OBJECTIVE WEATHER-MAP ANALYSIS

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ABSTRACT

Wind and pressure fields are fitted by third-degree polynomials in areas of the order of 10^6 square miles. Expressions involving derivatives of wind and pressure are computed and the question of computation of geostrophic deviations is re-examined. A method of connecting polynomials in separate areas is investigated. The following conclusions are drawn:

1. Isopleths and streamlines drawn from the polynomials greatly resemble subjective isopleths and streamlines. In all cases studied, the smoothing seems to be adequate.

2. Horizontal divergence and vertical velocities can be determined as well from the polynomials objectively as by other subjective methods. The errors of observation influence the magnitude of these quantities considerably, but usually do not affect the sign.

3. On the scale of these measurements, reliable pressure gradients can be obtained objectively; however, the Laplacian of pressure is very much affected by the technique of analysis and by observational errors.

4. Reliable values of the geostrophic deviations can be obtained only under favorable conditions. Hence any method of integration of the fundamental equations which requires knowledge of the geostrophic deviations is to be avoided.

1. Introduction

One of the aims of the meteorological computer project at Princeton is the application of the hydrodynamic equations to forecasting. Initial conditions for this problem would be furnished by the observations at a starting time, t_0. At this time it would be required to know the meteorological variables and some of their space derivatives at certain grid points. This could be accomplished by the normal modes of analysis, that is, by subjectively drawing isopleths, and approximating the derivatives by ratios of finite differences.

A disadvantage of this process is that it is subjective; another, that the data must first be plotted. These disadvantages could be avoided if the meteorological variables were objectively related to the space coordinates by analytic functions, for then the coded weather reports could be translated into initial conditions without human interference.

The problem is then to find an analytic function $p(x, y, z)$ which represents the distribution of the meteorological variable p in three dimensions. In all computations so far, however, the vertical dimension (or pressure) has been held constant; the "objective" analysis completed is thus analogous to the subjective analysis of constant-level or constant-pressure charts. Also, the analysis has been restricted to relatively small areas over which third-degree polynomials fit the data satisfactorily.

[1] Report of meteorological computer project, Contract No. N6 ORi-139 between the Office of Naval Research and the Institute for Advanced Study.

A third-degree polynomial can be written in the form

$$p(x, y) = \sum_{i,j} a_{ij}x^i y^j, \qquad (i + j \leqq 3),$$

where x and y are Cartesian coordinates and the coefficients a_{ij} are ten in number. The size of the area to which such a polynomial is to be fitted will depend on how much the observations are to be smoothed. A field of 10 observations can be fitted accurately by a third-degree polynomial, with no smoothing of data. Some smoothing is however desirable in all cases, since all observations are influenced by observational errors and more or less local eddies. The polynomials cannot possibly be expected to fit these eddies, but should rather represent the large scale features of the field to be analyzed.

The amount of smoothing will depend on the nature of the variable analyzed; variables whose observation is comparatively inaccurate or subject to "unrepresentative" fluctuations (winds, for example) should be smoothed more than variables which can be observed accurately (pressures, for example). If a pressure field is to be analyzed, 12–14 observations might be used to determine the 10 constants of the polynomial; in case of the east–west component of wind, presumably no less than 20 observations are required.

The problem of determining n coefficients from m observations ($m > n$) is usually handled by the method of least squares. This would yield the most probable polynomial if it is assumed that eddies and observational errors are distributed normally about the smoothed field. Generally, the condition of least squares for a cubic polynomial in x and y leads to ten normal equations with the ten values of a_{ij} as unknowns. Considerable algebra is involved in computing the coefficients in these equations and in their solution. The time needed to do these computations with standard computing machines is long compared to a forecast period; however, the computing time necessary for the projected electronic computer is not expected to be extravagant. The considerable computations necessary in the pilot study of objective analysis described herein were made by the Computation Laboratory of the Bureau of Standards at New York City.[2]

2. Analysis of a sample wind field

A situation (2200 GCT 2 December 1945) was chosen where the wind coverage in the eastern third of the United States was nearly complete up to 10,000 ft, with the number of wind reports varying from 54 at 3000 ft to 36 at 9000 ft. Cartesian coordinates x and y were defined on a Lambert conformal projection as follows: The y-axis was the 95th meridian, with the x-axis at right angles to it and passing through the point 30°N, 95°W. The u component (in the x-direction) and the v component (in the y-direction) of the wind were fitted by independent cubic polynomials at the 3-, 5-, 7-, and 9-thousand ft levels.

Fig. 1 compares the observed winds and the winds computed from the cubic at 3000 ft. Apparently the computed winds show little, if any, systematic difference from the observed winds. They are, of course, a great deal less erratic. Thus it would seem that the cubics fit the observations satisfactorily.

Fig. 2 shows the divergence of the same wind field, both as computed with conventional methods by two analysts (B and C) and as computed from the fitted cubic (A). (Lines of constant divergence on the latter are therefore conic sections.) Apparently the two sets of subjective measurement agree with each other no better than with the objective determination. In general features, all three maps are similar.[3]

[2] Directed by Arnold Lowen, under the immediate supervision of Jack Laderman.

[3] Bellamy (1949) has proposed quite different methods of objective measurement of divergence and vertical velocity, without preliminary analysis of the wind field. Unfortunately he does not give results which might be used for comparison.

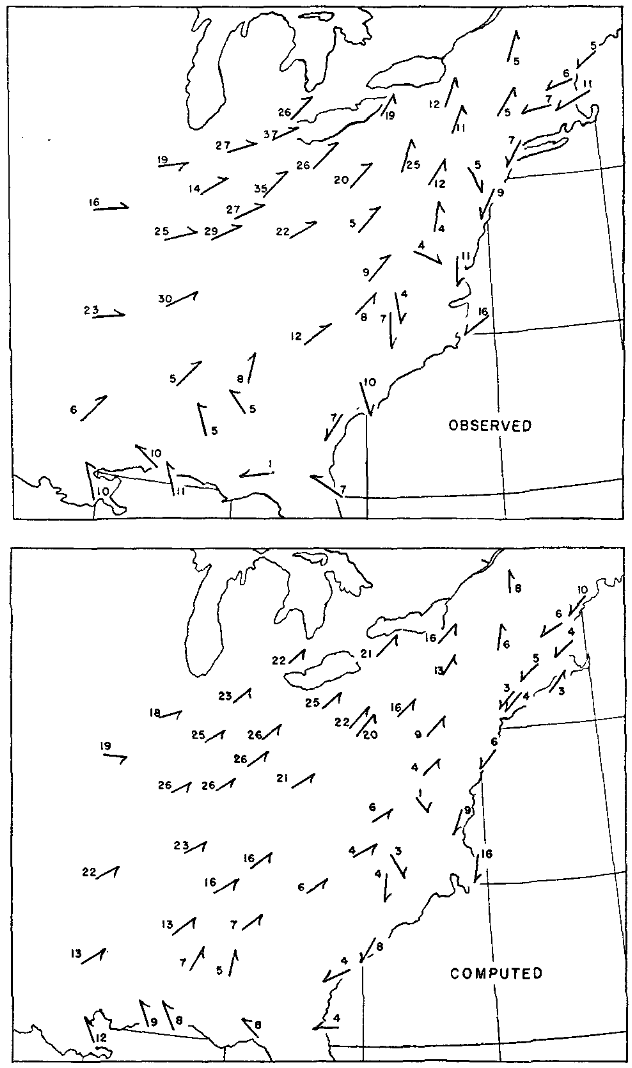

FIG. 1. Observed winds and winds computed from a cubic polynomial for 2200 GCT 2 December 1945 at 3000 ft. (Speeds are in mi hr^{-1}.)

Fig. 3 compares fields of vertical velocities for the same period at 10,000 ft, computed by subjective and objective methods. The objective computations of the vertical velocities have been obtained by adding the properly weighted values of divergence at 3-, 5-, 7-, and 9-thousand ft.[4] Again, lines of constant vertical

[4] For a summary of methods of computing vertical velocities in the atmosphere see Panofsky (1946).

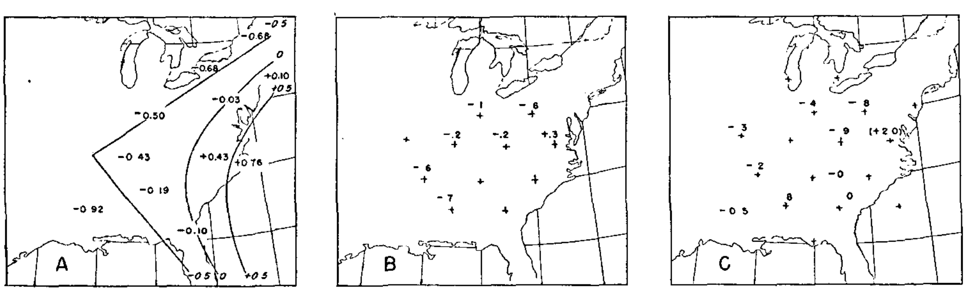

FIG. 2. Horizontal velocity-divergence of the wind field shown in fig. 1 in units of 10^{-5} sec^{-1} —objective (A) and subjective (B, C) analyses.

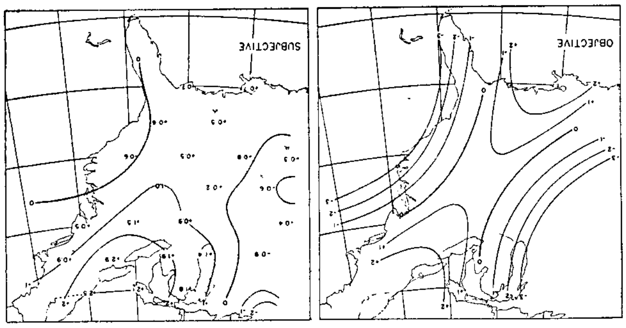

FIG. 3. Vertical velocities at 10,000 ft for 2200 GCT 2 December 1945 in units of cm sec^{-1}.

velocity (which are *instantaneous* values) are conic sections. The subjective vertical velocities were computed completely independently by the adiabatic method, which yields 12-hr average vertical velocities. The agreement is as good as can be expected.

3. Combination of separate areas

The objective analysis as described so far leads to independent polynomial expressions in separate areas of limited size. At the boundaries, the analyzed field is discontinuous, but this discontinuity has no physical reality since it depends on the arbitrary choice of the areas. Therefore a method must be devised which smooths out these discontinuities.

It is now postulated that the coefficients of the individual polynomials only apply at the center of gravity of the areas, or perhaps only at a centroid line. Between those points, the coefficients are assumed to vary by a simple law. If only two areas are analyzed, the coefficients can be taken constant along a certain direction and made to vary linearly in another; if more areas are to be connected, the coefficients of the individual polynomials are again polynomials in x and y, the order depending on the number of areas.

The initial data covering a hemisphere or the entire earth might be better represented by spherical harmonics, rather than by algebraic polynomials pieced together at the edges of separate and limited areas.

The process of piecing together was studied on a field of resultant vectors. Horizontal components of resultant vectors are defined by

$$R_x = \int_0^h u\, dz, \qquad R_y = \int_0^h v\, dz.$$

These can be obtained directly from pilot-balloon observations and have the property that their divergence is proportional to vertical velocity, which can be determined independently.

Accidental errors should have a considerably smaller effect on wind resultants than on winds at individual levels, and eddies located in particular layers will influence the resultants to a smaller extent. Therefore, the ratio of coefficients in the polynomials to the number of measurements from which these coefficients are to be determined would be greater. In other words, the observations require less smoothing.

The observations for 2200 GCT 8 December 1945 were chosen because resultant wind vectors $\boldsymbol{R}_{10}$ could be obtained at unusually many observing stations east of 100°W and because independently computed vertical velocities were also available for that time. Fifty-seven resultants were available in the midwestern and the eastern part of the United States.

The total area was divided into two sections, C and D, by a straight line. The criteria for choosing this line were two: (a) that the number of observing stations in each area was approximately the same, and (b) that the distribution of observing stations in each area did not show great variation of density. (A preliminary study had shown that observations in a section containing few data influence the final coefficient in the polynomial to such an extent that a small error in the observations will influence the values of the coefficients in the polynomials significantly.)

First, separate third-degree polynomials were fitted to the x and y components of the resultant vectors in both areas, and the divergences computed. Fig. 4 shows the distribution of vertical velocities determined from these values. As was to be expected, the field is discontinuous along the boundary AA′ between the areas C and D.

The two third-degree equations in the regions C and D were combined into a single fourth-degree equation by the assumption that the coefficients in the original cubics vary linearly at right angles to the line separating areas C and D. If the x' direction is defined as a direction at right angles to this dividing line, the coefficients a_{ij} are then given by an equation

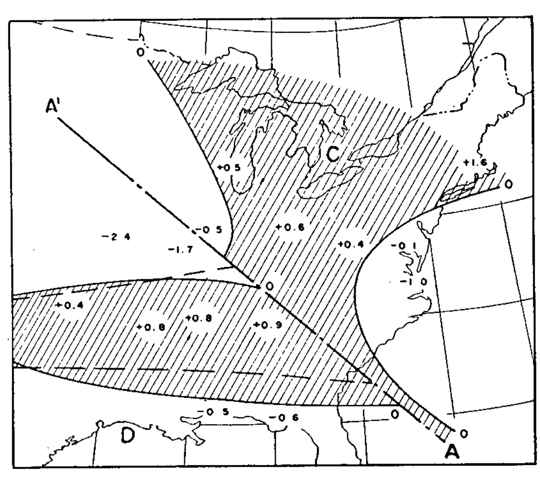

FIG. 4. Vertical velocities (cm sec^{-1}) at 10,000 ft for 2200 GCT 8 December 1945 obtained by independent analyses in separate areas C and D. Hatched areas indicate rising motion.

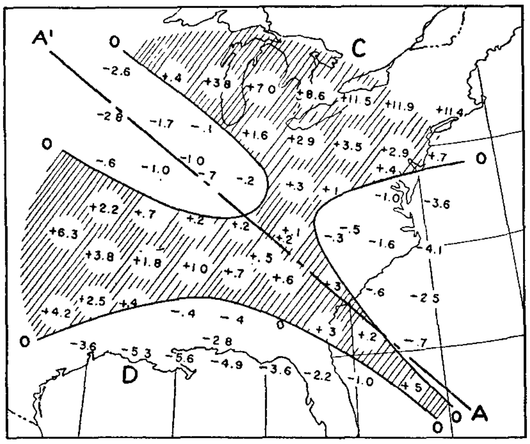

FIG. 5. Vertical velocities (cm sec^{-1}) at 10,000 ft for 2200 GCT 8 December 1945 obtained by combining independent analyses in separate areas C and D. Hatched areas indicate rising motion.

of the form

$$a_{ij} = m_{ij}x' + n_{ij}.$$

The constants m_{ij} and n_{ij} were computed under the assumption that the values of the a's were equal to the values determined for the individual cubics at the position of the centers of gravity of areas C and D.

Fig. 5 shows the distribution of "objective" vertical velocities obtained from the divergence of the fourth-degree equation. Apparently the smoothing process had a pronounced effect on the distribution of vertical velocities.

4. Influence of observational errors on objectively determined vertical velocity

Previous experience with subjective methods of measuring divergences of winds or resultant winds indicated that observational errors would have a large effect on the measured value. With objective techniques this effect can be estimated quantitatively.

The observations of R_x and R_y in region D were subjected to random errors estimated to be of the same magnitude as the errors of observation. The frequency of errors of given sizes (units of 10^4 m^2 min^{-1}) are summarized in the following table:

Error:	−8	−7	−6	−5	−4	−3	−2	−1	−0	1	2	3	4	5	6	7	8
Frequency:	1	1	1	2	2	2	3	3	3	3	3	2	2	2	1	1	1

Each possible error was written on one or more cards (depending on the frequency in the above table) and the cards were shuffled. A card was drawn at random and the error on the card was added algebraically to the first observation of R_x. The card was put back into the deck, the deck was shuffled again, and a new card was drawn. Its error was added to the second observation of R_x and so fourth, for all observations of R_x and R_y. Table 1 shows the vertical velocities computed from the altered observations at certain points compared with the original computations from fig. 4. This sample seems to indicate that the errors did not influence the sign significantly, but that the magnitudes are affected considerably.

In fig. 4, the dashed lines indicate the position of the zero lines as evaluated from the altered observations. Apparently the position of the zero lines is not very greatly affected, although the field of divergence, which was previously elliptic, is now hyperbolic.

5. Analysis of a contour field

Contour lines of a constant-pressure surface are rarely analyzed on the basis of reported elevations alone; winds usually are taken into consideration by the analyst. The majority of forecasters consider wind direction only and attempt to make the contour lines as nearly parallel to the wind as possible. The speed is usually used to indicate the reliability of the direction, but a minority of meteorologists make use of the wind speed to adjust the spacing of contour lines, using the geostrophic wind scale.

Both of these procedures can be incorporated into objective analysis. The quantity $\boldsymbol{v}\cdot\nabla h$ (where $\boldsymbol{v}$ is the horizontal wind and h the elevation of the pressure surface) is zero if wind and contour lines are parallel. Hence the condition $\Sigma(\boldsymbol{v}\cdot\nabla h)^2$ = minimum describes mathematically the subjective procedure of adjusting contour lines to fit the wind directions. Since the analysis has to fit winds and elevations simultaneously, the polynomial for h should be subjected to the condition:

$$a \sum_{s} (h - h_0)^2 + \sum_{s'} (\boldsymbol{v}_0\cdot\nabla h)^2 = \text{minimum}.$$

Here s denotes summation over the radiosonde stations and s′ over pilot-balloon stations. The subscript 0 denotes observed values and a is a constant which depends on the relative weight of winds and elevations. This constant has dimensions and depends therefore on the units used for elevation and wind.

If wind speed is to be used to space the contour lines in addition to wind direction and elevation, the following conditions may be used:

$$b \sum_{s} (h - h_0)^2 + \sum_{s'} (u_0 - u_{gs})^2 + \sum_{s'} (v_0 - v_{gs})^2 = \text{min}.$$

Here the subscript gs stands for geostrophic and b is a constant different from a in numerical value but not in dimensions.

TABLE 1. Vertical velocities in cm sec^{-1} at selected points at 10,000 ft at 2200 GCT 8 December 1945 computed from the divergence of resultant wind vectors, with and without added errors.

Station	Without error	With error
Big Springs	0.0	0.3
Kansas City	−2.4	−2.1
St. Louis	−1.7	−1.0
Memphis	0.0	0.8
Tallahassee	−0.6	−1.6
Jacksonville	−0.5	−1.4

TABLE 2. Horizontal derivatives of elevation (values to be multiplied by 10^{-5}) and horizontal Laplacian of elevation (10^{-13} cm^{-1}) at selected points on the 700-mb surface, 1500 GCT 25 March 1947.

Station	$\partial h/\partial x$ Without errors	$\partial h/\partial x$ With errors	$\partial h/\partial y$ Without errors	$\partial h/\partial y$ With errors	$\partial^2 h/\partial x^2 + \partial^2 h/\partial y^2$ Without errors	$\partial^2 h/\partial x^2 + \partial^2 h/\partial y^2$ With errors
Brunswick, Ga.	0.0	−0.9	−26.6	−25.3	26.0	19.2
New Orleans	−8.8	−7.0	−12.6	−15.1	−24.0	−5.0
Nashville	−14.7	−12.3	−25.1	−30.5	13.3	−11.9
Atlanta	−8.8	−7.5	−25.7	−25.8	19.7	3.7
Salisbury, N. C.	−2.5	−2.7	−26.1	−30.3	41.4	8.0

Fig. 6 shows the subjective (A) and objective (B, C) analyses for the 700-mb level at 1500 GCT 25 March 1947. There are 8 elevations and 14 winds, which were judged sufficient for the determination of the 10 constants of the elevation polynomial. Chart B is the result of fitting wind directions only. The constant a was chosen as 10^{-9} sec^{-2}; this means that a deviation of observed from fitted elevation of 30 ft has the same weight as, for example, a deviation of a wind from the contour lines by 18 degrees, when the spacing of these lines is 30 ft (100 km)$^{-1}$, or a deviation of 30 degrees with a spacing of 19 ft (100 km)$^{-1}$. Chart C shows the result when the observed wind speed was utilized also. The constant b was determined in such a way that a deviation of elevation of 30 ft had the same effect as a deviation of one of the Cartesian wind components from the geostrophic component of 5 m sec^{-1}. The numerical value of b was 0.29 sec^{-2}.

Both objective techniques give results very much alike; they differ less from each other than from the subjective analysis. Both objective techniques differ from the subjective analysis by a more even spacing and simpler pattern. It may be argued that the objective analyses are actually preferable in this case since the aim was to eliminate details superimposed on the broad field. The deviations of the observations from the analyzed field are small enough to be accounted for by errors and eddies. A change of the weight b to 0.10 did not change the analysis significantly.

Horizontal gradients of the field of h and its Laplacian derivative were also computed. The agreement in the gradients was tolerable; that in the Laplacian not good. This indicates that meteorological data at present are insufficient to yield reliable objective estimates of the distribution of the Laplacian of elevation over areas of the size of 10^6 mi^2. This conclusion was substantiated by an evaluation of the effect of observational errors on the derivatives of the same elevation field, based on the technique using wind directions only. Table 2 shows the derivatives of the height field, with and without errors added.

These values are based on the following distribution of errors:

For u and v (m sec^{-1})

Error	−20	−15	−10	−5	0	5	10	15	20
Weight	1	1	2	3	3	3	2	1	1

For h (decameters)

Error	−2	−1.5	−1.0	−0.5	0	0.5	1.0	1.5	2 0
Weight	1	1	2	3	3	3	2	1	1

The assumed errors in the wind components are possibly too large, hence the changes produced on the polynomials too great. Even so, it appears that computation of the Laplacian of pressure (or height) is not reliable, and should be avoided.

6. The deviation from geostrophic wind

The fundamental equations of meteorology can be transformed into a complete system of equations in which partial derivatives with respect to time can be integrated numerically, provided that the quantities on the right side of the equation are known with sufficient accuracy.

Two of the equations of this complete system are the two horizontal components of the vector equation of motion, solved for the time derivatives:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} + f(v - v_{gs}) + F_x, \quad (1)$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - w\frac{\partial v}{\partial z} - f(u - u_{gs}) + F_y. \quad (2)$$

The notation is standard: u, v and w are the three wind components in the directions x, y, and z, f is the Coriolis parameter, t is the time, the subscript gs stands for geostrophic, and F_x and F_y are the components of the force of friction in the x- and y-directions.

The terms $u - u_{gs}$ and $v - v_{gs}$, which appear prominently on the right side of (1) and (2), are the components of the "nongeostrophic wind." Many attempts have been made in the past to measure the

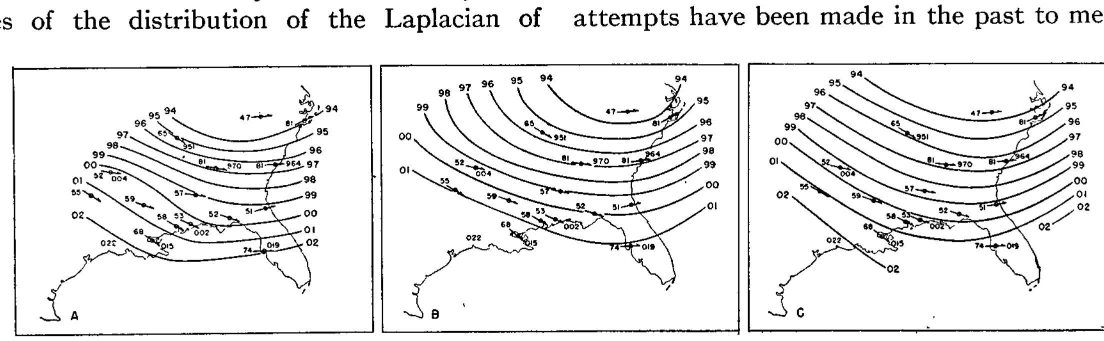

FIG. 6. Subjective (A) and objective (B, C) analyses of the height of the 700-mb surface at 1500 GCT 25 March 1947. Contours are labeled in hundreds of ft, winds are indicated in mi hr^{-1}, and observed height values are plotted in tens of ft.

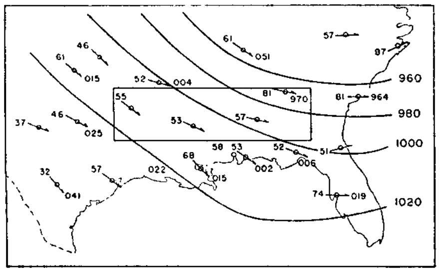

FIG. 7. Observed heights (tens of ft) and winds (mi hr^{-1}) at the 700-mb surface 1500 GCT 25 March 1947. The rectangle indicates area analysed in detail.

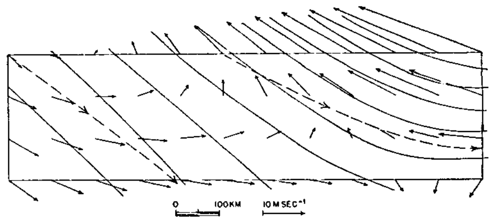

FIG. 9. Nongeostrophic winds at 700-mb as computed from quadratics for u and v and cubic for ϕ, 1500 GCT 25 March 1947. Long, dashed arrows represent streamlines. Solid lines are height contours spaced at 50-ft intervals.

deviations from geostrophic wind, the easiest method consisting of drawing smooth isobars, computing the geostrophic wind from the distance of the isobars and subtracting it from the reported wind observation. This method leads to erroneous results for two important reasons:

1. The isobars are usually not drawn independently of the wind.

2. The geostrophic wind is an average over the space between the isobars. Moreover, the isobars are smoothed and drawn from relatively distant observations. The winds, on the other hand, are local and not smoothed.

These difficulties can be overcome by:

1. Independent analysis of wind and pressure (or contour) fields.

2. Smoothing of the two fields, guided by similar principles.

Two situations were treated in this manner, (*A*) the wind and contour fields at 700 mb, 1500 GCT 25 March 1947, and (*B*) the wind and pressure fields at 10,000 ft, 1600 GCT 2 December 1944. The observations are given in figs. 7 and 8, together with subjective isopleths of h. In situation A, even the subjective analysis (fig. 6) indicates that the winds blow across the contour lines, toward lower elevations; also in the eastern part of the area, the winds are far below geostrophic.

If pressure (or elevation) is represented by a cubic in x and y, the geostrophic wind components are quadratics. Hence it was decided to fit quadratics to the observed wind component also (instead of cubics as in section 2). The areas were chosen in such a way that they contained 12 observations of pressure (or elevation). In both cases, about 25 wind reports were available in the same areas.

On both figs. 7 and 8, rectangles have been drawn to indicate the area analyzed in detail. They were chosen so as to be almost completely surrounded by both pressure (or elevation) and wind observations. Figs. 9 and 10 show the deviation vectors from geostrophic wind in the areas outlined in figs. 7 and 8, computed from the polynomials of p (or h), u and v. Isobars (or contour lines) are also given. In situation A, the geostrophic deviations point toward lower elevations, and are directed against the main flow in the eastern part, as expected. In situation B the geostrophic deviations form an anticyclonic whirl, centered slightly east of the wedge line.

The data used in this study were modified by random "errors," of a magnitude consistent with observational errors. The following table gives the errors and their weights:

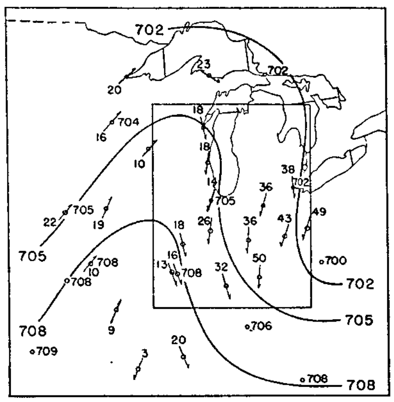

FIG. 8. Observed pressures (mb) and winds (mi hr^{-1}) on the 10,000-ft surface, 1600 GCT 2 December 1944. The rectangle indicates area analysed in detail.

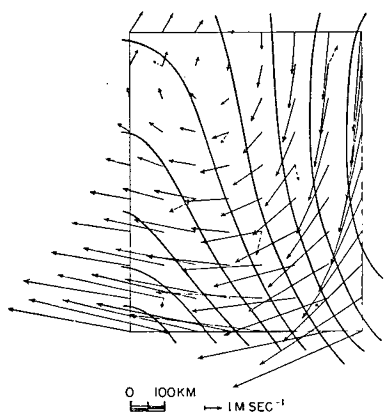

FIG. 10. Nongeostrophic winds at 10,000 ft as computed from quadratics for u and v and cubic for p, 1600 GCT 2 December 1944. Continuous lines are isobars (for each mb). Solid arrows are nongeostrophic winds computed from pressure and wind fields; dashed arrows represent theoretical geostrophic deviations.

For u and v (in m sec^{-1})

Error	−12	−8	−4	−2	0	2	4	8	12
Weight	1	1	2	3	3	3	2	1	1

For p (mb) and geopotential gh (10^6 erg g^{-1})

Error	−1.8	−1.2	−.8	−.4	0	.4	.8	1.2	1.8
Weight	1	1	2	3	3	3	2	1	1

Figs. 11 and 12 show the geostrophic deviations with errors applied. Apparently, the effect of the errors on the final result is much greater in situation B than in A. Indeed, it becomes likely that the geostrophic deviations of situation B (fig. 10) are extremely uncertain.

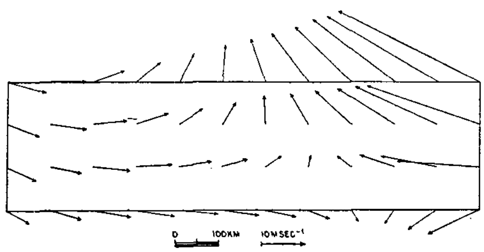

FIG. 11. Nongeostrophic winds at 700-mb as computed from quadratics for u and v and cubic for gh, 1500 GCT 25 March 1947 (with added error).

It is desirable to test the reliability of the computed nongeostrophic wind vectors shown in figs. 9 and 10 by independent measurements. Let $\boldsymbol{v}_n$ represent the nongeostrophic wind vector. Then the horizontal component of the equation of motion may be written:

$$\boldsymbol{v}_n = f^{-1}(\boldsymbol{F} - d\boldsymbol{v}/dt) \times \boldsymbol{k}, \qquad (3)$$

where $\boldsymbol{F}$ is the horizontal component of the force of friction and $\boldsymbol{k}$ is a unit vertical vector. In order to estimate the nongeostrophic wind vector to be expected it is first assumed that $\boldsymbol{F}$ is small compared to $d\boldsymbol{v}/dt$ at 700 mb and can be neglected. The term $d\boldsymbol{v}/dt$ indicates the change of the horizontal motion following a parcel in three dimensions. It can be computed most easily from isentropic charts if individual temperature changes can be assumed adiabatic.

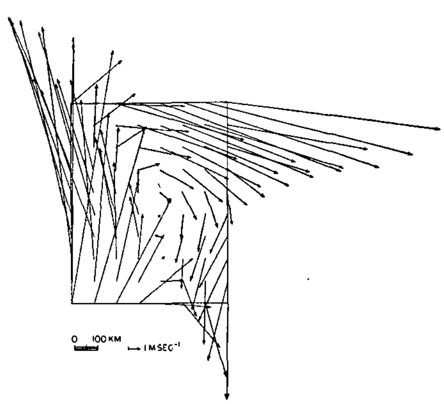

FIG. 12. Nongeostrophic winds at 10,000 ft as computed from quadratics for u and v and cubic for p, 1600 GCT 2 December 1944 (with added error).

In the case of situation A, isentropic charts were not available, and change of speed along an isobaric trajectory was substituted for that along an isentropic trajectory. The geostrophic deviations expected from these accelerations are indicated in fig. 13. The agreement between these and the observed geostrophic deviations (fig. 9) is tolerable; both the direction toward lower elevation, and the direction against the flow in the eastern part of the area agree. Better agreement is not likely to be expected because (a) observed geostrophic deviations were almost instantaneous while accelerations were 12-hr average changes of velocity up to the period analyzed (later winds were not available), and (b) friction and vertical motion were neglected. However an attempt to improve the agreement by including the effect of these two factors failed, perhaps due to the inadequacy of our knowledge of these quantities.

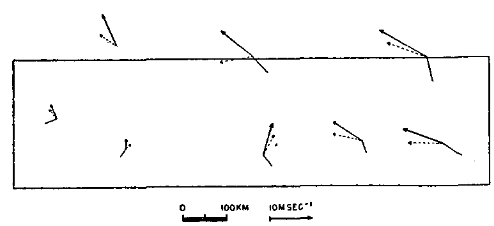

FIG. 13. Nongeostrophic winds at 700-mb as computed from objective analysis (solid arrows) and from theoretical considerations (dashed arrows) 1500 GCT 25 March 1947. Lines at tails of arrows represent direction of friction.

In the case of situation B isentropic trajectories could be constructed and 12-hr accelerations could be computed centered at the period of analysis. The nongeostrophic winds based on these measurements are entered as broken arrows in fig. 10. The agreement with the observed nongeostrophic winds is poor.

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