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An Experiment in Objective Analysis

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Abstract

An experiment in two-dimensional objective analysis, conducted at The Institute for Advanced Study is described. The method consists of fitting a second degree polynomial by the method of least squares to the meteorological data in a limited area around each point in a rectangular grid, and then interpolating for the height of the pressure surface at the grid point. The results of machine computations give analyses which are comparable in quality to the subjective analyses ordinarily encountered. When used in a three-parameter atmospheric model for a numerical forecast, the objective analyses led to a more accurate forecast than carefully prepared subjective analyses did. Also a satisfactory representation of the field of vertical stability (over North America) was obtained by the two-dimensional analyses, made for three different pressure surfaces.

The greatest obstacle encountered in producing a completely objective analysis arises from errors of computation and transmission of raw meteorological data. This obstacle has not been overcome, and necessitates a preliminary subjective examination of the data for detection of errors.

I. Introduction

Ever since the meteorology group was set up at Princeton with numerical weather prediction as one of its major aims, the problem of preparation of the initial data has been under consideration. It was realized that in the forecasting models the initial data would be required at a set of regularly spaced grid points whereas the observing stations have an almost random distribution.

One method of getting appropriate grid values would be via the normal process of analysis, namely by subjectively drawing isopleths and then by subjective interpolation obtain the values at grid points. This method has in fact been used to prepare the data presently being used. The preparation of this data, however, involved an experienced synoptic meteorologist in several days of work preparing data for one observation time. In the preparation of the data considerable care was taken to ensure continuity both in space and time and to remove any erroneous observations. This work would be very difficult to compress into a time compatible with any routine forecasting system. Two questions then arise. Is so much care necessary? Does the subjective analyst do anything that a computing machine could not also do in much less time?

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The only way to find answers to these questions is to propose alternatives to the methods used by the subjective analyst and to use these alternatives in the preparation of forecasts. This paper proposes one method of avoiding the subjective analysis and demonstrates its effects, firstly, on the type of analysis produced, and secondly, on numerical forecasts obtained from data so prepared.

The method used has evolved during the last few years as a result of informal discussions held at the Institute for Advanced Study (I.A.S.)[1]. Initially an attempt was made to fit polynomials to the observed data over fairly large areas. Such an attempt is described by PANOFSKY (1949). This method was not used in conjunction with a numerical forecast. Rather, as a result of certain discrepancies being revealed by the Panofsky investigation, the method described below was evolved.

Here we are primarily interested in obtaining an accurate representation of the data and not in smoothing. The only smoothing we require is that necessary to remove the random errors in the observations thus ensuring continuity in the horizontal derivatives of pressure. We seek a "most probable" representation of the data. If one attempts to fit a polynomial over a large area one automatically implies the use of a smoothing technique. If, however, we follow Charney's suggestion and only consider data immediately surrounding the grid point in question, we imply a minimum of smoothing. This procedure agrees with ordinary synoptic practice in that it does not pay any attention to data in remote parts of the region as would any method involving a Fourier analysis or polynomial fitting to the whole of the region.

Dr. J. SMAGORINSKY (1954) of the U.S. Weather Bureau has performed independently a considerable number of hand computations using a slight variant of the method described here with comparable accuracy. Smagorinsky's work, however, was definitely limited by his lack of a high speed computing machine, a facility which fortunately was available to us during the present experiments. It was felt at the I. A. S. that an attempt such as described below should be made for the following reasons. Firstly, if the attempt succeeded, we would be relieved of a lot of work in future data preparation. Secondly, it was thought that the basic two-dimensional interpolation process should be tested in a series of machine computations in order to discover its capabilities and limitations. It was further felt that any further extensions of machine computations of analyses should wait until this basic process had been fully tested.

[1] The other participants were Professor J. von Neumann, Mrs M. Smagorinsky, Drs J. Charney, G. Platzman, and J. Smagorinsky.

II. Method

The method is purely two-dimensional and is based on the fitting of a second order polynomial to the observations in an area surrounding the grid point at which the value of the pressure or height is required. In this paper we will be concerned with the height of given pressure surfaces expressed in terms of height deviation, D, from standard atmosphere. The polynomial can be expressed in the form:

$$D = a_{00} + a_{10}\,x + a_{01}\,y + a_{11}\,xy + a_{20}\,x^2 + a_{02}\,y^2$$

or in condensed form:

$$D = \sum_{i+j=0}^{i+j=2} a_{ij}\,x^i y^j \qquad (1)$$

Thus, if the origin of the coordinate system is taken to be the grid point under consideration, the D value we seek will be a_{00}. In order to determine the six coefficients a_{ij} we clearly require the equivalent of six independent observations of D at known positions (x, y).

The observations give us wind values as well as D values. The wind values are incorporated in the scheme by using the geostrophic assumption. Thus, one wind observation gives us values of $\partial D/\partial x$ and $\partial D/\partial y$ at a point. Provided that there is at least one D value within the region being considered, each wind observation is equivalent to two more D values or pieces of information. The errors introduced by the geostrophic assumption are not random. However, due to our lack of knowledge of the geostrophic deviations, we have had to assume that the errors from this source are random.

Since, in general, the observations from nearby stations are not in exact agreement, we usually require more than the minimum number of pieces of information and then fit the second order polynomial to these observations by the method of Least Squares[1]. Thus we choose the a_{ij} such that we minimize

$$S = \frac{1}{\sigma_D}\sum_{n=1}^{r}(D_n - D_n')^2 + \frac{1}{\sigma_w}\sum_{n=1}^{s}\left\{\left(\frac{\partial D_n}{\partial x} - \frac{\partial D_n'}{\partial x}\right)^2 + \left(\frac{\partial D_n}{\partial y} - \frac{\partial D_n'}{\partial y}\right)^2\right\}$$

where D_n, $\partial D_n/\partial x$, $\partial D_n/\partial y$ are from the observations and the corresponding primed quantities are those computed from the polynomial (1). σ_D is the standard deviation of the errors in the r observations of D and σ_w is the standard deviation of the s wind observations in the area over which the polynomial is being fitted. The ratio σ_D/σ_w is called the weighting factor. It can be shown that, if the errors have a Gaussian distribution, by choosing the a_{ij} in this way we get the most probable value of the required D. We thus have to solve the set of six linear algebraic equations:

$$\frac{\partial S}{\partial a_{ij}} = 0, \quad 0 \leqslant i + j \leqslant 2$$

for the coefficients a_{ij} and in particular the coefficient a_{00}. These equations can be written in the form:

$$y \cdot a = z$$

where y and z are a (6×6) matrix and a 6 component vector respectively, the elements of both being known from the observations and a is a vector whose elements are the unknowns a_{ij}.

By triangularizing the matrix y we can reduce the equations to the form:

$$y^\star \cdot a \equiv z^\star$$

in which $y^\star$ is a lower diagonal matrix.

Thus if $y^\star_{00}$, a_{00}, $z^\star_0$ are the leading elements of $y^\star$, a and $z^\star$ respectively we obtain directly that

$$D = a_{00} = z_0^\star / y_{00}^\star.$$

[1] A description of this method may be found in most books on statistics or numerical methods, e.g. Whittaker and Robinson's "Calculus of Observations".

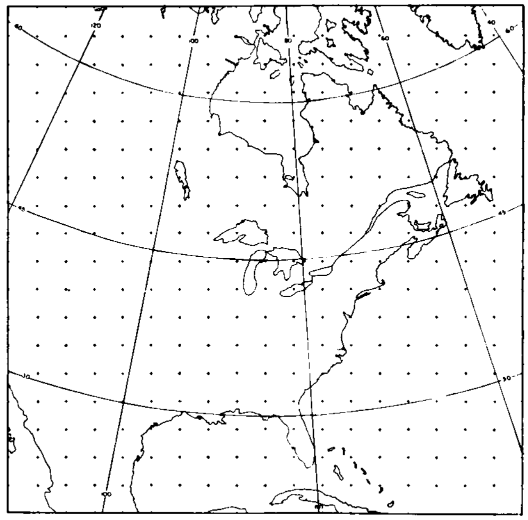

Fig. 1. Location of 19 × 19 grid of points used in computations.

III. Weighting Factor

It was not possible, as in some physical experiments, to look up the standard deviations of the errors in the observations in standard references, as the analysis must include data determined by a variety of different instruments and methods of observation. The error in measuring wind, for example, depends on the wind speed, and varies from one place to another over the map. Errors also arise from the introduction of the geostrophic assumption in the derivation of the expression for S. Consequently, the appropriate value of the weighting factor was determined by the trials described below.

A series of objective analyses was computed for the 500 mb surface for November 25, 1950, at 0300 GCT. The grid used is shown in Figure 1. Each analysis was made with a different weighting factor. The results of each analysis were then compared with the mean of three subjective analyses for the same map, made by three different analysts. The results considered were the D values for 222 points and $\nabla^2 D$, the finite difference Laplacian of these D values, for 152 points. The points used were selected prior to the experiment as being those where sufficient data were available to permit a fair comparison. The means of the standard deviations of D and $\nabla^2 D$ of the ob-

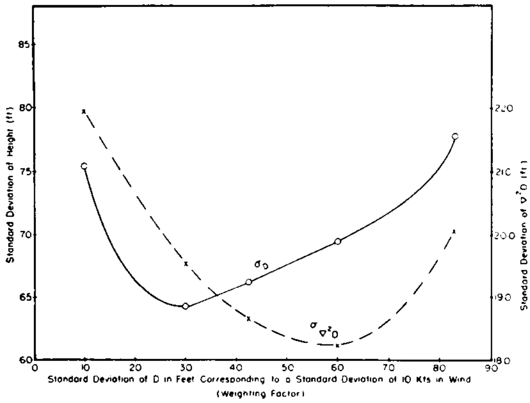

Fig. 2. Standard deviations between objective and subjective analyses for various weighting factors.

surrounding the small region in question. In a similar manner, the objective analysis program fits the quadratic surface as closely as possible to all wind and pressure data reported within a prescribed area surrounding the grid point. The limits of the area are defined by the relation

$$|x| + |y| = |a|,$$

where x and y are the coordinates of each piece of data with respect to the grid point at which D is to be computed. This defines a square (standing on one corner) with the grid point at the center. The search then consists of locating and assembling in the memory of the computer all the data reported within this area. These data are then used in the fitting of the quadratic surface and the computation of the D value at the grid point. The search is made for each grid point using a given sized area. When sufficient data are assembled for the computation, the D value is computed. If the data are inadequate in quantity, the point is passed and a search is made around the next point. In this way, the computation proceeds until all points in the grid have been considered. This leaves some points computed and some not computed.

The next step is to go back to the points which were skipped over because of lack of data, and to try to assemble more data for the computation. This is done by assembling all reported data *plus all previously computed data* which lie in the search area. The search area may be enlarged also. Thus, some of the points which were skipped the first time the grid was scanned can now be computed. (Points previously computed are now passed over and left unchanged.) Some uncomputed points are still left uncomputed, due to lack of data. After the machine has proceeded through the grid a second time, the search area is enlarged again and the above process is repeated. The search routine which is currently in use has evolved through many trials and is summarized in Table I.

The sudden increase in the size of the search area from the 4th to the 5th search is due to the fact that by the end of the 4th search the only areas remaining unanalyzed are those with practically no data at all. Searches 5 through 7 are designed to fill in these areas with values which are not unreasonable.

jective analyses vs. the subjective analyses are shown in Figure 2. It is evident from Figure 2 that the objective analysis has the best agreement with the subjective analyses, so far as height is concerned, when wind receives a relatively small weight. However, a heavier weighting on wind produces a better agreement when $\nabla^2 D$ is considered. It was believed best to use a weighting factor giving good results for vorticity calculations, in view of the fact that the objective analyses are principally for use in numerical prediction. It may be noted that in numerical prediction one is usually more interested in expressions such as $\nabla^2 D$ and other derivatives of D rather than the absolute value of D. Consequently, a weighting factor corresponding to a ratio of 50 ft. height error to 10 knots wind error was selected for routine use in upper air analyses.

The above type of test was not performed for other levels, but a smaller weighting factor, corresponding to a 30 ft. height error for a 10 knot wind error, was selected for use in 1,000 mb analyses. This decision was made because of the existence of greater errors in the geostrophic assumption near the ground. At high levels wind observations are more accurate with regard to direction than to speed. This would suggest that in future analysis schemes we should use separate weighting factors for the two parts of a wind observation.

IV. Search Procedure

An analyst, when drawing an isobar, looks at all the wind and pressure data in an area

Table I

Passage through grid	Size of search area (length of side of square = $a\sqrt{2}$)	Data included in search
1st	1,000 km	Reported data only
2nd	1,000 km	Reported data plus previously computed data
3rd	1,200 km	
4th	1,400 km	
5th	1,800 km	
6th	1,800 km	
7th	1,800 km	

The square search area was selected because of the great speed with which it can be used in the computation. However, it has become evident that with a rectangular grid, one cannot expand a search area in too large increments from one passage through the grid to the next, or discontinuities in the analysis develop. On the other hand, when reported data are scarce, very small increments of a square search area will prove unsatisfactory because no significant increase of data will be observed except when the increasing boundaries of the search area all at once pass a number of grid points containing previously computed data. Thus, it now appears that a circular search area would be meteorologically superior to a square one. The number of additional grid points enclosed with the expansions of the area would then increase more smoothly than with a square search area.

Strictly speaking, one needs six pieces of meteorological information (each wind gives two pieces of information and each height value gives one) to fit a quadratic surface in the manner described above in section II. However, in view of the fact that the data contain errors, a surplus of data is highly desirable in order that some smoothing can be done. Another reason for requiring surplus information is that, with some combinations of data, a nearly singular matrix $\mathbf{y}$ is encountered in the computation of the D value. A truly singular matrix has not yet been encountered in any of the analysis runs, but nearly-singular matrices have been seen. These give solutions for D which are computed in the non-significant digits of the data and are therefore meaningless. These can never be entirely avoided, but the probability of their occurrence can be reduced to a negligible figure by requiring enough data. Experience to date has indicated that this desired result can be obtained if a minimum of ten pieces of information is required as a prerequisite to the computation of the D value.

V. Machine Program

The machine program can be outlined in the following manner:

1. Part A code is read into the computer at the start of the analysis.
2. Part A reads in data cards, one by one, converts the data from decimal to binary, and stores it. The data cards contain the index number of the station (latitude and longitude for ships), the wind direction, and speed, and the height of the pressure surface to be analyzed for each reporting station. Part A then reads in part B code.
3. Part B code reads in the station locator cards, which have the latitude and longitude for the corresponding index number of each station which can possibly report. It matches this information with the data already in the memory and converts the index numbers of the data to latitude and longitude. All data are next converted to the coordinate system of the grid. Part B code then reads the code for parts C, D, and E into the machine.
4. Part C code is the search routine. Whenever enough data are found to enable the computation of a point, part C code transfers the control to part D.
5. Part D is the matrix code, i.e. the section which triangularizes the matrix $\mathbf{y}$, etc. It computes the D value at the grid point, using the data obtained in the search. It then transfers the control back to the search code.
6. Part E is the code which converts the results of the analysis from binary to decimal and punches the results on cards, which can then be tabulated.

It can be seen that the program is entirely automatic. The process of making the objective analysis thus consists of the following steps:

1) The reported meteorological data are punched on cards.

2) The cards are run through a printer to produce a listing of the data. This listing is then inspected by an analyst with the aid of a plotted map of the data for the detection of any serious errors in the data. This step is unfortunately necessary because of coding and communication errors. However, it need not delay the procedure unduly, since the maps can be plotted at the same time as the cards are being punched. It is not difficult to postulate an addition to the machine program which would check the data and reject those containing serious errors. This has not been put into practice, since it appears that such a procedure would lengthen the machine computations to an unreasonable time.

3) After any serious errors detected in the data are corrected, the data cards are added to the code cards, the resulting deck is placed in the computer, and the computation is started.

4) The machine computation then proceeds entirely automatically. At the present, slightly under $1^1/_2$ hours are required for the computation of the D values on one pressure surface for a 19×19 grid of points. It is known how to rewrite the code with one slight internal change which would reduce the above computation time to under one hour. Eventual reduction to a time of less than 35 minutes is visualized for the IAS computer.

5) The output cards from part E are usually printed to enable an inspection of the analysis. These cards are also used directly as input information for the forecast problem. The printed results at each grid point consist of:

a) the D value obtained from the analysis, in tens of feet,

b) a two digit number which tells the number of pieces of meteorological information used in the computation of the D value, and

c) a digit indicating at which passage through the grid the computation was made.

VI. Results

The subjective analyses available for comparison with the objective analysis of 500 mb, 25 November 1950, 0300 GCT, were prepared by three different individuals, and will be referred to as A, B, and C. Analyses A and B were prepared very carefully by experienced analysts, who took all the time they needed for their completion. Analysis C was made hurriedly under operational conditions. These analyses were then compared with an objective analysis (O) for the same map, using the weighting factor corresponding to fifty feet height error and ten knots wind error. The standard deviations of D and $\nabla^2 D$ in feet (for the same points referred to in section III) of the difference between the various analyses are shown in Table II.

Table II

Analysis pairs	A-B	A-C	B-C	O-A	O-B	O-C
Std. Dev. D.	50	67	60	69	72	68
Std. Dev. $\nabla^2 D$	13	17	16	18	20	18

It can be seen from this table that the objective analysis did not agree with the subjective analyses quite as well as the subjective analyses agreed with each other. However, the relatively close agreement between A and B is something which cannot ordinarily be expected between subjective analyses. It should not be supposed from these results that the objective analysis was inferior to the subjective analyses, since in this case the forecast is the ultimate test of the worth of the analysis.

Previously a 24-hour numerical forecast had been made on the IAS computer for the intense cyclogenesis of November 24—25, 1950 starting from 1,500 GCT, November 24. The input data consisted of the 1,000 and 500 mb charts prepared by analyst B (above). The same forecast was then made from objective analyses. The two resulting forecasts, expressed in the form of 24-hour forecast changes of height of the 1,000 mb surface, were then correlated with each other yielding a correlation coefficient of 0.96. Both forecasts predicted with fair accuracy the cyclogenesis which developed, neither forecast being observably superior to the other. From this experiment it was concluded that where a good data coverage is found, e.g. over North America, two-dimensional objective analyses, prepared for two different levels, are satisfactory for use in numerical prediction.

When three pressure surfaces in the vertical are to be analyzed for use in a forecast, a new problem is encountered. This is the problem of representing accurately the horizontal variation of vertical stability. This variation is the additional feature treated in the three-parameter forecasting model, as compared with the two-parameter model. In a test of the representation of stability by the above-described objective analysis scheme, maps were prepared for 1,500 GCT, November 24, 1950, of a quantity directly related to vertical stability, namely

$$\triangle^2 D = D_{400} - 2D_{700} + D_{1000}$$

where the subscripts indicate the pressure surface at which the D value was read from the maps. This quantity was computed from the data at the three pressure surfaces at each grid point from the objective and also from the subjective analyses. From the grid point values, values of $\triangle^2 D$ were interpolated at the locations of all reporting radiosonde stations. The values of $\triangle^2 D$ were also calculated from the actual radiosonde reports. These were used as verification of the values obtained from the objective and from the subjective analyses. The values of $\triangle^2 D$ obtained by objective analysis when correlated with the values from the actual observations yielded a correlation coefficient of 0.90. The correlation coefficient between the values obtained from subjective analysis and the reports was 0.94. From this it was concluded that the two-dimensional scheme of objective analysis gives an adequate representation of the horizontal variation of vertical stability where the data coverage is good and when the pressure surfaces analyzed are at least 300 mb apart. An improvement over these results could probably be obtained with a more elaborate scheme of objective analysis. It is doubtful, however, whether or not the slight improvement which might be expected would justify the extra computation involved.

The next experiment consisted of using the maps prepared for 1,500 GCT, 24 November 1950, in two forecasts with a three-parameter model (3)—one to be made from the subjective analyses and the other from the objective analyses. The results, at 400 and 1,000 mb, of these forecasts are presented in Figures 3 and 4. The 700 mb results are very similar to these. These figures show the surprising result that the forecast made from the objective analyses verified better than the forecast made from the subjective analyses. Further confirmation of this is given by the correlation coefficients of forecast change vs. observed change at 400 mb, where the difference between the two forecasts was greatest. These are given in Table III.

Table III

	Correlation coefficient
Forecast change vs. observed change—subjective analysis....	0.63
Forecast change vs. observed change—objective analysis.....	0.79
Forecast change—objective analysis vs. forecast change—subjective analysis..............	0.82

The results described above can be attributed to the fact that when the objective analyses are made, the geostrophic approximation is *forced* on the analyses, thus giving better results when a quasi-geostrophic model is used to make the forecast. This forcing is most marked in the vicinity of pronounced troughs or lows at middle and high levels in the troposphere, where the speed and curvature of the flow are strong. It takes the form of diminishing the analyzed geostrophic wind from the actual geostrophic wind in the areas where the observed winds are markedly sub-geostrophic. This is due to the fact that observed winds as well as all the heights of the pressure surface are used in the objective analysis. The smoothing process incorporated into the objective analysis also contributes to the same effect, particularly when the half-wavelength of the wind pattern is not much larger than the dimension of the search area. The quasi-geostrophic prediction model, using the geostrophic approximation for measurement and advection of vorticity, then has a more useful representation of these quantities than it otherwise would.

VII. Future Extensions of Objective Analysis

A logical step in the extension of the above analysis scheme would be to use a forecast made from an earlier map in the analysis over areas where reported data are sparse or non-existent. This can be compared to the activities of an analyst who consults the previous map when analyzing an area which has poor data coverage. The program which

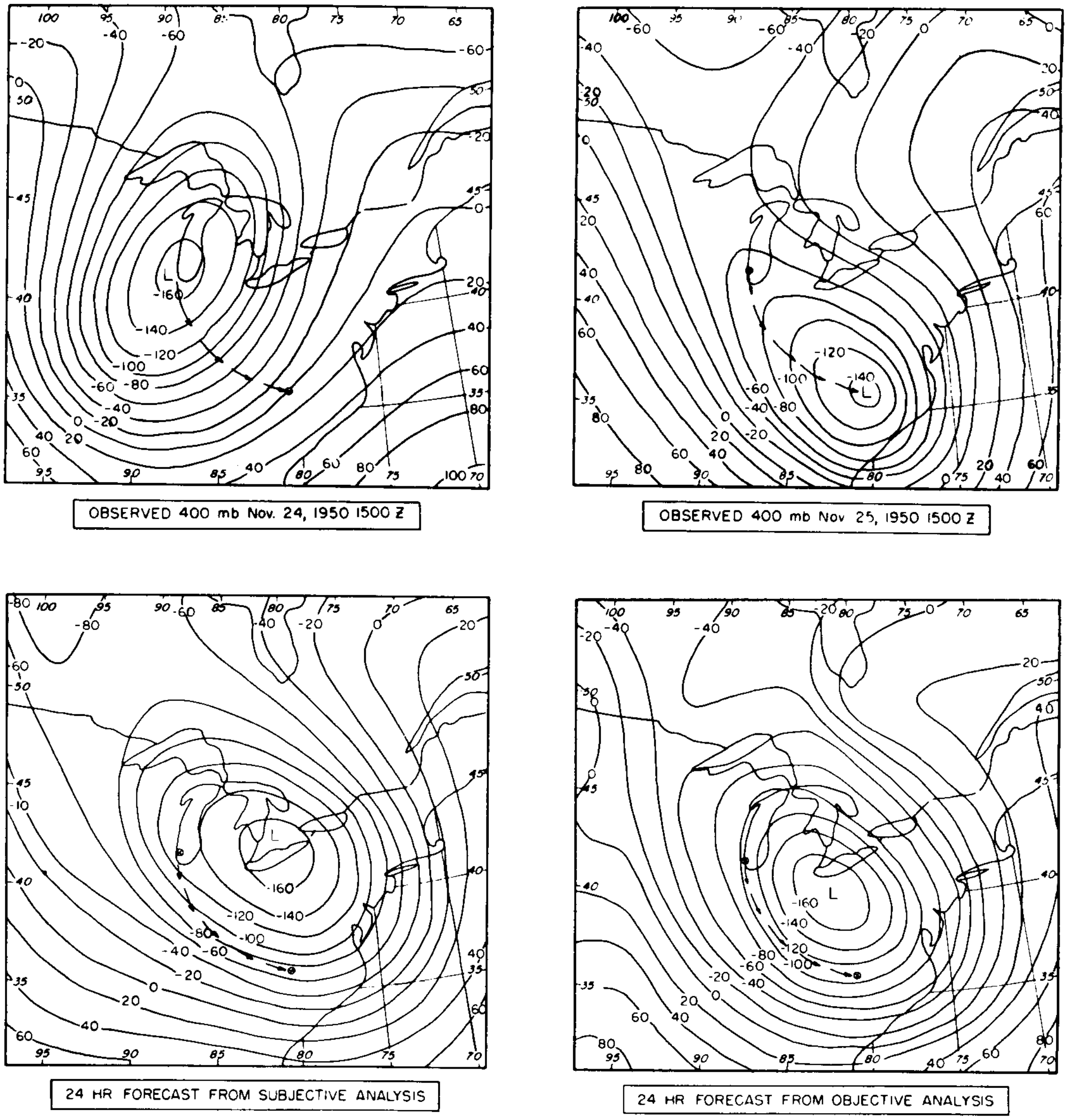

Fig. 3. 400 mb initial, forecast, and verification maps. The dashed line indicates the observed 24-hour movement of the low center.

has been drawn up (and partly coded for the computer) to accomplish this objective goes through the following steps:

1) The analysis is started in the manner described earlier. However, after the search routine has been completed with the use of a certain area the analysis is stopped, leaving unanalyzed the areas where data are very sparse.

2) The forecast D values for the pressure surface under consideration are read into the computer.

3) At all points where the analysis was completed, differences ($\triangle D$) between analyzed and forecast D values are obtained by subtraction. At the points where the analysis was not completed, no differences can be formed.

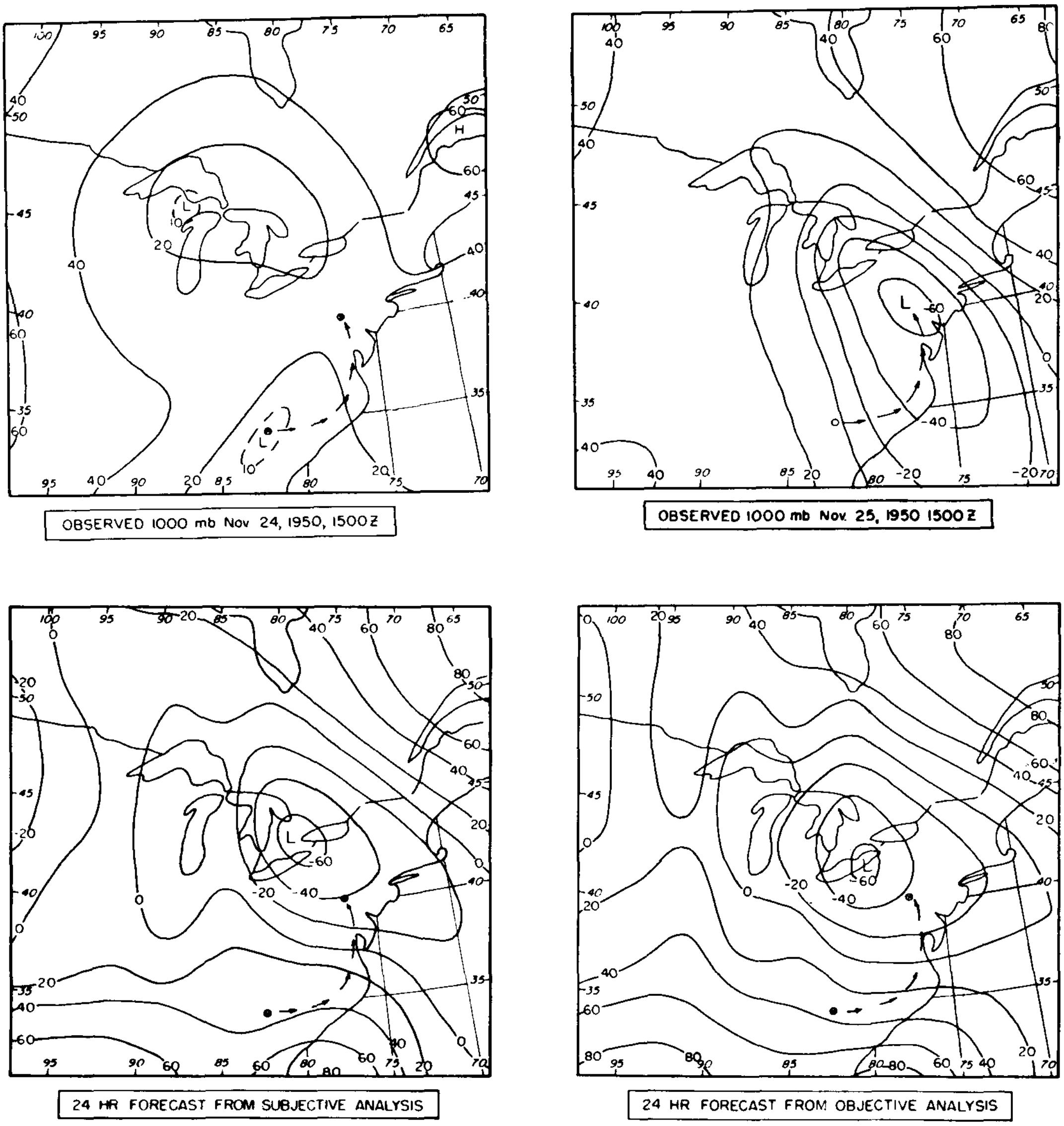

Fig. 4. 1,000 mb initial, forecast, and verification maps. The dashed line indicates the observed 24-hour movement of the low center.

4) The field of $\triangle D$ is filled in at the uncompleted locations by fitting a quadratic surface to the completed points around the uncompleted ones and interpolating for the uncompleted points. This completes the $\triangle D$ field. The values of $\triangle D$ are then added to the forecast D values, giving a "corrected" forecast D value at each point where no analysis could be made.

5) The corrected forecast D values are given a certain weight and treated as data. The original analysis code then goes over the reported data, previously computed data, and the weighted and corrected forecast values. It fits the quadratic surface to all these values in the vicinity of the unanalyzed point and interpolates for the height of the pressure surface at the point in question.

VIII. Conclusions

The results obtained above lead to the conclusion that a simple two-dimensional scheme of objective analysis is feasible and satisfactory for the purpose of transforming raw data into input data for a two- or three-parameter numerical prediction model. In fact such a scheme appears to be better suited for use in a quasi-geostrophic prediction model than information obtained from subjective analysis. The principal difficulty encountered so far is introduced by incomplete and erroneous meteorological observations. The detection and correction of the errors at present must be done subjectively, i.e., a skilled analyst must inspect the data very carefully for omissions and errors. One could argue that this process practically amounts to a subjective analysis. It should be pointed out that in the preparation of input data for a forecast from a subjective analysis, the analysis itself is only the beginning of the process. The reading, recording, and checking of interpolated information at the grid points is the most tedious and lengthy part of the process. All this is unnecessary when an objective analysis method is used. It is clear, however, that the reduction and eventual elimination of errors in computation and transmission of meteorological data is a subject which must receive high priority in future research and development before a truly objective analysis is possible.

IX. Acknowledgements

Some of the preliminary work in laying out and coding the machine program was done by Lt. A. L. Stickles, U.S. Navy, while at the Institute for Advanced Study. The principal burden of writing the code for the machine computations was borne by Mr James Cooley of the Institute for Advanced Study. This work was performed under Contract No. N-6-ORI-139, T. O. 1 between the Office of Naval Research and the Institute for Advanced Study.

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