Mesoscale Meteorology

METR 4433 | Exam #2 | Spring 2015

Solutions to Jeremy's questions

1 Single and Multicell Storms (30 points)

(a) What is the optimal environmental condition for long-lasting squall lines according to RKW theory? Why? (10 points)

Solution

According to RKW theory, the optimal condition for long lasting squall lines is when there is a balance between the horizontal vorticity produced by the cold pool and the opposite horizontal vorticity associated with the ambient low-level vertical wind shear on the downshear flank of the system. In other words, the shear magnitude in the low-level inflow should be equal to the cold pool propagation speed. This is considered the optimal condition because it supports the formation of an updraft with near-zero net vorticity above the gust front, hence resulting in a deep upright updraft.

(b) Related to RKW theory, we found that $c/\Delta u$ was an important ratio in understanding the strength and development of squall lines, where c is the density current propagation speed and Δu is the low-level environmental shear. In simple terms, describe what it means when: $c/\Delta u < 1$, $c/\Delta u = 1$, $c/\Delta u > 1$. (10 points)

Solution

- $c/\Delta u < 1$ signifies that the ambient shear is too strong relative to the cold pool.
- $c/\Delta u = 1$ represents the optimal state for deep lifting by the cold pool.
- $c/\Delta u > 1$ signifies that the cold pool is too strong for the ambient shear.

(c) Physically describe how those three scenarios ($c/\Delta u < 1$, $c/\Delta u = 1$, $c/\Delta u > 1$) relate to the three stages of early two-dimensional squall line evolution. Hint: be sure to mention updrafts and how each scenario supports the particular stage of development. (10 points)

Solution

- During the initiation phase, there exists environmental shear but no cold pool. Thus, $c/\Delta u < 1$ and the updraft is tilted downshear.
- During the strong cell stage, a cold pool is formed, which in turn spawns new cells. Those cells also produce outflow that strengthen the cold pool. Eventually, the cold pool circulation becomes strong enough to balance the horizontal vorticity associated with the ambient shear. This stage corresponds to $c/\Delta u = 1$ and represents the optimal condition according to RKW theory - where updrafts are deep and upright.

- As the cold pool continues to strengthen, the cold pool circulation eventually overwhelms the ambient vertical wind shear vorticity ($c/\Delta u > 1$). Cells begin to tilt upshear and advect rearward over the cold pool and the squall line weakens.

2 Cold Pools (30 points)

(a) What important role does thunderstorm outflow play in multicell storm development? (10 points)

Solution
The most important role of the cold pool is to provide low-level lifting.

(b) Imagine a cold pool from a multicell storm that generates a positive $1\ \mathrm{mb}$ pressure perturbation at the surface. What constant potential temperature deficit must exist between the cold pool and a constant environmental potential temperature of $300\ \mathrm{K}$ in order for it to reach a depth of $400\ \mathrm{m}$? Assume that the environmental density is equal to $1\ \mathrm{kg\ m^{-3}}$. Hint: The Boussinesq approximated vertical equation of motion is given by:

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial z} + g\left(\frac{\theta'}{\overline{\theta}}\right)$$

Assume the pressure perturbation at the surface is caused solely by the hydrostatic effect of the heavier overlying air/fluid inside the cold pool and that the pressure perturbation at the top of the cold pool is 0. (10 points)

Solution
Here we assume that the flow is stationary in the cold pool and the pressure perturbation is caused due to hydrostatic effects. Thus, $\partial w/\partial t = 0$, and we can rewrite the equation and integrate from the surface to the top of the cold pool.

$$\begin{aligned}
\frac{1}{\rho_0}\int_0^h \frac{\partial p'}{\partial z}dz &= g\left(\frac{\theta'}{\overline{\theta}}\right)\int_0^h dz \\
\frac{1}{\rho_0}\left\{p'(h) - p'(0)\right\} &= g\left(\frac{\theta'}{\overline{\theta}}\right)h \\
-\frac{\Delta p}{\rho_0} &= g\left(\frac{\theta'}{\overline{\theta}}\right)h \\
\theta' &= -\frac{\overline{\theta}\Delta p}{\rho_0 g h} \\
\theta' &= -\frac{(300\ \mathrm{K})(100\ \mathrm{Pa})}{(1\ \mathrm{kg\ m^{-3}})(9.8\ \mathrm{m\ s^{-2}})(400\ \mathrm{m})} \\
&\boxed{\theta' = -7.7\ \mathrm{K}}
\end{aligned}$$

Thus, the deficit is $7.7\ \mathrm{K}$, meaning the cold pool has a temperature of $292.3\ \mathrm{K}$.

(c) In class we considered an idealized model in the gust-front-following coordinate system. In this system, we derived a special Bernoulli equation, given by:

$$\frac{u^2}{2} + \frac{p'}{\rho_0} = \text{constant}$$

This relationship is valid along a streamline that follows the lower boundary from far upstream (where $u = U$ and $p' = 0$) to a point right behind the gust front (where $u = 0$ and $p' = \Delta p$). What is the propagation speed of the above cold pool? (10 points)

Solution

Since the Bernoulli equation is constant, we can set the upstream and gust front values equal to one another.

$$\left(\frac{u^2}{2} + \frac{p'}{\rho_0}\right)_{\text{upstream}} = \left(\frac{u^2}{2} + \frac{p'}{\rho_0}\right)_{\text{cold pool}}$$

$$\frac{U^2}{2} = \frac{\Delta p}{\rho_0}$$

$$U = \sqrt{\frac{2\Delta p}{\rho_0}}$$

$$U = \sqrt{\frac{2(100 \text{ Pa})}{1 \text{ kg m}^{-3}}}$$

$$\boxed{U = 14.1 \text{ m s}^{-1}}$$

Second In-class Exam

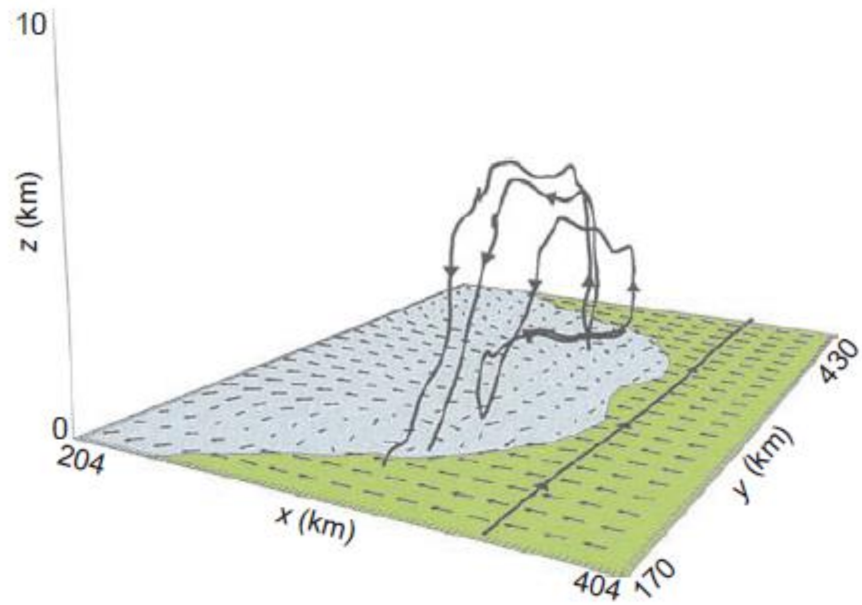
Mesoscale Meteorology (METR 4433)
April 16, 2015

Solution to Ming Xue's questions

3. (Total 20%) Bow Echo

a) (10%) What is the primary source of vorticity in the bookend vortices in bow echo systems, and why book end vortices are significant?

- In a mature bow echo, bookend vortices are the result of vortex lines being **generated baroclinically in/at the leading edge of the cold pool of** a squall line and subsequently being drawn/tilted upward by the leading updraft, yielding counter-rotating line-end vortices
- The bookend vortices **enhance the rear-inflow jet and initiate the bowing process.**
- (optional comments) Bow echoes' extreme intensity is due in large part to their relatively small size. In particular, the smaller distance between the bookend vortices enhances the focusing effect on the mid-level flow between the vortices, which can significantly strengthen the rear-inflow jet.
- Figure is optional.



b) (10%) Describe, with the help of diagrams, how rear inflow jet is promoted within a bow echo system, as viewed in a vertical cross section through the bow echo.

The development of rear inflow jet can be explained from the pressure perturbation/pressure gradient force point of view or from the vorticity/circulation point of view, as illustrated by the two slides below, respective. Students only have to discuss on of them.

For the first point of view, the key point is the development of low pressure underneath the tilted warm updraft. The larger is the CAPS, the larger the warming and pressure lowering tend to be.

This creates an inward horizontal pressure gradient force that pushes air from the rear towards the convective region, forming the rear inflow jet.

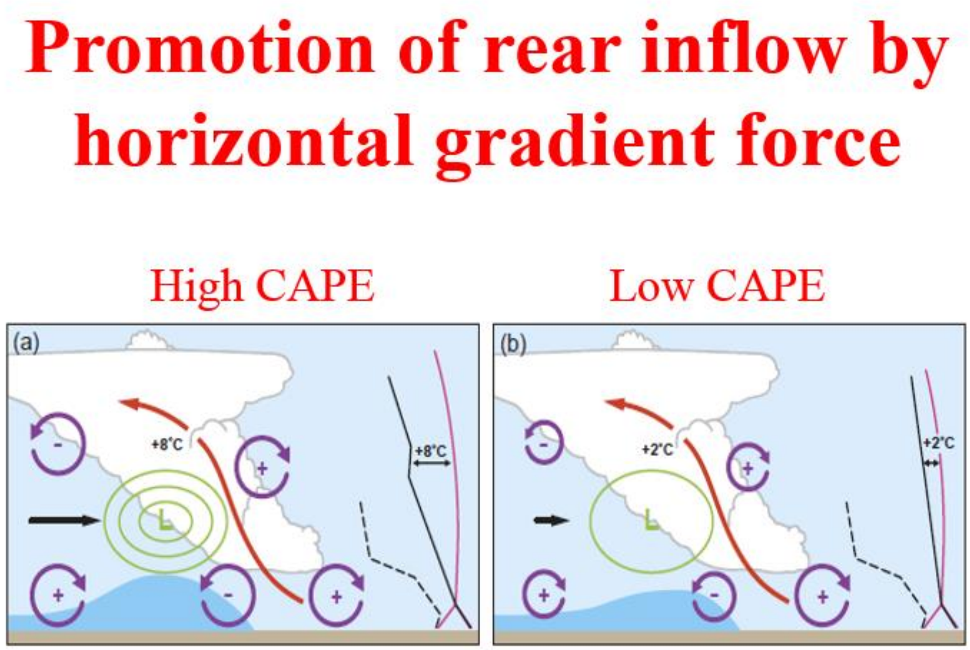
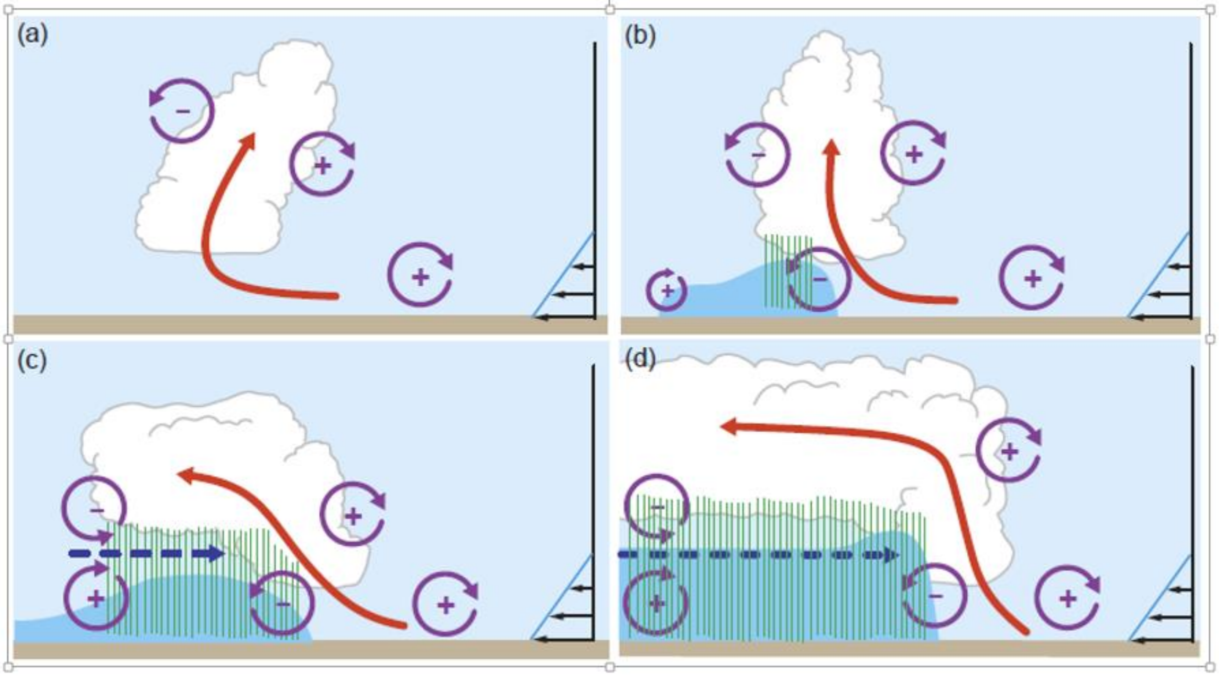


Figure 9.22 Illustration of how a tilted updraft induces midlevel rear inflow in an (a) high-CAPE and (b) low-CAPE environment. The low pressure is largely a consequence of hydrostatics, which also implies that the magnitude of the midlevel pressure deficit, and thus the rear inflow, increase as the updraft buoyancy (a function of environmental CAPE) increases. Purple arrows indicate the senses of the horizontal vorticity at various locations. Soundings are schematically presented along the right-hand side of each panel. (Adapted from an image provided by the Cooperative Program for Operational Meteorology, Education, and Training [COMET].)

With the vorticity point of view, the key point is the development of counter rotating circulations at the back edge of the rear-ward-spreading warm cloud, and the cold pool, and the two circulations act to enhance rear inflow air towards the convective region. Again, the larger is the CAPS, and the deeper is the cold pool, the stronger is the effect.

Vorticity Explanation of Rear Inflow Jet Formation



The updraft current is denoted by the thick double-lined flow vector, with the rear-inflow current in (c) and (d) denoted by the thick dashed vector. The shading denotes the surface cold pool. The thin, circular arrows depict the most significant sources of horizontal vorticity, which are either associated with the ambient shear or are generated within the convective system. Regions of lighter or heavier rainfall are indicated by the more sparsely or densely packed vertical lines, respectively. The scalloped line denotes the outline of the cloud (adapted in Weisman 1993 from Weisman 1992).

4. (Total 20%) Supercell Storm

(a) (7%) In what kind of environmental conditions do supercell storms typically occur? Why such conditions are needed for supercells to develop?

- A supercell storm is defined as a thunderstorm with a deep rotating updraft. Mention **rotating updraft** as being essential feature of supercell.
- Supercells form in environments with **strong vertical wind shear, and sufficient CAPE.**
- The **main source of updraft rotation in supercells is the horizontal vorticity in the environment** associated with vertical environmental shear. The vorticity is **tilted into the vertical** by the updraft to cause updraft rotation. **CAPE is necessary to develop and support strong updraft.** The vertical shear also allows a 3 dimensional updraft-downdraft circulation structures to establish so that updraft and downdraft do not interfere with each other. The last point is optional.

(b) (7%) Why do supercell storms tend to be stronger than regular storms? Give physical explanations, not give list certain features.

- The mid-level rotation in the updraft or mid-level mesocyclone creates negative pressure perturbations, due to centrifugal force or based on the diagnostic pressure equation, and whereby creates additional upward pressure gradient force in addition to the buoyancy force associated with the CAPE, hence support stronger updraft. Optional point: the 3D circulation structure also prevents interference of downdraft with updraft, making the updraft more steady and long lasting.

(c) (6%) What is the primary source of vertical rotation in supercell storms, and describe the processes responsible for the creation of such rotation.

- The primarily source of vertical rotation is the **low-level environmental shear,** which carried **horizontal vorticity**. Such horizontal vorticity is **tilted** into the vertical direction by the updraft, creating vertical vorticity. A figure like the following is helpful but not required.