the outer circle the corresponding area for a height observation.

observation, the outer circle the corresponding area for a height observation. The verification of the forecasts was done over the area inside the dashed line. The dotted line indicates the area used in the comparison presented in table I, (the values in brackets).



Fig. 6. Numerical analysis of the 500 mb contours on September 26, 1500 GMT, 1954. Heights given in decameters at the stations. Contour lines tabelled in m. Tellus VII (1955), 3

This experiment showed that it is possible to save about  $3^{1/2}$  hours in comparison with the manual preparation of the analysis. The possibility of clerical errors is also reduced. Further, it may be regarded as an advantage that the same set of observations and a given forecast will give a unique solution of the analysis; it is in other words objective.

The most important question, however, is if this numerical procedure gives us as true a picture of the actual conditions as the conventional method.

This is quite difficult to test. As the manual analysis generally represents considerable synoptic knowledge and experience, it is not unreasonable to require that the numerical analysis show no considerable deviations from the manual analysis.

In order to test this we have compared the numerical analyses (N) with two independent conventional analyses  $(C_1 \text{ and } C_2)$  of the same synoptic situations. The analyses  $C_1$  and  $C_2$ were made in daily service at the Swedish Weather Service (SMHI) and at the Institute of Meteorology of the Stockholm University. The root mean square of the differences  $N-C_1$ ,  $N-C_2$ , and  $C_1-C_2$  were computed and are shown in Table I. The comparison was made over the whole map except in certain areas close to the boundaries where some of the conventional analyses were incomplete. The comparison was also made over a small area (indicated in fig. 5) where the station density is comparatively dense.

In computing the preliminary field 12<sup>h</sup> barotropic forecasts were used in all analyses except for 22/1 and 23/1 0300 GMT, 1955, where 24<sup>h</sup> barotropic forecasts were used. These forecasts were computed from numerically analyzed maps when it was possible. Thus, in the period from 26/9 1500 GMT to 29/9 0300 GMT, 1954 which consists of 6 consecutive analyses, 12 hours apart, it was only necessary to use a conventionally analyzed map for computing the 12<sup>h</sup> forecast from 26/9 0300 GMT 1954. In spite of this the goodness of the analyses did not decrease during the period (cf. tables). Table I shows that generally the differences between the two conventional analyses are approximately the same as the differences between the numerical and conventional analyses. It can further be seen th at the analysis is comparable to the conventional analysis also in regions with relatively sparse observations.

Another possible test is to compute the root mean square of the differences between analyzed and observed heights at the stations. The result of this investigation is shown in Table II. If we had obtained the true analyses this root mean square difference should be approximately the same as the standard error in height

Date	$\sqrt{\frac{\sum (N-C_1)^2}{p}}_{(m)}$	$\sqrt{\frac{\Sigma(N-C_2)^2}{p}}_{(m)}$	$\sqrt{\frac{\Sigma(C_1-C_2)^2}{p}}_{(m)}$
$\begin{array}{c} 26/9 & -54 & 15 & \dots & \\ 27/9 & -54 & 03 & \dots & \\ 27/9 & -54 & 15 & \dots & \\ 28/9 & -54 & 03 & \dots & \\ 28/9 & -54 & 03 & \dots & \\ 29/9 & -54 & 03 & \dots & \\ 23/5 & -55 & 15^1 & \dots & \\ \end{array}$	23 (24)	22 (24)	29 (30)
	26 (19)	27 (11)	29 (21)
	26 (19)	26 (22)	28 (18)
	27 (26)	26 (25)	23 (18)
	31 (25)	28 (20)	25 (18)
	25 (23)	22 (15)	25 (22)
	23 (11)	14 (11)	22 (12)
Mean	26 (21)	24 (18)	26 (20)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	25 (24)	23 (23)	23 (18)
	41 (21)	48 (29)	36 (26)
	44 (36)	50 (37)	36 (29)

Table I.

The values in brackets refer to the comparison in the small region.

<sup>1</sup> This analysis was done on an operational basis.

<sup>2</sup> This analysis was obtained by recomputing the analysis, now using the first analysis as the preliminary field.

<sup>3</sup> In computing these analyses a 24<sup>h</sup> forecast was used for the construction of the preliminary field.

Tellus VII (1955), 3

Та	ble	п.
_		

Date	$\sqrt{rac{\Sigma (N-O)^2}{q}}_{(m)}$	Number of stations (q)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	23 22 23 24 22 22 22 20	230 231 228 233 213 237 247
Mean	22	
$\begin{array}{r} 28/9 - 54 \ 03^2 \\ 22/1 - 55 \ 03^3 \\ 23/1 - 55 \ 03^3 \end{array}$	19 26 27 as for table I	233 223 209



observations. Unfortunately the observation errors are not very well known. From the comparison of different types of radiosondes in Payerne 1950 (NYBERG, 1952) and an investigation by RAAB and RODSKJER (1950) one can estimate it to be 20—30 m at the 500 mb level. The results in Table II therefore seem to indic ite that the smoothing in the numerical

Fig. 7. Geostrophically computed 500 mb windprofile (the curve) compared with wind observations (dots).

analysis which is responsible for the differences between the analysis and observations, is of the right order of magnitude. In order to study the smoothing we recomputed one of the analyses (Jan. 23, 0300 GMT, 1955). In this



Fig. 8 a. Numerical analysis of the 500 mb contours on September 27, 0300 GMT, 1954. Tellus VII (1955), 3







we introduced one error by increasing the height value at Helsingfors with 270 m. The largest difference between the two analyses was 35 m which occurred in two gridpoints. The difference then decreased roughly by 10 m per gridunit. The decrease was a little less in the direction where the station network was less dense.

This indicates that in areas where the station density is relatively good, the method is relatively insensitive to moderate errors at individual stations.

It is interesting to investigate how the computed height gradients agree with the observed winds. As an example we show here the profile of a jet stream over the English Channel on the 27th of September 1954 at 0300 GMT. In fig. 8 a the position of this cross-section A-B is shown. In fig. 7 the solid curve represents the computed geostrophic velocities in gridpoints close to this line. The dots represent observed winds close to the line. Generally the deviations are not more than about 5 knots. It is of special interest that the peak of the jet stream seems to be very well represented, considering that finite differences over 600 km were used in computing the geostrophic wind. On the whole the computed profile seems to be smoother than the observed one. This is, however, not unreasonable, considering the influence of observation errors and local wind variations, which cannot be represented by the finite differences in the wind computation. In one case, at the station Bordeaux in France, this smoothing results in an almost complete neglection of an observed wind. It is not easy to prove that this neglection of the wind is justified. If the observation were right, it would, however, indicate a relative anticyclonic shear much stronger than the cyclonic shear on the northern side of the jet, and this seems to be unlikely.

Another and perhaps the most important test is how the forecasts based on a numerical

Fig. 8 d. 500 mb contours on September 30, 0300 GMT 1954.

Tellus VII (1955), 3

Fig. 8 b. 72h forecast of 500 mb contourfield from September 27, 0300 GMT, 1954. The initial map numerically analyzed.

Fig. 8c. 72h forecast of 500 mb contourfield from September 27, 0300 GMT, 1954. The initial map conventionally analyzed.

	24 <sup>h</sup> forecast				48 <sup>h</sup> forecast						72 <sup>h</sup> forecast							
Date	N	ımeri Anal.	cal	Conventional Anal.			Numerical Anal.			Conventional Anal.			Numerical Anal.			Conventional Anal.		
	r	<i>є</i> (m)	$\frac{\varepsilon}{\sigma_x}$	r	<i>е</i> (m)	$\frac{\varepsilon}{\sigma_x}$	r	е (m)	$\frac{\varepsilon}{\sigma_x}$	r	е (m)	$\left  \frac{\varepsilon}{\sigma_x} \right $	r	е (m)	$\frac{\varepsilon}{\sigma_x}$	r	<b>е</b> (m)	$\frac{\varepsilon}{\sigma_x}$
27/9—54 03 28/9—54 03 29/9—54 03	0.72 0.78 0.82	38 39 39	0.79 0.52 0.57	0.53 0.88 0.83	47 39 39	0.89 0.52 0.57	0.71 0.77 0.86	63 61 61	0.77 0.67 0.54	0.64 0.83 0.80	73 57 69	0.84 0.56 0.63	0.74 0.73 0.70	88 87 113	0.85 0.71 0.79	0.49 0.76 0.67	98 82 117	0.96 0.67 0.81
Mean	0.77	39	0.64	0.75	42	0.67	0.78	62	0.66	0.76	66	o.68	0.72	96	0.78	0.64	99	0.81
28/9-54 03 <sup>2</sup>	0.84	37	0.52	o.88	39	0.52	0.77	61	0.67	0.83	57	0.56	0.72	88	0.71	0.76	82	0.67
Footnote, the same as for table I.																		

	Tabl	e III
--	------	-------

analysis compare with those based on a conventional analysis. For this test it is, however, necessary to have abundant material. It is quite possible that in single cases a good analysis may give a worse numerical forecast than a forecast based on a bad analysis, due to errors in the model or errors in the verification map.

We have made 24, 48 and 72 hour forecasts from three of the analyses discussed above. Besides the correlation coefficient (r), we have computed the root mean square of the differences between observed and computed changes  $\varepsilon$  and the ratio  $\frac{\varepsilon}{\sigma_x}$ , where  $\sigma_x$  is the root mean square of the observed changes.

These results were compared with the results from numerical forecasts based on the same maps but analyzed with conventional methods (Table III).

As far as one can draw conclusions from



Fig. 9. Numerical analysis of the 500 mb contours on May 23, 1500 GMT, 1955. Tellus VII (1955), 3

these few results there seems to be no significant difference between the goodness of the forecasts based on our analyses and the conventional analyses.

One of the  $72^{h}$  barotropic forecasts from a numerically analyzed map (fig. 8 a) is shown in fig. 8 b. The corresponding forecast from a conventionally analyzed map is shown in fig. 8 c. The verification map which also is conventionally analyzed is shown in fig. 8 d.

One of the analyses (28 September 0300 GMT, 1954) was recomputed. This time the first analysis was used as the preliminary field. As can be seen from Table I, the difference between the first and second analysis is comparatively small. In certain areas, however, the analysis was improved, especially where the forecast was unsatisfactory. One example: The observed height at the Russian station (28–900) was 5,470 m. In the conventional analysis there was a closed low with its center close to a gridpoint half a gridunit south of that station. The first preliminary height at this gridpoint was 5,580 m, i.e. about 110 m higher than the conventional analysis. The first computed analysis gave the height 5,536 m. The difference between the conventional analysis and the numerical one thus was 66 m. In the second analysis the height in the same gridpoint was 5,514 m. Thus the recomputation reduced the height further 22 m.

Close to Stockholm a similar improvement of the analysis was obtained. The 24<sup>b</sup> forecast was slightly better (cf. Table III) when the second analysis was used, while the 48<sup>h</sup> and 72<sup>h</sup> forecasts did not change appreciably.

in angener Ner en in

#### Acknowledgement

The authors wish to thank Professor C.-G. Rossby for his great interest and encouragement. To Dr. B. Bolin, who suggested this work, we are greatly indebted for reading the manuscript. We also thank Dr. G. Dahlquist and Mr. E. Charasch for valuable discussions.

Finally we wish to thank Miss S. Back, who prepared all the tapes for the machine computations and Mrs. B. Dahlborg who carried out the drafting.

#### REFERENCES

- ELIASSEN, A., 1954: An Attempt to formulate the Problems encountered in the Study of the Requirements of Upper Air Network for Numerical Forecasting. *Report from the Institute of Theoretical Astrophysics*, *Blindern, Norway*.
- 1954: Provisional Report on Calculation of Spatial Covariance and Autocorrelation of the Pressure field. Institute for Weather and Climate Research, The Norwegian Academy of Science and Letters. Publication and Reports July 1, 1953—June 30, 1954.
- GILCHRIST, B. and CRESSMAN, G. P., 1954: An Experiment in Objective Analysis. Tellus 6, pp 309-318.
- NYBERG, A., 1952: On the comparison of Radiosonde Data in Payerne May 1950. Sveriges Meteorologiska och Hydrologiska Institut, Communication, Series B, No.9.
- PANOFSKY, J., 1949: Objective Weather Map Analysis. Journal of Meteorology, 6, pp 386—392.
  RAAB, L. and RODSKJER, N., 1950: A Study of the Ac-
- RAAB, L. and RODSKJER, N., 1950: A Study of the Accuracy of Measurements of the Väisälä Radiosonde, Arkiv för Geofysik, Bd 1, Nr 2.

Tellus VII (1955), 3