Successive Correction (Cressman) Objective Analysis Procedure

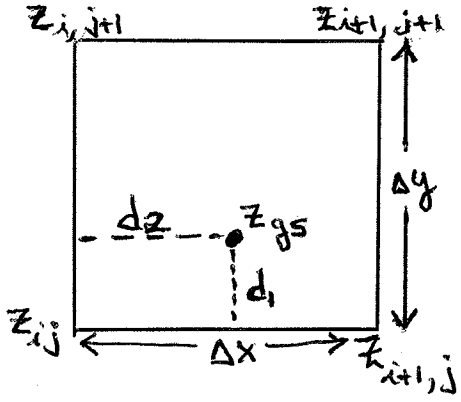
The following is a step-by-step procedure for performing a univariate successive-correction objective analysis.

1. Station data are obtained and checked for gross errors (quality control)

2. A first guess (background) field z_g is obtained and supplied to the grid

3. The background field is interpolated to the stations:

   a. For stations within the grid domain boundaries, use bilinear interpolation. If we let $\Delta x = \Delta y = 1$, the formula can be written (see diagram to right):



$$z_{gs} = [\, z_{ij}\,(1-d_1)(1-d_2) + z_{i+1,j}\,(1-d_1)\,d_2 + z_{i,j+1}\,d_1(1-d_2) + z_{i+1,j+1}\,(d_1)\,d_2\,] \Big/ \Delta x\,\Delta y$$

   b. We can't use this formula to obtain background values at the stations that are outside the grid. One approach to do this is to use the Cressman weight function in the following way:

$$z_{gs} = \frac{\sum_{i=1}^{N} w(r_i)\, z_{gi}}{\sum_{i} w(r_i)}, \qquad w(r_i) = \frac{R^2 - r_i^2}{R^2 + r_i^2}, \quad r_i \le R$$

   where N is the number of obs used. Note that this is extrapolation.

4. Calculate the observation increments (the deviations or corrections between the observations and background values at the station locations).

5. Sum weighted observation increments at the grid points. The formula is:

$$za_{ij} = zg_{ij} + \frac{\sum_{k=1}^{K} w(r_k)\,[\, z_{os} - z_{gs}]_k}{\sum_{k=1}^{K} w(r_k)}$$

   where K is the number of obs inside the influence area defined by radius R.

6. Repeat steps 3 to 5 for scans $\nu = 2, 3, \ldots\ldots N$. Usually $R^{\nu+1} \leq R^{\nu}$. N is usually 3-4, or, until the analysis is "pleasing".

Summary of the Cressman Scheme (as used here)

Advantages

1. Does make use of a background field to compute observation increments (corrections)
2. Method is straightforward and very fast
3. Can be multivariate (winds and heights can be used together)
4. Procedure is "tuneable" (by varying R, ν, etc. such that pleasing results are obtained.

Disadvantages

1. Observational errors not accounted for. Thus it is not well suited for a diverse mix of observation types.
2. Does not account for uneven distribution of data; analysis is poor near data holes and domain edges.
3. Scale of result varies with observational density (when iterated). That is, the response function is different at different points.
4. "Optimum" values of R, ν, etc. are determined *ad hoc* (i.e. – by trial and error)

More properties of SCM (Daley, p. 64-71)

Recall the expression for the analysis $f_A(\vec{r}_i)$, eq. (4), from Lecture 7. This shows the effect of one particular observation at k on the analysis. However, if we have K_i observations in an influence region, each made with a different instrument, we need to rewrite eq. (4) in the following way:

$$f_A(\mathbf{r}_i) - f_B(\mathbf{r}_i) = \frac{\sum_{k=1}^{K_i} E_O^{-2}(k)\, w(\tilde{r})\, [f_o(\mathbf{r}_k) - f_B(\mathbf{r}_k)]}{\sum_{k=1}^{K_i} E_o^{-2}(k)\, w(\tilde{r}) + E_B^{-2}} \qquad (11)$$

If we have a single observation type, E_o is not a function of k, and (11) simplifies to

$$f_A(\mathbf{r}_i) - f_B(\mathbf{r}_i) = \frac{\sum_{k=1}^{K} w(\tilde{r})\, [f_o(\mathbf{r}_k) - f_B(\mathbf{r}_k)]}{\sum_{k=1}^{K_i} w(\tilde{r}) + \varepsilon_o^2} \qquad (12)$$

where $\varepsilon_o^2 = E_o^2 / E_b^2$, the ratio of the expected observational error variance to the expected background error variance.

Note that for the Cressman scheme, $E_o(k)$ is just E_o. If we do not use E_b, we have just

$$f_A(\mathbf{r}_i) - f_B(\mathbf{r}_i) = \frac{\sum_{k=1}^{K} w(\tilde{r})\, [f_o(\mathbf{r}_k) - f_B(\mathbf{r}_k)]}{\sum_{k=1}^{K_i} w(\tilde{r})} \qquad (13)$$

This equation is similar to eq. (10) without the use of winds or the "A" factor.

If we have a <u>single observation</u>, eq. (13) shows that:

(i) $f_A(\vec{r}_i) = f_o(\vec{r}_i)$ <u>if</u> k is at the i grid point

(ii) $f_A(\vec{r}_i) = f_B(\vec{r}_i) + [f_o(\vec{r}_k) - f_B(\vec{r}_k)]$, $k \neq i$

(iii) $f_A(\vec{r}_i) = f_o(\vec{r}_k)$ for any ob within R if there is no background field.

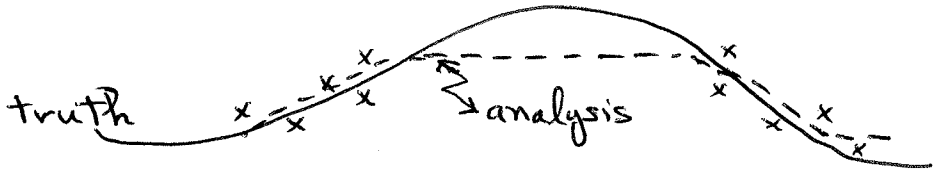
Other properties of a <u>single pass</u> of a SCM in terms of the *a posteriori* weight W_{ik} :

a. $\sum_{k=1}^{K_i} W_{ik} \leq 1$

b. $W_{ik} \geq 0$, $1 \leq k \leq K_i$ [If $w(\tilde{r})$ never < 0, as in Cressman or Barnes weights]

c. $| f_A(\mathbf{r_i}) - f_B(\mathbf{r_i}) | \leq \max | f_O(\mathbf{r_i}) - f_B(\mathbf{r_i}) |$

that is, the analysis increment is always $\leq$ the observational increment, indicating that no "meaningful" extrapolation is possible. This can be illustrated via:



d. W_{ik} is independent of the density (relative separation) of the observations. Thus too much weight is given to obs. in high data density regions, and too little to obs. in sparse data regions

These 4 properties are restrictive. They do not hold for statistical schemes, nor do they hold for iterated SCM schemes, as we shall see.

Iteration correction scheme - SCM

Assume: (i) Background error is homogeneous
(ii) All obs. within R made with same instrument
(iii) Observational errors are spatially uncorrelated

We start with eq. (12) written in matrix form:

$$f_A^1(\mathbf{r_i}) = f_B(\mathbf{r_i}) + \underline{W}_i^T [\underline{f}_o - \underline{f}_B] \qquad (14)$$

where $\underline{W}_i^T$ is a (1 x K) row vector of *a postoriori* weights, with elements

$$W_{ik} = \frac{w(r_{ik})}{\sum_{k=1}^{K_i} w(r_{ik}) + \varepsilon_o^2} \qquad (15)$$

and

$$r^2_{ik} = (x_k - x_i)^2 + (y_k - y_i)^2 \sim r^2$$

Note that f_A^1 represents the first iteration. The second iteration is given by

$$f_A^2(\mathbf{r_i}) = f_A^1(\mathbf{r_i}) + \underline{W}_i^T [\underline{f}_o - \underline{f}_A^1] \qquad (16)$$

where $\underline{f}_A^{\,1}$ are analysis (now background) values at the stations obtained by "forward interpolation" using $f_A^{\,1}(\vec{r}_i)$ grid values. The general iterated formula is

$$f_A^{\,j+1}(\mathbf{r}_i) = f_A^{\,j}(\mathbf{r}_i) + \underline{W}_i^{\,T}\left[\underline{f}_o - \underline{f}_A^{\,j}\right] \qquad (17)$$

Special example illustrating properties of iterated systems

Assume K observations and that $\underline{f}_A$ uses all K obs. at each grid point (i.e. – a "global" analysis).

Also assume that the obs. and grid points coincide; thus no forward interpolation is required. Thus the i subscript can be replaced everywhere by k in eq. (17) and we have

$$f_A^{\,j+1}(\mathbf{r}_k) = f_A^{\,j}(\mathbf{r}_k) + \underline{W}_k^{\,T}\left[\underline{f}_o - \underline{f}_A^{\,j}\right] \qquad (18)$$

Now write eq. (18) for *all* grid points (normally, a sum over i, but here it is k):

$$\underline{f}_A^{\,j+1} = \underline{f}_A^{\,j} + \underline{\underline{W}}^{T}\left[\underline{f}_o - \underline{f}_A^{\,j}\right] \qquad (19)$$

where $\underline{\underline{W}}^{T}$ is a real, square (K, K) matrix of *a posterori* weights.

To illustrate the properties of the analysis provided by (19), consider Fig. 3.2 in Daley.

Here we have a periodic domain in x, $-\pi \le x \le \pi$, with K = 20 equally-spaced observations indicated by the solid dots. Apply the Cressman weight function with $R = \pi/4$ (thus 5 obs. are used at each grid point).

Also assume $\varepsilon_o^{\,2} = 0.25$ and background field values = 0. Note that this is different from assuming no background field at all, since then $\varepsilon_o^{\,2} = 0$.

The result of this analysis for j = 0, 1 and 2 iterations is shown in Fig. 3.2(a). It shows that the iterated analysis does converge to the observations. [This is proved mathematically in Sec. 3.5.]

What are the weights $\underline{\underline{W}}^{T}_{j+1}$ after the j+1 iteration? Eq. (15) gives the formula for one iteration, so we need to derive the formula for $\underline{\underline{W}}^{T}_{j+1}$ To do so, add $-\underline{f}_o$ to eq. (19):

$$\underline{f}_A^{\,j+1} - \underline{f}_o = \underline{f}_A^{\,j} - \underline{f}_o + \underline{\underline{W}}^{T}\left[\underline{f}_o - \underline{f}_A^{\,j}\right]$$

or

$$\underline{f}_A^{\,j+1} - \underline{f}_o = \left[\underline{\underline{I}} - \underline{\underline{W}}^{T}\right]\left[\underline{f}_A^{\,j} - \underline{f}_o\right]$$

Now apply eq. (19) again for $\underline{f}_A^j$

$$\underline{f}_A^{j+1} - \underline{f}_o = [\underline{\underline{I}} - \underline{\underline{W}}^T][\underline{f}_A^{j-1} + \underline{\underline{W}}^T(\underline{f}_o - \underline{f}_A^{j-1}) - \underline{f}_o]$$

or

$$\underline{f}_A^{j+1} - \underline{f}_o = [\underline{\underline{I}} - \underline{\underline{W}}^T]^2[\underline{f}_A^{j-1} - \underline{f}_o]$$

Continuing use of (19) until the $\underline{f}_A$ term on the RHS becomes $\underline{f}_A^0$, we have

$$\underline{f}_A^{j+1} - \underline{f}_o = [\underline{\underline{I}} - \underline{\underline{W}}^T]^{j+1}[\underline{f}_A^0 - \underline{f}_o]$$

But by assumption, $\underline{f}_A^0 = \underline{f}_B$, the original background field. So, making this substitution inside the brackets, and then adding $\pm \underline{f}_B$ to the RHS, we have

$$\underline{f}_A^{j+1} = \underline{f}_o + \underline{f}_B - \underline{f}_B - [\underline{\underline{I}} - \underline{\underline{W}}^T]^{j+1}[\underline{f}_o - \underline{f}_B]$$

or

$$\underline{f}_A^{j+1} = \underline{f}_B + \{\underline{\underline{I}} - [\underline{\underline{I}} - \underline{\underline{W}}^T]^{j+1}\}[f_o - f_B]$$

Therefore the iterated analysis equation is

$$\underline{f}_A^{j+1} - \underline{f}_B = \underline{\underline{W}}^T(j+1)[\underline{f}_o - \underline{f}_B] \qquad (20)$$

where

$$\underline{\underline{W}}^T(j+1) = \underline{\underline{I}} - [\underline{\underline{I}} - \underline{\underline{W}}^T]^{j+1}$$

A plot of $\underline{\underline{W}}^T(j+1)$ for j = 1, 2 and 5 is given in Fig. 3.2(b). When j = 1, we have the case where the Cressman weight is used in eq. (15).

When j > 1, the $W_{kk}(j)$ weight increases; i.e. – the weight at the "center" observation increases (this increases the degree of detail in the analysis). Note that the weights $W_{lk}(j)$, l = k, away from the center can become negative.

Actually, it can be shown that none of the restricted properties (a) to (d) on p. 3 – 4 for a single iteration scheme necessarily hold for an iterated scheme, j > 1.

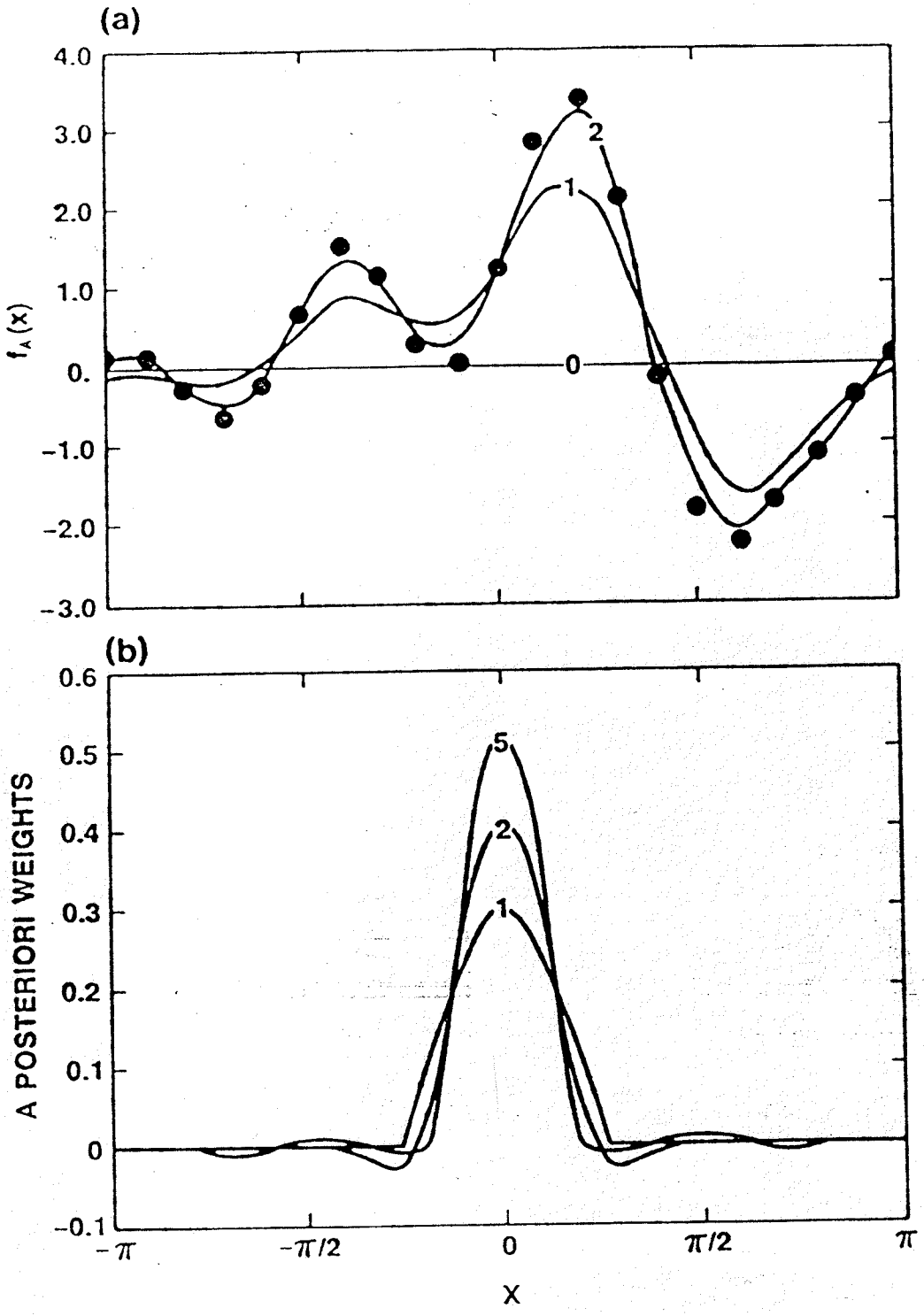


Figure 3.2 Illustration of the iteration process for the SCM algorithm for 20 equally spaced observations on a one-dimensional periodic domain: (a) shows the analyzed values as a function of iteration number; (b) shows the a posteriori weights.

for $j > 1$, the weights $\underline{W}^T(j)$ can become nonzero for $|x_k - x_l| > R$. For this regular periodic network $\underline{W}^T$ happens to be symmetric, but this is not generally so.

Exercise 3.2 demonstrates that property (3.1.13) also does not apply when $j > 1$. In fact, none of the properties (3.1.10–13) necessarily hold when $j > 1$.

The convergence of the iteration cycle (3.2.6) is discussed in Section 3.5. Before