MESOSCALE METEOROLOGY

METR 4433 Exam #1 Spring 2015

Solutions

1 Scale Analysis (20 points)

(a) The hydrostatic approximation is appropriate for the meso-β scale, but not for meso-γ (storm) scale. Prove this by performing a scale analysis on the appropriate vertical equations of motion, given by

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \qquad \text{(meso-}\beta\text{ scale)}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\overline{\rho}}\frac{\partial p'}{\partial z} + \frac{\theta'}{\overline{\theta}}g \qquad \text{(meso-}\gamma\text{ scale)}$$

Solution

For the meso-β scale

$$\begin{array}{ccccc}
\dfrac{\partial w}{\partial t} & +u\dfrac{\partial w}{\partial x} & +w\dfrac{\partial w}{\partial z} & = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial z} & -g \\[2ex]
\dfrac{W}{T} & \dfrac{VW}{L} & \dfrac{WW}{H} & \dfrac{\Delta p}{\rho H} & g \\[2ex]
\dfrac{1}{10^4} & \dfrac{10 \times 1}{10^5} & \dfrac{1 \times 1}{10^4} & \dfrac{10^5}{1 \times 10^4} & 10 \\[2ex]
10^{-4} & 10^{-4} & 10^{-4} & 10 & 10
\end{array}$$

For the meso-γ scale

$$\begin{array}{ccccc}
\dfrac{\partial w}{\partial t} & +u\dfrac{\partial w}{\partial x} & +w\dfrac{\partial w}{\partial z} & = -\dfrac{1}{\overline{\rho}}\dfrac{\partial p'}{\partial z} & +\dfrac{\theta'}{\overline{\theta}}g \\[2ex]
\dfrac{W}{T} & \dfrac{VW}{L} & \dfrac{WW}{H} & \dfrac{\Delta p}{\overline{\rho} H} & \dfrac{\Delta\theta}{\theta_0}g \\[2ex]
\dfrac{10}{10^3} & \dfrac{10^2}{10^4} & \dfrac{10^2}{10^4} & \dfrac{10^2}{0.5 \times 10^4} & \dfrac{1 \times 10}{300} \\[2ex]
10^{-2} & 10^{-2} & 10^{-2} & 2 \times 10^{-2} & 3 \times 10^{-2}
\end{array}$$

The pressure gradient and gravity terms are the dominant scales for the meso-β scale, which means they roughly balance each other and, thus, the system may be considered roughly hydrostatic. However, they are of equal magnitude to the other terms for the meso-γ scale, which means they do not balance each other and, thus, the system is non-hydrostatic.

(b) Give one example of atmospheric phenomena that fall into each scale.

Solution

Examples of meso-β phenomena include, but are not limited to: mountain disturbances, lake disturbances, large convective systems, squall lines, nocturnal low-level jet, etc. Examples of meso-γ phenomena include: most thunderstorms, large cumulus, urban effects, etc.

2 Mountain Waves (20 points)

For a constant mean horizontal flow of speed $\overline{u}$ and constant static stability N over a low sinusoidal mountain whose height is given by $h(x) = h_m \sin(kx)$, the solution of perturbation vertical velocity $w^{'}$ can be expressed (after applying the upper boundary condition) as

$$w^{'}(x,z) = A_1 \cos(kx + mz) + A_2 \sin(kx + mz) \; ,$$

where $k = 2\pi/L_x$ is the wavenumber of the terrain, L_x is the mountain length scale, $m = \sqrt{(l^2 - k^2)}$, and $l = N/\overline{u}$ is the Scorer parameter.

(a) Solve for A_1 and A_2 to arrive at the solution of $w^{'}$ for the case where $l > k$. (hint: recall that the lower boundary condition is given by $w^{'}(x,0) \approx \overline{u}\partial h/\partial x$).

Solution

Applying the lower boundary yields

$$w^{'}(x,0) \approx \overline{u}\frac{\partial h}{\partial x} = \overline{u}kh_m \cos(kx) \; .$$

Now we compare this result with the general solution and at $z = 0$

$$w^{'}(x,0) = \overline{u}kh_m \cos(kx) = A_1 \cos(kx) + A_2 \sin(kx) \; .$$

This implies $A_1 = \overline{u}kh_m$ and $A_2 = 0$. Thus, the solution is

$$w^{'} = \overline{u}kh_m \cos(kx + mz) \; .$$

(b) Characterize the wave that your solution describes.

Solution

This describes a wave that propagates vertically without loss of amplitude.

(c) Rewrite the condition $l > k$ in terms of an advection time scale and a buoyancy time scale. Briefly discuss what your expression implies about the relative importance of each term.

Solution

$$l > k \quad \rightarrow \quad \frac{N}{\overline{u}} > k \quad \rightarrow \quad \frac{N}{\overline{u}} > \frac{2\pi}{L_x} \quad \rightarrow \quad NL_x > 2\pi\overline{u} \quad \rightarrow \quad \frac{(L_x/\overline{u})}{(2\pi/N)} > 1.$$

The numerator $(L_x/\overline{u})$, represents the advection time of an air parcel passing over one wavelength of the terrain, while the denominator $(2\pi/N)$ represents the period of buoyancy oscillation due to stratification. This means that the time an air parcel takes to pass over the terrain is more than it takes for vertical oscillation due to buoyancy force. In other words, buoyancy force plays a larger role than the horizontal advection.

3 Atmospheric Boundary Layer (20 points)

(a) Define the atmospheric boundary layer.

Solution

According to Stull (1988): *"Planetary boundary layer is the part of troposphere that is directly influenced by the presence of the earth surface and responds to surface forcings with a timescale of about an hour or less."*

According to Sorbjan (2012): *"The lowest portion of the atmosphere, which extensively exchanges mass (water), momentum, and heat with the Earth's surface."*

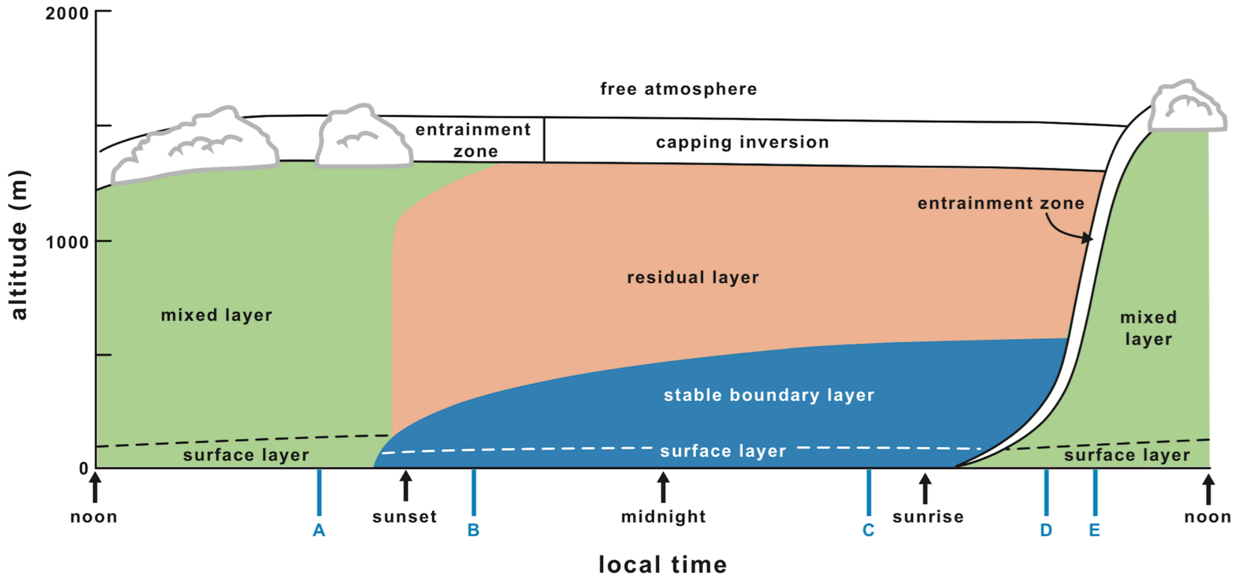
According to our course notes and the textbook: *"The boundary layer is a disturbance of the lower atmosphere induced by the underlying surface of the Earth. In other words, the boundary layer is an interfacial layer between the troposphere and the ground."*

Anything along these lines is acceptable.

(b) Make a schematic showing the diurnal cycle of the boundary layer. Make sure to clearly label relevant parts of your schematic (hint: this is a plot that extends from noon to noon, which shows the various sublayers and how they change after sunrise and sunset).

Solution

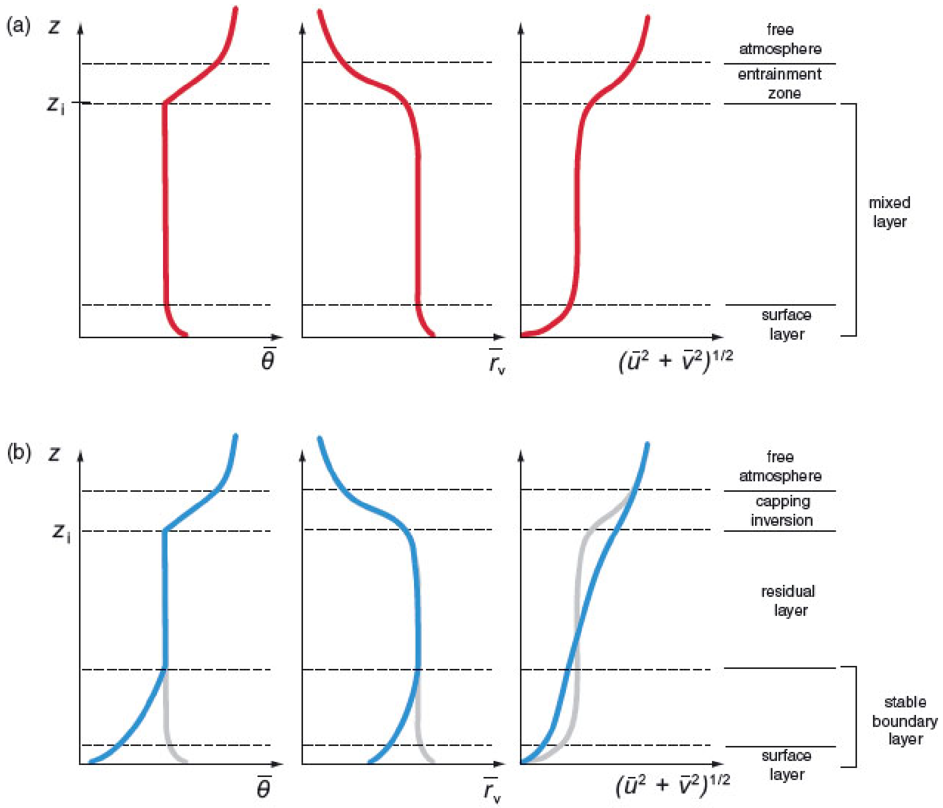
Something like this.



(c) Sketch the typical vertical profiles of potential temperature and wind associated with the atmospheric boundary layer of a sunny late afternoon and of a clear-sky early morning.

Solution

Something like these (the left and right panels). The top is daytime, the bottom is nighttime (pre-sunrise). If you chose post-sunrise, I allowed for variation.



4 Dryline (20 points)

(a) Define the dryline.

Solution

From the course notes: *"A narrow zone of strong horizontal moisture gradient at and near the surface."*

(b) During quiescent conditions over the U.S. Southern Great Plains, we often observe eastward propagation of the dryline during the day and westward retreat of the line at night. Explain this commonly observed diurnal movement. Use schematics (if necessary) to help you explain.

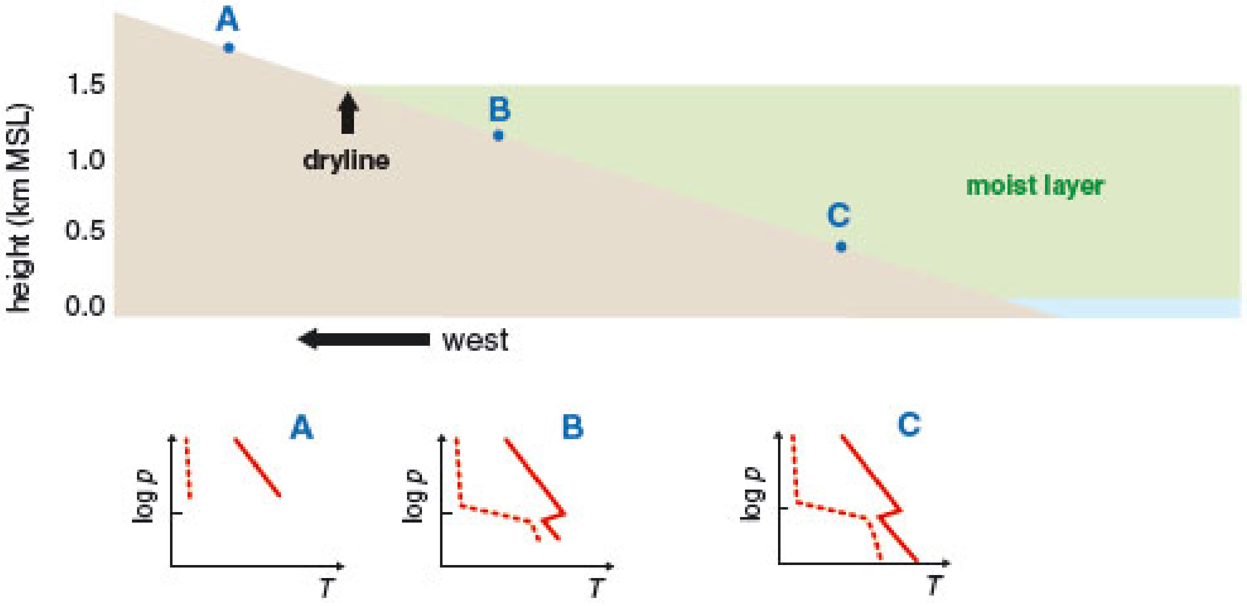
Solution

Something that encapsulates the following

- As the sun rises, surface heating near the dry line is greater than that of the surface to the east in the deeper moist air. Thus, it takes less surface sensible heating to destabilize the shallow mixed layer just to the east of the initial dry line position.

- The heating reduces static stability just to the east of the dryline, which leads to vertical mixing once the lapse rate becomes dry adiabatic.
- This mixing brings dry air (and westerly momentum) downward, and the position of the dry line moves eastward.
- Continued heating "mixes out" deeper and deeper moist layers, leading to an apparent eastward *propagation* of the dryline.
- Eventually, the heating is insufficient to mix out the moist layer and propagation stops.
- During evening hours, dry air to the west cools faster than the moist air to the east, leading to a radiation inversion. The increased static stability inhibits vertical mixing.
- Winds slow and back to a southeasterly direction in response to the pressure gradient force associated with the western lee trough.
- The moist air is advected westward, and thus surface stations will experience an east to west dry line passage.

You might have (but were not required) drawn something like this.



(c) Give one reason why drylines are often the focus of convection.

Solution
Recall that drylines are associated with capping inversions on the moist side. The strong stability of the cap acts to suppress deep convection. This can allow the low-level heat and moisture to build up so that the atmosphere is strongly conditionally unstable. Thus, capping inversions allow for the generation of large CAPE values and subsequent explosive thunderstorm development. Accordingly, the dryline is the westernmost boundary of moist, convectively unstable air. The area along it and immediately east is the first region susceptible to convection encountered by the vertical motion associated with various disturbances.

Below are examples of several such disturbances. Any one of these would suffice.

- Surface convergence between winds with easterly component east of the dry line and westerly component west of the dry line leads to vertical motion.
- Gravity waves may form on or near the dry line, possibly triggering the first release of potential instability.
- Dryline bulges provide an even greater focus for surface moisture convergence.
- "Underrunning" air (southerly flow below the cap) can move northward until the cap is weaker and/or large-scale or mesoscale forcing is strong enough to release the instability. Convective temperature may be reached just east of the dry line.

5 Low-Level Jet (20 points)

(a) Why is the U.S. Southern Great Plains low-level jet of meteorological interest?

Solution
In class, we discussed several such reasons

- Increased northward transport of moisture at jet level.
- Increased low-level convergence at nose of jet.
- Involved in sustaining convection at night.
- Partly responsible for nighttime thunderstorm maximum observed in the Great Plains.

(b) Name one of the theories used to explain the formation and evolution of the low-level jet.

Solution
The two theories we covered in class that are used to explain the formation and evolution of the low-level jet were the *inertial oscillation* (or the Blackadar theory) and *baroclinicity over sloping terrain* (or the Holton theory).