

The Barnes Objective Analysis Procedure

Reading: Koch *et al.*, (1983) paper (handed out); Daley, p. 90-93.

We will study the Barnes scheme in detail (2-3 lectures) because it is commonly used in research and software packages (e.g., GEMPAK; McIDAS) for diagnostic analysis of meteorological data.

A Barnes objective analysis refers to the use of a Gaussian filter as the distance-dependent weight function:

$$w_m = \exp [- r_m^2 / \kappa] \quad (1)$$

where r_m is the distance between the m^{th} observation and the grid point, and κ is a parameter governing the shape of the resulting response function (see Fig. 3.3 in Daley).

κ has units of r^2 (e.g., m^2 if doing an analysis in x-y space), and should be related to the average spacing of the observations (and also the data distribution if it is non-uniform).

E.g., if have dense observations, κ should become smaller so that there is a smaller weight w_m for a given r_m (*i.e.* – only data closest to grid point are weighted highly)

The Barnes scheme can be designed to include (exclude) the scales you want (don't want) to retain by the choice of κ and number of scans.

A response function for eq. (1) can be obtained analytically by assuming sinusoidal fields and continuous data (Barnes, 1973; Pauley and Wu, 1990). The response function for one pass is

$$D_0 = \exp [- \kappa_0 (\pi/\lambda)^2] \quad (2)$$

where λ is wavelength.

Thus the analyzed field $g_0(i, j)$ can be related to the original data field $f(x, y)$ via

$$g_0(i, j) = D_0 f(x, y) \quad (3)$$

Figure 1 in Koch *et al.* (shown at bottom of the next page) is a plot of D_0 vs a normalized wavelength $\lambda^* = \lambda/L$, where $L = 2\Delta n$, where Δn could be interpreted as the mean station separation. The curves are for various values of the normalized weight parameter $\kappa_0^* = \kappa_0 / L^2$. This figure shows how different wavelengths are filtered during the first pass as a function of the weight parameter. The plot shows that we can get a fairly

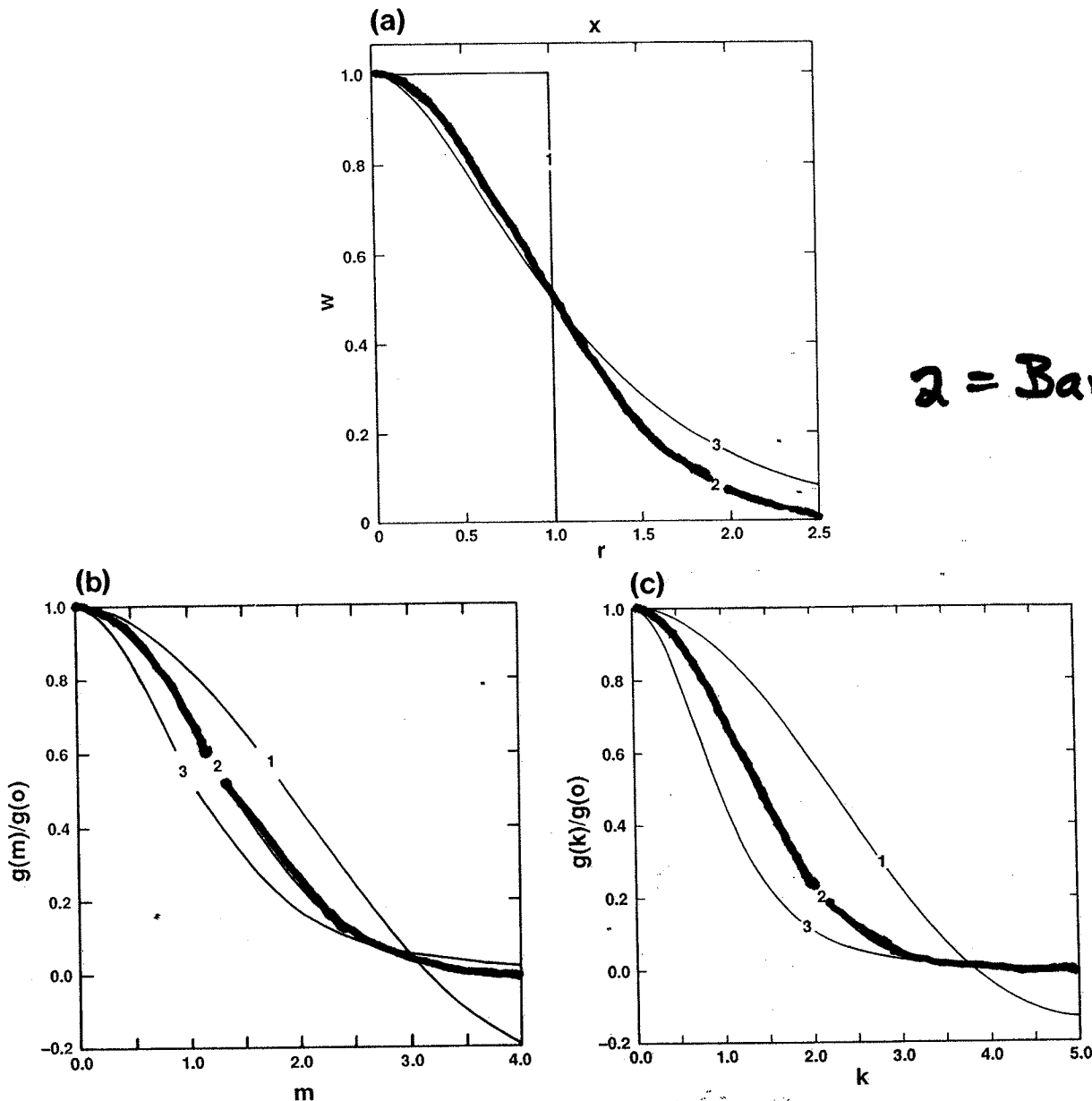


Figure 3.3 (a) Weight functions and (b,c) transfer functions. (b) is the one-dimensional case; (c) is the axisymmetric case.

Now consider the weight function given by (3.1.9),

$$w(x) = \exp\left(-\frac{x^2}{2L^2}\right) \quad (3.3.13)$$

which is plotted as curve 2 of Figure 3.3(a), with L chosen so that $w(1) = 0.5$. Equation (3.3.13) can be transformed using (3.3.9). The transformation involves well-known exponential integrals that can be found in integral tables (Abramovitz and Stegun 1965). In this case $g(0) = 1/\sqrt{2\pi}$ and

$$\frac{g(m)}{g(0)} = \exp\left(-\frac{m^2 L^2}{2}\right) \quad (3.3.14)$$

smooth analysis (short wavelengths are filtered out) for $\kappa_0^* = 1$ or greater, which means that $\kappa_0 \geq (2\Delta n)^2$.

Barnes (1964) applied eq. (1) several times to allow the scheme to “recover” (retain) more amplitude in the medium scale waves that are overly smoothed in the first pass. However, we still want the amplitude of the $2\Delta n$ wavelength to be near zero.

Barnes (1973) and Koch *et al* (1983) show that we can obtain our “desired analysis” in 2 scans by modifying κ_0 on the second pass via $\kappa_1 = \gamma\kappa_0$, where γ is a “convergence factor”, $0 < \gamma < 1$. The smaller γ is, the closer one fits the observations.

The analysis equation for this second pass is :

$$g_1(i, j) = g_0(i, j) + D_1 [f(x, y) - g_0(x, y)] \quad (4)$$

where the $g_0(x, y)$ is obtained by interpolating the $g_0(i, j)$ field from eq. (3) to the stations, and where

$$D_1 = \exp [- \gamma \kappa_0 (\pi / \lambda)^2] = D_1^{\gamma} \quad (5)$$

is the response function for the modified weight function

$$w'_m = \exp [- r_m^2 / \gamma \kappa_0] \quad (6)$$

Since $\gamma \kappa_0 < \kappa_0$, w'_m is smaller for any r_m , and less weight is given to more distant observations than before, thus forcing the analysis closer to the nearby obs.

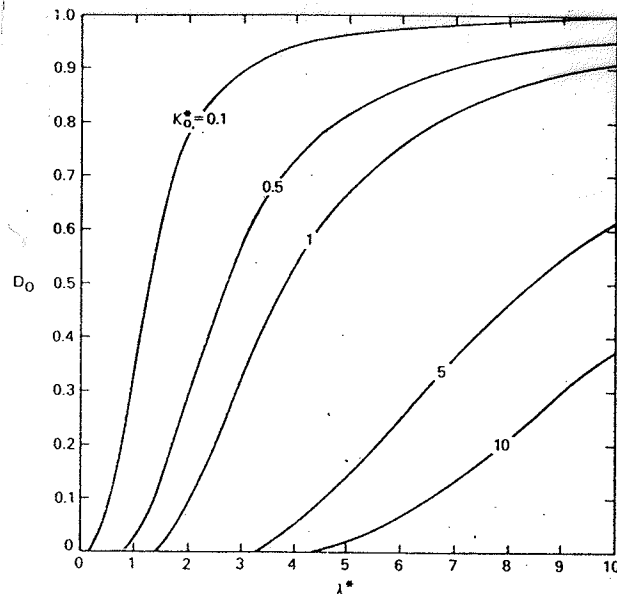


FIG. 1. Relationship of first pass response D_0 to dimensionless wavelength $\lambda^* = \lambda/L$ for various values of dimensionless weight parameter $\kappa_0^* = \kappa_0/L^2$, in which L is an arbitrary scale length based upon the data distribution (e.g., $L = 2\Delta n$).

Note, however, that D_1 is not the total response to both passes. To get the total response D_1^* , we substitute eqs. (3) and (5) into (4) to get:

$$g_1(i, j) = D_0 f(x, y) + D_0^\gamma [f(x, y) - D_0 f(x, y)]$$

where we have assumed for the last term that the response on the (x, y) data field is the same as on the (i, j) gridded field. This can be rearranged to get

$$g_1(i, j) = D_0 f(x, y) [1 + D_0^{\gamma-1} - D_0^\gamma]$$

Therefore,

$$D_1^* = D_0 [1 + D_0^{\gamma-1} - D_0^\gamma] \quad (7)$$

The change in the response from the first to the 2nd pass for two different values of γ is shown in Fig. 3 in Koch *et al.* Note that $\gamma = 1$ means that the 2nd pass was made with no accelerated convergence. The greatest increase in response occurs at the shorter wavelengths. However, we want to make sure that we don't significantly increase the response at the $2\Delta n$ wavelength, which is usually noisy. Here, the increase is about 40%, which is worrisome; - ok if have accurate, well-distributed data; not ok if not the case.

Since D_1^* depends on γ , which we specify, and D_0 , which is a function of k_0 , the main task w.r.t. designing the desired response is to devise a procedure to determine K_0 .

First, though, we summarize the Barnes analysis scheme as presented by Koch *et al.*:

1. Assuming we have no background field, the first pass is

$$g_0(i, j) = \frac{\sum_{m=1}^M W_m f(x_m, y_m)}{\sum_{m=1}^M W_m} \quad (8)$$

where the weight function W_m is given by eq. (1), and M is the number of observations within the cutoff radius R_c .

2. Interpolate $g_0(i, j)$ to stations to get $g_0(x_m, y_m)$
3. Compute the observation increment $f'(x_m, y_m) = f_m(x_m, y_m) - g_0(x_m, y_m)$
4. The second pass is then obtained from

$$g_1(i, j) = g_0(i, j) + \frac{\sum_{m=1}^M W_m' f'(x_m, y_m)}{\sum_{m=1}^M W_m'} \quad (9)$$

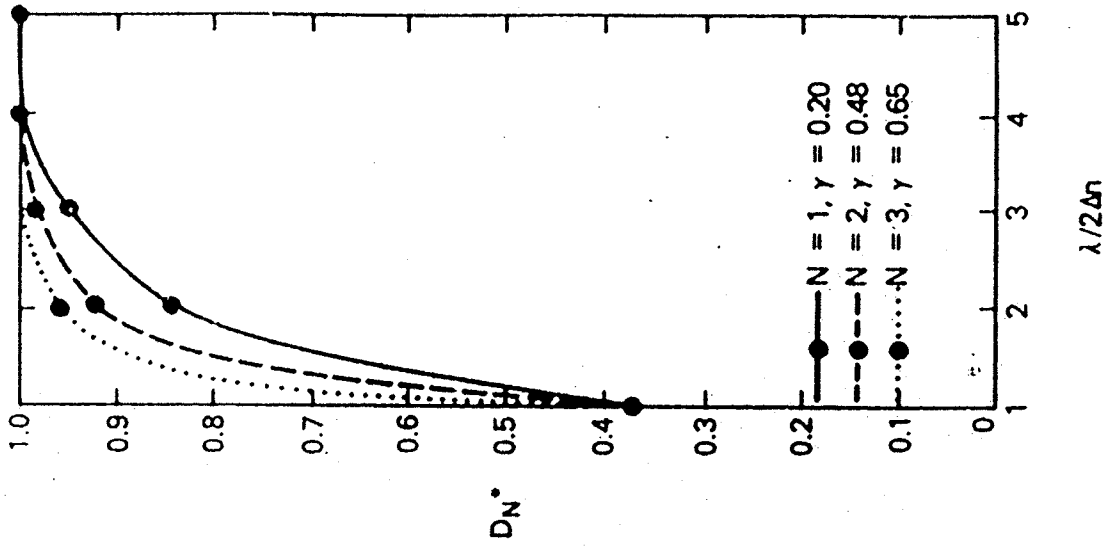


FIG. A2. Effective enhancement of filter response D_N^* produced by making multiple passes, as function of resolvable wavelengths $\lambda > 2\Delta n$.

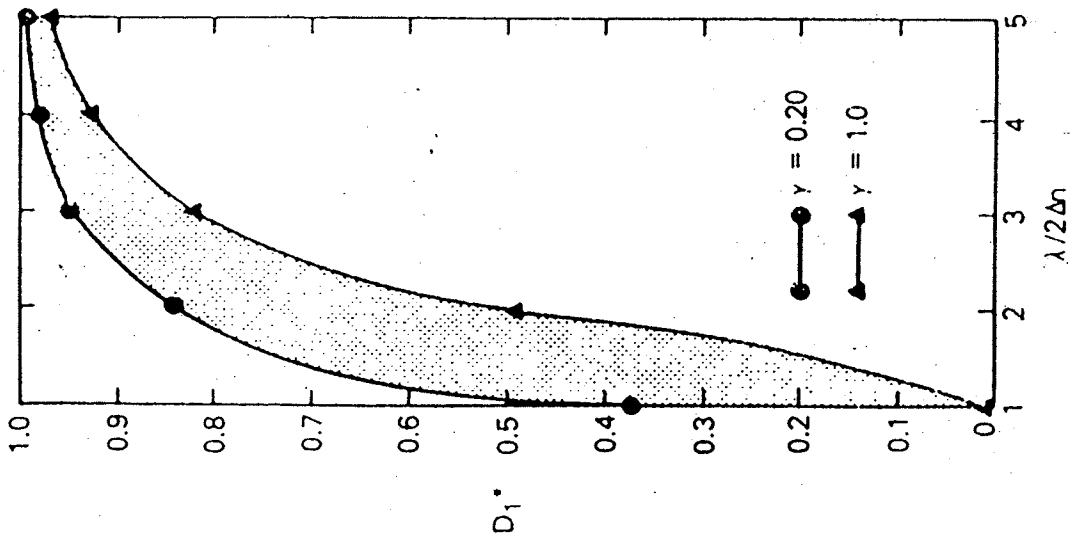


FIG. 3. Permissible second (correction) pass filter responses as functions of multiples of the $2\Delta n$ wavelength in the interactive Barnes scheme.

where the weight function is

$$w'_m = \exp \left[- r_m^2 / \gamma \kappa_0 \right], \quad (10)$$

with $0 < \gamma \leq 1$.

Thus, control over what you want to accomplish with the Barnes analysis is obtained by *a priori* choices of κ_0 , γ , Δx , R_c , and the number of passes.

Tailoring the Barnes Scheme to Your Needs (according to Koch *et al*, 1983)

(a) Choice of domain

See text for discussion . One important point is to choose your objective analysis domain to be smaller than the data domain.

(b) Calculation of Δn

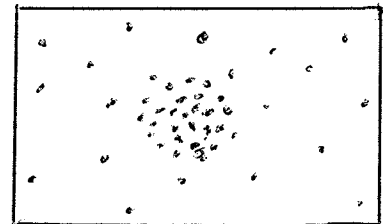
The mean distance between stations Δn plays an important role in the tailoring of the Barnes scheme. In Koch *et al*, Δn_c is the value that one computes from your station data. The quantity Δn_r is the spacing one obtains by assuming an even distribution of the M observations over the domain area A , which is given by

$$\Delta n_r = A^{1/2} \left[\frac{1 + M^{1/2}}{M-1} \right], \text{ which goes to } \left(\frac{A}{M} \right)^{1/2} \text{ for large } M.$$

For example, if $M = 100$, $\Delta n_r = 1/9 A^{1/2}$, whereas the simple formula gives $1/10 A^{1/2}$.

If you have poor data distribution, then $\Delta n_c \ll \Delta n_r$, and a value of Δn larger than Δn_c should be selected. E.g., if the distribution looks like:

we have $\Delta n_r > \Delta n_c$, and we would need to use a value closer to Δn_r to avoid excessive noise in the analysis if we were using only one value of κ_0 . However, we could do either of the following:



- (i) Produce two analyses, one with $\Delta n = \Delta n_r$, and a second one just in the data dense region only with $\Delta n = \Delta n_c$ in that region.
- (ii) Compute Δn at every grid point from nearby stations, thus changing κ_0 at every grid point (as well as the response function).