# MESOSCALE METEOROLOGY

METR 4433 Spring 2015

**Derivation Explanation From Chapter 2 Lecture Notes (22 January 2015)**

Today I messed up the derivation when discussing the physical reasons for the difference between wave solutions depending on how $l$ compares with $k$ (oops, what an idiot). I apologize for the wasted time. Here is the proper derivation, explained. I hope it is clear.

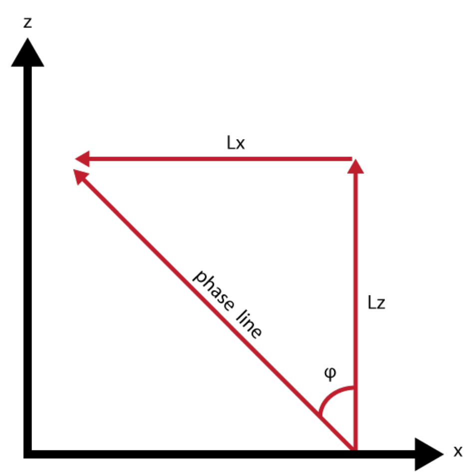
When $m$ is real, $l > k$, or $N > \overline{u}k$. Here, $\overline{u}k$ is known as the *intrinsic frequency*, which is the frequency that a wave would have if observed in a reference frame moving with the mean wind. If we rearrange our previous definition of $m$ to solve for $\overline{u}$, we get

$$\overline{u} = \pm\sqrt{\frac{N^2}{k^2 + m^2}}.$$

Using this expression, the inequality becomes

$$N > \pm\frac{Nk}{\sqrt{k^2 + m^2}}.$$

When $m$ is real, the total wave number may be regarded as a vector $\vec{k} \equiv (k, m)$, directed perpendicular to lines of constant phase, and in the direction of phase increase, whose components are $k = 2\pi/L_x$ and $m = 2\pi/L_z$. If we consider the case where $l > k$, the waves tilt upstream with height as shown in the figure below.



Basic geometry shows that

$$\cos\phi = \frac{L_z}{\sqrt{L_x^2 + L_z^2}},$$

where $L_x = 2\pi/k$ and $L_z = 2\pi/m$. Substitution yields

$$\cos\phi = \frac{\dfrac{2\pi}{m}}{\sqrt{\dfrac{4\pi^2}{k^2} + \dfrac{4\pi^2}{m^2}}} = \frac{\dfrac{2\pi}{m}}{\sqrt{\dfrac{4\pi^2 m^2 + 4\pi^2 k^2}{m^2 k^2}}} = \frac{\dfrac{2\pi}{m}\dfrac{mk}{2\pi}}{\sqrt{k^2 + m^2}} = \frac{k}{\sqrt{k^2 + m^2}}$$

Thus, the inequality becomes

$$N > N\cos\phi.$$

In other words, the intrinsic frequency must be less than the stratification frequency. With buoyancy as the restoring force, the atmosphere can support oscillations with frequencies less than or equal to $N$ for angles with respect to the vertical varying between $90\,\mathrm{deg}$ (purely horizontal) and $0\,\mathrm{deg}$ (purely vertical). To relate this to the mountain wave problem, we must realize that the flow over the terrain is driving an oscillation at a frequency with a magnitude equal to $\overline{u}k$. As long as this frequency is less than $N$ we can find a slanted path along which the oscillation can be supported. Once the frequency exceeds $N$, there is no real angle $\phi$ that satisfies the inequality. Thus, no such slanted path is possible and the waves simply decay with height, which is the case where $m$ is imaginary.