Mesoscale Meteorology

METR 4433 | Problem Set #2 | Spring 2015

Assigned: Mar. 5, Due: Mar 26

1.) Dynamics (25 points)

The vertical equation of motion in an inviscid, Boussinesq atmosphere is given by

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g. \tag{1}$$

Let the dependent thermodynamic variables be given by

$$p(t,x,y,z) = \overline{p}(z) + p^{'}(t,x,y,z)$$
$$\rho(t,x,y,z) = \rho_0 + \rho^{'}(t,x,y,z),$$

where the first terms on the right-hand side denote the basic state, which is in hydrostatic balance, with a subscript "o" indicating a constant value.

Show that the Eq. (1) can be written

$$\frac{dw}{dt} = -\frac{1}{\rho_0}\frac{\partial p^{'}}{\partial z} + g\left(\frac{\theta^{'}}{\overline{\theta}} + 0.61 q_v^{'}\right), \tag{2}$$

where q_v is the water vapor mixing ratio and the prime indicates a pressure perturbation from a basic state which is a function only of height (hint: linearize the pressure gradient force as we have previously done and use the virtual temperature to account for moisture effects).

2.) Thermodynamics – Adverse Pressure Gradients in Cloud Updrafts (25 points)

Consider a hypothetical air parcel that is moving vertically in an atmosphere with a constant potential temperature of 300 K. If the parcel is 4 K warmer than the environment at the ground and starts from a state of rest,

(a) Using the appropriate perturbation equation of motion (neglect friction, pressure, and moisture effects), find the vertical velocity of the parcel at an altitude of 4 km.

(b) What is the magnitude and direction of the vertical perturbation pressure gradient force per unit mass (units of ms^{-2}) if the parcel attains an upward vertical velocity of 30 ms^{-1} at an altitude of 4 km?

(c) How long did it take for the parcel to reach an altitude of 4 km (hint: use $F = ma$ and recall that our two forces per unit mass are buoyancy and pressure gradient)?

3.) Density Currents (25 points)

Assume that the cold outflow from a squall line is propagating at a speed of $15\ \mathrm{ms}^{-1}$ into a calm environment. If the temperature within the outflow is 10 K cooler than the environment, with the environment having a temperature of 305K,

(a) Estimate the mean depth h of the density current (hint: start with Eq. (2) and neglect moisture effects).

(b) Estimate the pressure rise behind the gust front (in millibars) if the environmental density ρ_0 at the ground is $1.1\ \mathrm{kg\ m}^{-3}$.

4.) Buoyancy (25 points)

For the parcel in Problem 2, determine the liquid water mixing ratio (in $\mathrm{g/kg}$) that would be required to make this parcel neutrally buoyant (hint: consider water loading, ignore vapor buoyancy).