9.4 MOUNTAIN WAVES

Section 7.5.2 showed that stably stratified air forced to flow over sinusoidally varying surface topography creates oscillations, which can be either vertically propagating or vertically decaying, depending on whether the intrinsic wave frequency relative to the mean flow is less than or greater than the buoyancy frequency. Most topographic features on the surface of the earth do not, however, consist of regularly repeating lines of ridges. In general the distance between large topographic barriers is large compared to the horizontal scales of the barriers. Also, the static stability and the basic state flow are not usually constants, as was assumed in Section 7.5.2, but may vary strongly with height. Furthermore, nonlinear modifications of mountain waves are sometimes associated with strong surface winds along the lee slopes of ridges. Thus, mountain waves are significant features of mesoscale meteorology.

9.4.1 Flow over Isolated Ridges

Just as flow over a periodic series of sinusoidal ridges can be represented by a single sinusoidal function, that is, by a single Fourier harmonic, flow over an isolated ridge can be approximated by the sum of a number of Fourier components (see Section 7.2.1). Thus, any zonally varying topography can be represented by a Fourier series of the form

$$h_M(x) = \sum_{s=1}^{\infty} \mathrm{Re}\left[h_s \exp\left(ik_s x\right)\right] \tag{9.29}$$

where h_s is the amplitude of the sth Fourier component of the topography. We can then express the solution to the wave equation (7.46) as the sum of Fourier components:

$$w(x,z) = \sum_{s=1}^{\infty} \mathrm{Re}\left\{W_s \exp\left[i\left(k_s x + m_s z\right)\right]\right\} \tag{9.30}$$

where $W_s = ik_s\overline{u}h_s$, and $m_s^2 = N^2/\overline{u}^2 - k_s^2$.

Individual Fourier modes will yield vertically propagating or vertically decaying contributions to the total solution (9.30) depending on whether m_s is real or imaginary. This in turn depends on whether k_s^2 is less than or greater than $N^2/\bar{u}^2$. Thus, each Fourier mode behaves just as the solution (7.48) for periodic sinusoidal topography. For a narrow ridge, Fourier components with wave numbers greater than $N/\bar{u}$ dominate in (9.29), and the resulting disturbance decays with height. For a broad ridge, components with wave numbers less than $N/\bar{u}$ dominate and the disturbance propagates vertically. In the wide mountain limit where $m_s^2 \approx N^2/\bar{u}^2$, the flow is periodic in the vertical with a vertical wavelength of $2\pi m_s^{-1}$, and phase lines tilt upstream with height as shown in Fig. 9.7.

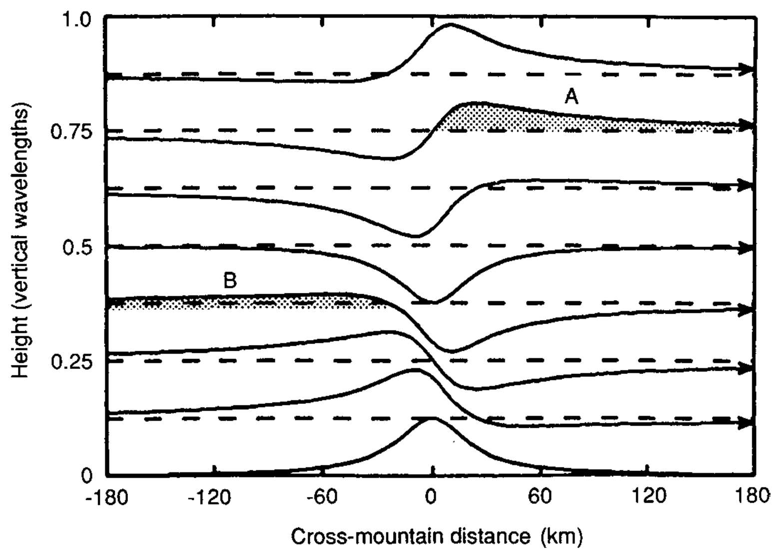

Fig. 9.7 Streamlines of flow over a broad isolated ridge showing upstream phase tilt with height. The pattern is periodic in height and one vertical wavelength is shown. Orographic clouds may form in the shaded areas where streamlines are displaced upward from equilibrium either upstream or downstream of the ridge if sufficient moisture is present. (After Durran, 1990.)

Vertically propagating gravity waves generated by flow over broad topography can produce clouds both upstream and downstream of the topography depending on variations of the moisture distribution with altitude. In the example shown in Fig. 9.7 the positions labeled A and B indicate regions where streamlines are displaced upward downstream and upstream of the ridge, respectively. If sufficient moisture is present, orographic clouds may then form in region A or B as suggested by the shading in Fig. 9.7.

9.4.2 Lee Waves

If $\bar{u}$ and N are allowed to vary in height, then (7.46) must be replaced by

$$\left(\frac{\partial^2 w'}{\partial x^2} + \frac{\partial^2 w'}{\partial z^2}\right) + l^2 w' = 0 \tag{9.31}$$

where the *Scorer parameter*, l, is defined as

$$l^2 = N^2 \Big/ \overline{u}^2 - \overline{u}^{-1} d^2 \overline{u} \Big/ dz^2$$

and the condition for vertical propagation becomes $k_s^2 < l^2$.

If the mean cross mountain wind speed increases strongly with height, or there is a low-level stable layer so that N decreases strongly with height, there may

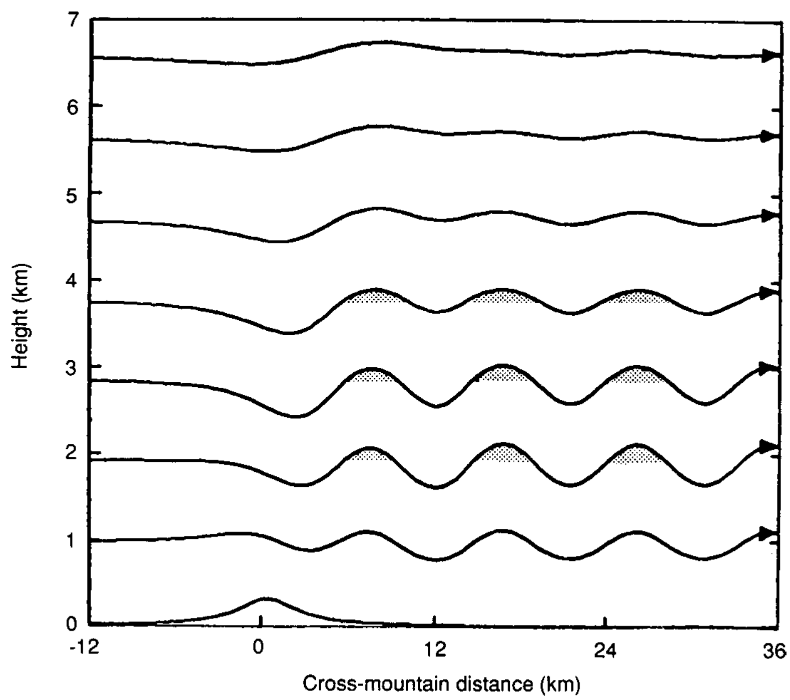

Fig. 9.8 Streamlines for trapped lee waves generated by flow over topography with vertical variation of the Scorer parameter. Shading shows locations where lee wave clouds may form. (After Durran, 1990.)

be a layer near the surface in which vertically propagating waves are permitted, which is topped by a layer in which the disturbance decays in the vertical. In that case vertically propagating waves in the lower layer are reflected when they reach the upper layer. Under some circumstances the waves may be repeatedly reflected from the upper layer and the surface downstream of the mountain, leading to a series of "trapped" lee waves as shown in Fig. 9.8.

Vertical variations in the Scorer parameter can also modify the amplitude of waves that are sufficiently long to propagate vertically through the entire troposphere. Amplitude enhancement leading to wave breaking and turbulent mixing can occur if there is a *critical level* where the mean flow goes to zero $(l \to \infty)$.

9.4.3 Downslope Windstorms

Strong downslope surface winds are occasionally observed along the lee slopes of mountain ranges. Although partial reflection of vertically propagating linear gravity waves may produce enhanced surface winds under some conditions, it appears that nonlinear processes are essential to account for observed windstorms associated with stable flow over topography.

To demonstrate the role of nonlinearity, we assume that the troposphere has a stable lower layer of undisturbed depth h topped by a weakly stable upper layer and assume that the lower layer behaves as a barotropic fluid with a free surface $h(x,t)$. We assume that disturbances have zonal wavelengths much greater than the layer depth. The motion of the lower layer may then be described by the shallow water equations of Section 7.3.2, but with the lower boundary condition replaced by

$$w(x, h_M) = Dh_M \big/ Dt = u\partial h_M \big/ \partial x$$

where h_M again denotes the height of the topography.

We first examine the linear behavior of this model by considering steady flow over small-amplitude topography. The linearized shallow water equations (7.20) and (7.21) then become

$$\overline{u}\frac{\partial u'}{\partial x} + \frac{g\delta\rho}{\rho_1}\frac{\partial h'}{\partial x} = 0 \tag{9.32}$$

$$\overline{u}\frac{\partial \left(h' - h_M\right)}{\partial x} + H\frac{\partial u'}{\partial x} = 0 \tag{9.33}$$

Here $\delta\rho/\rho_1$ is the fractional change in density across the interface between the layers, $h' = h - H$, where H is the mean height of the interface and $h' - h_M$ is the deviation from H of the thickness of the lower layer.

The solutions for (9.32) and (9.33) can be expressed as

$$h' = -\frac{h_M\left(\overline{u}^2 \big/ c^2\right)}{\left(1 - \overline{u}^2 \big/ c^2\right)}, \quad u' = \frac{h_M}{H}\left(\frac{\overline{u}}{1 - \overline{u}^2 \big/ c^2}\right) \tag{9.34}$$

where $c^2 \equiv (gH\delta\rho/\rho_1)$ is the shallow water wave speed. The characteristics of the disturbance fields h' and u' depend on the magnitude of the mean-flow *Froude number*, defined by the relation $\mathrm{Fr}^2 = \overline{u}^2 \big/ c^2$. When $\mathrm{Fr} < 1$, the flow is referred to as *subcritical*. In subcritical flow, the shallow water gravity wave speed is greater than the mean-flow speed, and the disturbance height and wind fields are out of phase. The interface height disturbance is negative, and the velocity disturbance is positive over the topographic barrier as shown in Fig. 9.9a. When $\mathrm{Fr} > 1$, the flow is referred to as *supercritical*. In supercritical flow the mean flow exceeds the shallow water gravity wave speed. Gravity waves cannot play a role in establishing the steady-state adjustment between height and velocity disturbances because such waves are swept downstream from the ridge by the mean flow. In this case the fluid thickens and slows as it ascends over the barrier (Fig. 9.9b). It is also clear from (9.34) that for $\mathrm{Fr} \sim 1$ the perturbations are no longer small and the linear solution breaks down.

The nonlinear equations corresponding to (9.32) and (9.33) in the case $\delta\rho = \rho_1$ can be expressed as

$$u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} = 0 \tag{9.35}$$

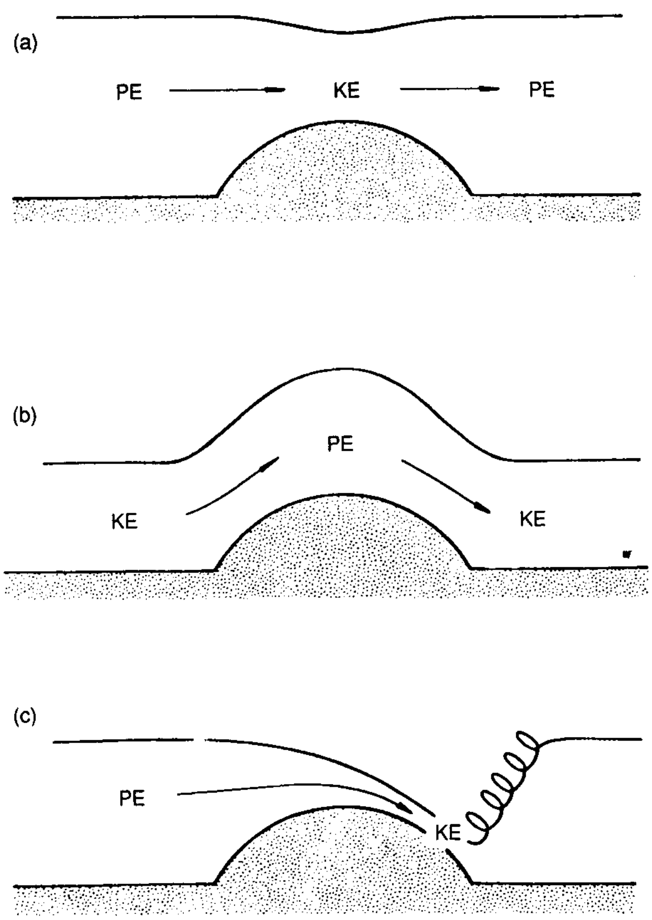

Fig. 9.9 Flow over an obstacle for a barotropic fluid with free surface. (a) Subcritical flow (Fr < 1 everywhere). (b) Supercritical flow (Fr > 1 everywhere). (c) Supercritical flow on lee slope with adjustment to subcritical flow at hydraulic jump near base of obstacle. (After Durran, 1990.)

$$\frac{\partial}{\partial x}\left[u\left(h - h_M\right)\right] = 0 \tag{9.36}$$

Equation (9.35) may be integrated immediately to show that the sum of kinetic and potential energy, $u^2/2 + gh$, is constant following the motion. Thus, energy conservation requires that if u increases h must decrease, and vice versa. In addition, (9.36) shows that the mass flux, $u(h - h_M)$, must also be conserved. The direction of the exchange between kinetic and potential energy in flow over a ridge is determined by the necessity that both (9.35) and (9.36) be satisfied.

Multiplying (9.35) by u and eliminating $\partial h/\partial x$ with the aid of (9.36) gives

$$\left(1 - \mathrm{Fr}^2\right)\frac{\partial u}{\partial x} = \frac{ug}{c^2}\frac{\partial h_M}{\partial x} \tag{9.37}$$

where the shallow water wave speed and the Froude number are now defined using the local thickness and velocity of the fluid:

$$c^2 \equiv g(h - h_M)\,; \quad \mathrm{Fr}^2 \equiv u^2/c^2$$

From (9.37) it is clear that the flow will accelerate on the upslope side of the ridge ($\partial u/\partial x > 0$ where $\partial h_M/\partial x > 0$) if the Froude number is less than unity, but will decelerate if the Froude number is greater than unity.

As a subcritical flow ascends the upslope side of a topographic barrier, Fr will tend to increase both from the increase in u and the decrease in c. If $\mathrm{Fr} = 1$ at the crest, then from (9.37) the flow will become supercritical and continue to accelerate as it descends the lee side until it adjusts back to the ambient subcritical conditions in a turbulent hydraulic jump as illustrated in Fig. 9.9c. In this case very high velocities can occur along the lee slope, as potential energy is converted into kinetic energy during the entire period that a fluid column traverses the barrier. Although conditions in the continuously stratified atmosphere are clearly more complex than in the shallow water hydraulic model, numerical simulations have demonstrated that the hydraulic model provides a reasonable conceptual model for the primary processes occurring in downslope windstorms.

9.5 CUMULUS CONVECTION

Mesoscale storms associated with cumulus convection represent a large fraction of all meteorologically important mesoscale circulations. Before considering such systems it is necessary to examine a few of the essential thermodynamic and dynamical aspects of individual cumulus clouds. The subject of cumulus convection is extremely complex to treat theoretically. Much of this difficulty stems from the fact that cumulus clouds have a complex internal structure. They are generally composed of a number of short-lived individual rising towers, which are produced by elements of ascending buoyant air called *thermals*. Rising thermals *entrain* environmental air and thus modify the cloud air through mixing. Thermals are nonhydrostatic, nonsteady, and highly turbulent. The buoyancy of an individual thermal (i.e., the difference between its density and the density of the environment) depends on a number of factors, including the environmental lapse rate, the rate of dilution by entrainment, and drag by the weight of liquid water in cloud droplets. A detailed discussion of the dynamics of thermal convection is beyond the scope of this text. This section utilizes a simple one-dimensional cloud model and focuses primarily on the thermodynamic aspects of moist convection. Convective storm dynamics is considered in Section 9.6.