Introduction to Mesoscale Meteorology: Part II

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Overview

Administrative Follow-Up
- Homework
- Exams
- Term Project

Introduction to Mesoscale, Continued
- Synoptic Scale
- Mesoscale
- Classification of Scales
- Definition of Scales via Scale Analysis on Equations of Motion
- Summary

Homework

Homework 1

- Distributed on February 3rd
- Due on February 17th

Homework 2

- Distributed on March 3rd
- Due on March 24th (extra week for Spring Break)

Homework 3

- Distributed on April 9th
- Due on April 23rd
- The dates for this homework may change slightly

Exams

Exam 1

- In class on Tuesday, February 24th from 11:30 am - 12:45 pm
- This will cover the first three chapters of material (intro, mountain waves, boundary layer)

Exam 2

- Time to be announced later
- This will cover the fourth chapter of material (convection, single/multi-cell storms, squall lines, supercells, tornadoes, etc.)

Final Exam

- Thursday May 7 in NWC 5600 from 10:30 am - 12:30 pm

Term Project

- Project will be a review of a mesoscale topic of your choice
- Distributed on Tuesday, March 10 - more info then
- Due on the last day of class (April 30th)

Synoptic Scale

- Midlatitude synoptic-scale motions are primarily driven by baroclinic instability.
- Baroclinic instability most likely with disturbances $\sim 3 \times L_R$

$$L_R = NH/f \sim 1000 \text{ km} - \text{Rossby radius of deformation}$$

$N \rightarrow$ Brunt-Väisälä frequency

$H \rightarrow$ scale height of the atmosphere

$f \rightarrow$ Coriolis parameter

Synoptic Scale

- L_R can be thought of as the length scale at which the velocity vector of a gravity wave is rotated such that it is perpendicular to the pressure gradient
- In other words, L_R is the scale at which rotational effects become important
- For scales $\sim L_R$, velocity and pressure fields both adjust to maintain balance between momentum and mass fields
- For scales $\gg L_R$, velocity field adjusts to the pressure field during geostrophic adjustment
- For scales $\ll L_R$, pressure field adjusts to the velocity field during geostrophic adjustment

Synoptic Scale

- Synoptic scale ($\gtrsim L_R$) is characterized by near geostrophic balance for straight flow
- Flow accelerations and ageostrophic motions are small
- Curved flow imbalances result in centripetal acceleration such that flow is approximately parallel to curved isobars, *i.e.* the gradient wind balance.

Mesoscale

- For scales $< L_R$, pressure gradients can be much larger than on the synoptic scale, while Coriolis acceleration remains of similar magnitude to that on the synoptic scale
- This leads to both large flow accelerations and ageostrophic motions
- Thus, gradient wind balance does not hold

Mesoscale

- Thus, the mesoscale is dynamically the scale for which both ageostrophic advection and Coriolis acceleration are important
- Unlike the synoptic scale, the mesoscale can be forced by a variety of drivers
  - topography
  - thermal instability
  - symmetric instability
  - barotropic instability
  - Kelvin-Helmholtz instability, and more

Mesoscale

Compared with synoptic scale disturbances (*e.g.*, extratropical cyclones), mesoscale phenomena are more short-lived. The mesoscale Lagrangian timescales are bound by:

The buoyancy oscillation

- $2\pi/N \sim 10\ \mathrm{min}$
- simple gravity waves

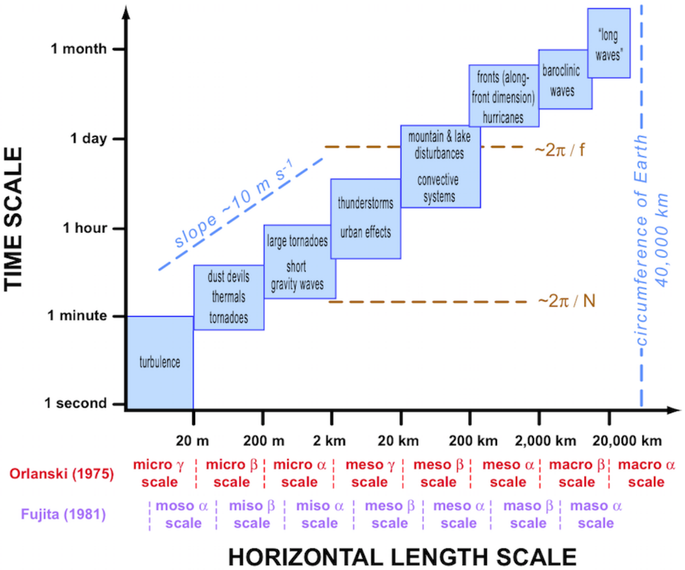
A pendulum day

- $2\pi/f \sim 17\mathrm{h}$
- inertial oscillations, such as low-level ageostrophic wind component (leads to nocturnal low-level wind max)

A Rough Definition

- Let us use a more qualitative definition and try to relate the mesoscale to something more concrete.
- Roughly consider that the word *mesoscale* defines meteorological events having spatial dimensions of <u>the order of one state</u>.
- Thus, individual thunderstorms or cumulus clouds are excluded since their scale is on the order of a few kilometers.
- Similarly, synoptic-scale cyclones are excluded since their scale is on the order of several thousands of kilometers.

Classification of Scales



Classification of Scales

- For purposes of this class, we will use Orlanski's definitions
- Our primary focus will be *meso*-β (or classical mesoscale) and *meso*-γ (or small or convective scale)
- We will also cover *meso*-α (*e.g.*, hurricanes) and *micro*-α scale (*e.g.*, tornadoes).

Time Scales

Some temporal scales in the atmosphere are obvious.

- diurnal cycle
- annual cycle
- inertial oscillation period - due to Earth's rotation, the Coriolis parameter f
- advective time scale - time taken to advect over certain distance

Space Scales

Some space scales in the atmosphere are obvious.

- global - related to earth's radius
- scale height of the atmosphere - related to the total mass of the atmosphere and gravity
- scale of fixed geographical features - mountain height, width, width of continents, oceans, lakes

Scale Analysis

Scale analysis is a very useful method for establishing the importance of various processes in the atmosphere and terms in the governing equations.

Based on the relative importance of these processes/terms, we can deduce much of the behavior of motion at such scales.

Scale Analysis

Consider this simple example:

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv$$

What is this equation? Can you identify the terms in it?

Scale Analysis

It's of course the horizontal momentum equation

$$
\begin{aligned}
u, v &\rightarrow \text{zonal, meridional wind components} \\
p &\rightarrow \text{pressure} \\
\rho &\rightarrow \text{density} \\
f &\rightarrow \text{Coriolis parameter} \\
\frac{du}{dt}, \frac{\partial u}{\partial t} &\rightarrow \text{Lagrangian, Eulerian time derivatives} \\
u\frac{\partial u}{\partial x}, w\frac{\partial u}{\partial z} &\rightarrow \text{advection} \\
-\frac{1}{\rho}\frac{\partial p}{\partial x} &\rightarrow \text{horizontal pressure gradient force} \\
fv &\rightarrow \text{Coriolis acceleration}
\end{aligned}
$$

Scale Analysis

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv$$

With scale analysis, we

- assign a characteristic value for each of the variables,
- estimate the magnitude of each term, and
- determine their relative importance

Scale Analysis

For example

$$\frac{du}{dt} \sim \frac{\Delta u}{\Delta t} \sim \frac{V}{T} \, ,$$

where ,

- V is the velocity scale (typical magnitude or amplitude if described as a wave component), and
- T the time scale (typical length of time for velocity to change by Δu, or the period for oscillations

Scale Analysis

$$\frac{du}{dt} \sim \frac{\Delta u}{\Delta t} \sim \frac{V}{T} \ ,$$

A point of emphasis

- The typical magnitude of change defines the scale, which is not always the same as the magnitude of the quantity itself.
- The absolute temperature is a good example, as is surface pressure.

Scale Analysis

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv$$

The utility of scale analysis

- For different scales, the terms in the equation of motion have different importance.
- This leads to different behavior of the motion, meaning there is a dynamic significance to the scale!

Scale Analysis: Synoptic Scale

Let's do the scale analysis for *synoptic scale motion* using the horizontal momentum equation

- $V \sim 10 \ \mathrm{ms}^{-1}$
- $W \sim 0.1 \ \mathrm{ms}^{-1}$
- $L \sim 1000 \ \mathrm{km} = 10^6 \ \mathrm{m}$
- $H \sim 10 \ \mathrm{km} = 10^4 \ \mathrm{m}$
- $T \sim L/V \sim 10^5 \ \mathrm{s}$
- $f \sim 10^{-4} \ \mathrm{s}^{-1}$
- $\rho \sim 1 \ \mathrm{kg\,m}^{-3}$
- Δp in horizontal $\sim 10 \ \mathrm{mb} = 1000 \ \mathrm{Pa}$

Scale Analysis: Synoptic Scale

Δp in horizontal $\sim 10\ \mathrm{mb} = 1000\ \mathrm{Pa}$

- Note, again, that it is the typical variation that determines that typical scale, not the value itself
- Using the scale of $1000\ \mathrm{mb}$ will give you the wrong result

Scale Analysis: Synoptic Scale

$$\frac{\partial u}{\partial t} \qquad + u\frac{\partial u}{\partial x} \qquad + w\frac{\partial u}{\partial z} \qquad\qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad\qquad + fv$$

Scale Analysis: Synoptic Scale

$$\frac{\partial u}{\partial t} \qquad +u\frac{\partial u}{\partial x} \qquad +w\frac{\partial u}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad +fv$$

$$\frac{V}{T} \qquad \frac{VV}{L} \qquad \frac{WV}{H} \qquad \frac{\Delta p}{\rho L} \qquad fV$$

Scale Analysis: Synoptic Scale

$$\frac{\partial u}{\partial t} \quad + u\frac{\partial u}{\partial x} \quad + w\frac{\partial u}{\partial z} \quad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \quad + fv$$

$$\frac{V}{T} \quad \frac{VV}{L} \quad \frac{WV}{H} \quad \frac{\Delta p}{\rho L} \quad fV$$

$$\frac{10}{10^5} \quad \frac{10 \times 10}{10^6} \quad \frac{0.1 \times 10}{10^4} \quad \frac{10^3}{1 \times 10^6} \quad 10^{-4} \times 10$$

Scale Analysis: Synoptic Scale

$$\frac{\partial u}{\partial t} \qquad +u\frac{\partial u}{\partial x} \qquad +w\frac{\partial u}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad +fv$$

$$\frac{V}{T} \qquad \frac{VV}{L} \qquad \frac{WV}{H} \qquad \frac{\Delta p}{\rho L} \qquad fV$$

$$\frac{10}{10^5} \qquad \frac{10 \times 10}{10^6} \qquad \frac{0.1 \times 10}{10^4} \qquad \frac{10^3}{1 \times 10^6} \qquad 10^{-4} \times 10$$

$$10^{-4} \qquad 10^{-4} \qquad 10^{-4} \qquad 10^{-3} \qquad 10^{-3}$$

Scale Analysis: Synoptic Scale

$$\frac{\partial u}{\partial t} \qquad + u\frac{\partial u}{\partial x} \qquad + w\frac{\partial u}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad + fv$$

$$10^{-4} \qquad 10^{-4} \qquad 10^{-4} \qquad 10^{-3} \qquad 10^{-3}$$

It turns out that the time tendency and advection terms are one order of magnitude smaller for synoptic scale flows.

The pressure gradient force and Coriolis force are in rough balance. What kind of flow do you get in this case? *quasi-geostrophic* flow.

Scale analysis confirms our previous dynamical reasoning.

Scale Analysis: Synoptic Scale

Let's do the scale analysis for *synoptic scale motion* using the vertical momentum equation

- Δp over vertical length scale $H \sim 1000 \text{ mb} = 10^5 \text{ Pa}$

Scale Analysis: Synoptic Scale

$$\frac{\partial w}{\partial t} \qquad + u\frac{\partial w}{\partial x} \qquad + w\frac{\partial w}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial z} \qquad -g$$

Scale Analysis: Synoptic Scale

$$\frac{\partial w}{\partial t} \qquad +u\frac{\partial w}{\partial x} \qquad +w\frac{\partial w}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial z} \qquad -g$$

$$\frac{W}{T} \qquad \frac{VW}{L} \qquad \frac{WW}{H} \qquad \frac{\Delta p}{\rho H} \qquad g$$

Scale Analysis: Synoptic Scale

$$\frac{\partial w}{\partial t} \qquad + u\frac{\partial w}{\partial x} \qquad + w\frac{\partial w}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial z} \qquad -g$$

$$\frac{W}{T} \qquad \frac{VW}{L} \qquad \frac{WW}{H} \qquad \frac{\Delta p}{\rho H} \qquad g$$

$$\frac{0.1}{10^5} \qquad \frac{10 \times 0.1}{10^6} \qquad \frac{0.1 \times 0.1}{10^4} \qquad \frac{10^5}{1 \times 10^4} \qquad 10$$

Scale Analysis: Synoptic Scale

$$
\begin{array}{ccccc}
\dfrac{\partial w}{\partial t} & +u\dfrac{\partial w}{\partial x} & +w\dfrac{\partial w}{\partial z} & = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial z} & -g \\[2ex]
\dfrac{W}{T} & \dfrac{VW}{L} & \dfrac{WW}{H} & \dfrac{\Delta p}{\rho H} & g \\[2ex]
\dfrac{0.1}{10^5} & \dfrac{10 \times 0.1}{10^6} & \dfrac{0.1 \times 0.1}{10^4} & \dfrac{10^5}{1 \times 10^4} & 10 \\[2ex]
10^{-6} & 10^{-6} & 10^{-6} & 10 & 10
\end{array}
$$

Scale Analysis: Synoptic Scale

$$\frac{\partial w}{\partial t} \qquad +u\frac{\partial w}{\partial x} \qquad +w\frac{\partial w}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial z} \qquad -g$$

$$10^{-6} \qquad 10^{-6} \qquad 10^{-6} \qquad 10 \qquad 10$$

Vertical acceleration $\ll$ vertical pressure gradient and gravitational terms, which are of the same order of magnitude.

The balance between these two terms gives the *hydrostatic balance* - a very good approximation for synoptic scale flows.

Also, the vertical motion is much smaller than the horizontal motion.

Scale Analysis: Synoptic Scale

We saw a good example of horizontal scale determining the dynamics of motion!

In summary, synoptic (and up) scale flows are

- quasi-two-dimensional (because $w << u$)
- hydrostatic (we can see it by performing scale analysis for vertical equation of motion)
- Coriolis force is a dominant term in the equation of motion and it is in rough balance with the PGF
- the flow is quasi-geostrophic

Scale Analysis: Mesoscale

What's all this synoptic scale stuff? What about the mesoscale?

Okay. Let's do the scale analysis for *mesoscale motion* using the horizontal momentum equation

- $V \sim 10 \text{ ms}^{-1}$
- $W \sim \mathbf{1 \ ms^{-1}}$ (↑)
- $L \sim \mathbf{100 \ km = 10^5 \ m}$ (↓)
- $H \sim 10 \text{ km} = 10^4 \text{ m}$
- $T \sim \boldsymbol{L/V} = \mathbf{10^4 \ s}$ (↓)
- $f \sim 10^{-4} \text{ s}^{-1}$
- $\rho \sim 1 \text{ kg m}^{-3}$
- Δp in horizontal $\sim \mathbf{1 \ mb = 100 \ Pa}$ (↓)

Scale Analysis: Mesoscale

$$\frac{\partial u}{\partial t} \qquad + u\frac{\partial u}{\partial x} \qquad + w\frac{\partial u}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad + fv$$

Scale Analysis: Mesoscale

$$\frac{\partial u}{\partial t} \qquad + u\frac{\partial u}{\partial x} \qquad + w\frac{\partial u}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad + fv$$

$$\frac{V}{T} \qquad \frac{VV}{L} \qquad \frac{WV}{H} \qquad \frac{\Delta p}{\rho L} \qquad fV$$

Scale Analysis: Mesoscale

$$\frac{\partial u}{\partial t} \qquad +u\frac{\partial u}{\partial x} \qquad +w\frac{\partial u}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad +fv$$

$$\frac{V}{T} \qquad \frac{VV}{L} \qquad \frac{WV}{H} \qquad \frac{\Delta p}{\rho L} \qquad fV$$

$$\frac{10}{10^4} \qquad \frac{10 \times 10}{10^5} \qquad \frac{1 \times 10}{10^4} \qquad \frac{10^2}{1 \times 10^5} \qquad 10^{-4} \times 10$$

Scale Analysis: Mesoscale

$\dfrac{\partial u}{\partial t}$	$+u\dfrac{\partial u}{\partial x}$	$+w\dfrac{\partial u}{\partial z}$	$= -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}$	$+fv$
$\dfrac{V}{T}$	$\dfrac{VV}{L}$	$\dfrac{WV}{H}$	$\dfrac{\Delta p}{\rho L}$	fV
$\dfrac{10}{10^4}$	$\dfrac{10 \times 10}{10^5}$	$\dfrac{1 \times 10}{10^4}$	$\dfrac{10^2}{1 \times 10^5}$	$10^{-4} \times 10$
10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}

Scale Analysis: Mesoscale

$$\frac{\partial u}{\partial t} \qquad + u\frac{\partial u}{\partial x} \qquad + w\frac{\partial u}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad + fv$$

$$10^{-3} \qquad 10^{-3} \qquad 10^{-3} \qquad 10^{-3} \qquad 10^{-3}$$

All terms in the equation are of the same magnitude

None of them can be neglected

We no longer have geostrophy!

Scale Analysis: Mesoscale

Let's do the scale analysis for *mesoscale motion* using the vertical momentum equation

- The hydrostatic approximation is still reasonable for the mesoscale

Scale Analysis: Mesoscale

$$\frac{\partial w}{\partial t} \qquad +u\frac{\partial w}{\partial x} \qquad +w\frac{\partial w}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial z} \qquad -g$$

Scale Analysis: Mesoscale

$$\frac{\partial w}{\partial t} \quad + u\frac{\partial w}{\partial x} \quad + w\frac{\partial w}{\partial z} \quad = -\frac{1}{\rho}\frac{\partial p}{\partial z} \quad - g$$

$$\frac{W}{T} \quad \frac{VW}{L} \quad \frac{WW}{H} \quad \frac{\Delta p}{\rho H} \quad g$$

Scale Analysis: Mesoscale

$$
\begin{array}{ccccc}
\dfrac{\partial w}{\partial t} & +u\dfrac{\partial w}{\partial x} & +w\dfrac{\partial w}{\partial z} & = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial z} & -g \\[2em]
\dfrac{W}{T} & \dfrac{VW}{L} & \dfrac{WW}{H} & \dfrac{\Delta p}{\rho H} & g \\[2em]
\dfrac{1}{10^4} & \dfrac{10 \times 1}{10^5} & \dfrac{1 \times 1}{10^4} & \dfrac{10^5}{1 \times 10^4} & 10
\end{array}
$$

Scale Analysis: Mesoscale

$\dfrac{\partial w}{\partial t}$	$+u\dfrac{\partial w}{\partial x}$	$+w\dfrac{\partial w}{\partial z}$	$= -\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}$	$-g$
$\dfrac{W}{T}$	$\dfrac{VW}{L}$	$\dfrac{WW}{H}$	$\dfrac{\Delta p}{\rho H}$	g
$\dfrac{1}{10^4}$	$\dfrac{10 \times 1}{10^5}$	$\dfrac{1 \times 1}{10^4}$	$\dfrac{10^5}{1 \times 10^4}$	10
10^{-4}	10^{-4}	10^{-4}	10	10

Scale Analysis: Mesoscale

$$\frac{\partial w}{\partial t} \qquad + u\frac{\partial w}{\partial x} \qquad + w\frac{\partial w}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial z} \qquad -g$$

$$10^{-4} \qquad 10^{-4} \qquad 10^{-4} \qquad 10 \qquad 10$$

Vertical acceleration $\ll$ vertical pressure gradient and gravitational terms, which are of the same order of magnitude.

The balance between these two terms gives the *hydrostatic balance* - still a good approximation for mesoscale flows.

The vertical motion is much smaller than the horizontal motion.

Scale Analysis: Mesoscale

Scale analysis confirms our previous dynamical reasoning.

In summary, for mesoscale flows:

- non-geostrophic (*i.e.*, ageostrophic component is significant)
- quasi-two-dimensional (because $w \ll u$)
- nearly hydrostatic
- Coriolis force is non-negligible

This can be a *dynamic definition of the mesoscale*, or in Orlanski's definition, the **meso-β** scale.

Scale Analysis: Convection/Storm Scale

We can go one step further down the scale, looking at cumulus convection or even supercell storms.

Let's do the scale analysis using the horizontal momentum equation

- $V \sim 10 \text{ ms}^{-1}$
- $W \sim \mathbf{10 \text{ ms}^{-1}}$ $(\uparrow)$
- $L \sim \mathbf{10 \text{ km} = 10^4 \text{ m}}$ $(\downarrow)$
- $H \sim 10 \text{ km} = 10^4 \text{ m}$
- $T \sim \boldsymbol{L/V = 10^3} \textbf{ s}$ $(\downarrow)$
- $f \sim 10^{-4} \text{ s}^{-1}$
- $\rho \sim 1 \text{ kg m}^{-3}$
- Δp in horizontal $\sim 1 \text{ mb} = 100 \text{ Pa}$

Scale Analysis: Convection/Storm Scale

$$\frac{\partial u}{\partial t} \qquad +u\frac{\partial u}{\partial x} \qquad +w\frac{\partial u}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad +fv$$

Scale Analysis: Convection/Storm Scale

$$\frac{\partial u}{\partial t} \qquad + u\frac{\partial u}{\partial x} \qquad + w\frac{\partial u}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad + fv$$

$$\frac{V}{T} \qquad \frac{VV}{L} \qquad \frac{WV}{H} \qquad \frac{\Delta p}{\rho L} \qquad fV$$

Scale Analysis: Convection/Storm Scale

$$
\begin{array}{ccccc}
\dfrac{\partial u}{\partial t} & +u\dfrac{\partial u}{\partial x} & +w\dfrac{\partial u}{\partial z} & = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} & +fv \\[2ex]
\dfrac{V}{T} & \dfrac{VV}{L} & \dfrac{WV}{H} & \dfrac{\Delta p}{\rho L} & fV \\[2ex]
\dfrac{10}{10^3} & \dfrac{10 \times 10}{10^4} & \dfrac{10 \times 10}{10^4} & \dfrac{10^2}{1 \times 10^4} & 10^{-4} \times 10
\end{array}
$$

Scale Analysis: Convection/Storm Scale

$$
\begin{array}{ccccc}
\dfrac{\partial u}{\partial t} & +u\dfrac{\partial u}{\partial x} & +w\dfrac{\partial u}{\partial z} & = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} & +fv \\
\dfrac{V}{T} & \dfrac{VV}{L} & \dfrac{WV}{H} & \dfrac{\Delta p}{\rho L} & fV \\
\dfrac{10}{10^3} & \dfrac{10 \times 10}{10^4} & \dfrac{10 \times 10}{10^4} & \dfrac{10^2}{1 \times 10^4} & 10^{-4} \times 10 \\
10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-3}
\end{array}
$$

Scale Analysis: Convection/Storm Scale

$$\frac{\partial u}{\partial t} \qquad +u\frac{\partial u}{\partial x} \qquad +w\frac{\partial u}{\partial z} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad +fv$$

$$10^{-2} \qquad 10^{-2} \qquad 10^{-2} \qquad 10^{-2} \qquad 10^{-3}$$

Coriolis force is an order of magnitude smaller than the other terms

Coriolis can be neglected when studying cumulus convection that lasts for an hour or so.

Again, the acceleration term is as important as the PGF.

Scale Analysis: Convection/Storm Scale

Let's do the scale analysis using the vertical momentum equation - which are now based on the Bousinessq equations of motion.

- $\nabla \cdot \vec{u} \approx 0$
- $\rho =$ constant everywhere, except when paired with gravity
- viscosity, thermal diffusivity, and specific heat are assumed constant

The Boussinesq approximation is used because it is the residual between the PGF and buoyancy force terms. Therefore we want to estimate the terms in terms of the deviations/perturbations from the hydrostatically balanced base state.

Scale Analysis: Convection/Storm Scale

$$\frac{\partial w}{\partial t} \qquad + u\frac{\partial w}{\partial x} \qquad + w\frac{\partial w}{\partial z} \qquad = -\frac{1}{\overline{\rho}}\frac{\partial p'}{\partial z} \qquad\qquad + \frac{\theta'}{\overline{\theta}} g$$

Scale Analysis: Convection/Storm Scale

$$\frac{\partial w}{\partial t} \qquad +u\frac{\partial w}{\partial x} \qquad +w\frac{\partial w}{\partial z} \qquad = -\frac{1}{\overline{\rho}}\frac{\partial p'}{\partial z} \qquad +\frac{\theta'}{\overline{\theta}}g$$

$$\frac{W}{T} \qquad \frac{VW}{L} \qquad \frac{WW}{H} \qquad \frac{\Delta p}{\overline{\rho}H} \qquad \frac{\Delta\theta}{\theta_0}g$$

Scale Analysis: Convection/Storm Scale

$$\frac{\partial w}{\partial t} \quad +u\frac{\partial w}{\partial x} \quad +w\frac{\partial w}{\partial z} \quad = -\frac{1}{\overline{\rho}}\frac{\partial p'}{\partial z} \quad +\frac{\theta'}{\overline{\theta}}g$$

$$\frac{W}{T} \quad \frac{VW}{L} \quad \frac{WW}{H} \quad \frac{\Delta p}{\overline{\rho} H} \quad \frac{\Delta \theta}{\theta_0}g$$

$$\frac{10}{10^3} \quad \frac{10 \times 10}{10^4} \quad \frac{10 \times 10}{10^4} \quad \frac{10^2}{0.5 \times 10^4} \quad \frac{1 \times 10}{300}$$

Scale Analysis: Convection/Storm Scale

$$
\begin{array}{ccccc}
\dfrac{\partial w}{\partial t} & +u\dfrac{\partial w}{\partial x} & +w\dfrac{\partial w}{\partial z} & = -\dfrac{1}{\overline{\rho}}\dfrac{\partial p'}{\partial z} & +\dfrac{\theta'}{\overline{\theta}}g \\[2ex]
\dfrac{W}{T} & \dfrac{VW}{L} & \dfrac{WW}{H} & \dfrac{\Delta p}{\overline{\rho} H} & \dfrac{\Delta\theta}{\theta_0}g \\[2ex]
\dfrac{10}{10^3} & \dfrac{10\times 10}{10^4} & \dfrac{10\times 10}{10^4} & \dfrac{10^2}{0.5\times 10^4} & \dfrac{1\times 10}{300} \\[2ex]
10^{-2} & 10^{-2} & 10^{-2} & 2\times 10^{-2} & 3\times 10^{-2}
\end{array}
$$

Scale Analysis: Convection/Storm Scale

$$
\begin{array}{ccccc}
\dfrac{\partial w}{\partial t} & +u\dfrac{\partial w}{\partial x} & +w\dfrac{\partial w}{\partial z} & = -\dfrac{1}{\overline{\rho}}\dfrac{\partial p'}{\partial z} & +\dfrac{\theta'}{\overline{\theta}}g \\
10^{-2} & 10^{-2} & 10^{-2} & 2\times 10^{-2} & 3\times 10^{-2}
\end{array}
$$

Here we are using the vertical mean density as the density scale in PGF term.

Clearly, the vertical acceleration term is now important.

Thus, the hydrostatic approximation is no longer applicable.

Scale Analysis: Convection/Storm Scale

In summary, for convective/storm scale:

- three-dimensional
- nonhydrostatic
- ageostrophic
- Coriolis force is negligible

In Orlanski's definition, this is the **meso-γ** scale.

Scale Analysis: Small Scales

As one goes further down to the micro-scales, the basic dynamics becomes similar to the **meso**-γ flows - the flow is

- three-dimensional
- nonhydrostatic
- ageostrophic
- Coriolis force is negligible

Know whether hydrostatic is reasonable *a prioiri*

The analysis I showed made assumptions about whether the hydrostatic approximation was appropriate. Here's how to do that.

We want to assess the scales of vertical acceleration and PGF. You can show that

$$\frac{\mathcal{O}\left(\frac{dw}{dt}\right)}{\mathcal{O}\left(-\frac{1}{\bar{\bar{\rho}}}\frac{\partial p'}{\partial z}\right)} \sim \left(\frac{H}{L}\right)^2$$

where

$$\frac{H}{L} \rightarrow \text{aspect ratio}$$

Know whether hydrostatic is reasonable *a prioiri*

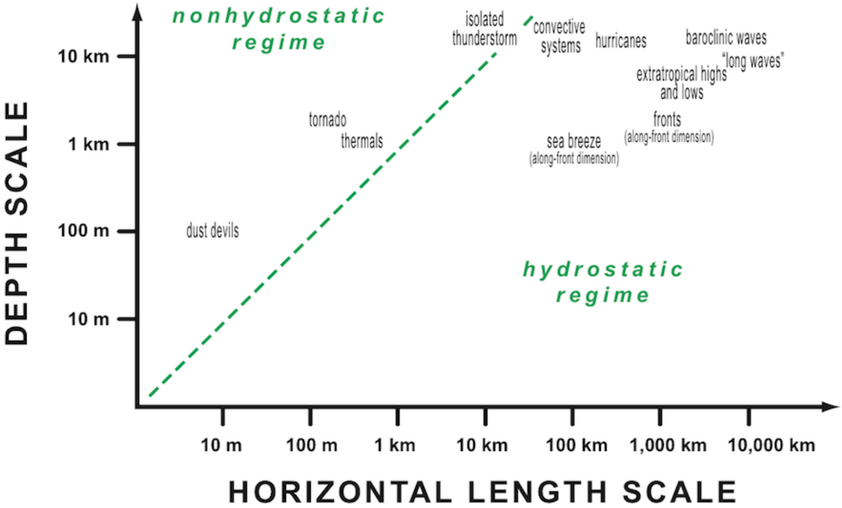
$H/L \ll 1 \rightarrow$ hydrostatic (at least a reasonable approximation)

$H/L \gtrsim 1 \rightarrow$ nonhydrostatic

From our cases:
Synoptic: $H/L = 10^4/10^6 = 10^{-2} \rightarrow$ hydrostatic
Meso-β: $H/L = 10^4/10^5 = 10^{-1} \rightarrow$ hydrostatic
Meso-γ: $H/L = 10^4/10^4 = 1 \rightarrow$ nonhydrostatic

Read for yourself Markowski and Richardson (page 9) for complete details.

Know whether hydrostatic is reasonable *a prioiri*



Summary

There are more than one way to define the scales of weather systems.

- Time/space scales of the system.
- Physically meaningful non-dimensional parameters (*e.g.*, Rossby number).

Summary

The most important thing to know is the key characteristics associated with weather systems/disturbances at each of these scales!

Don't get caught up in arbitrary names scientists give to the scales. Focus on physical features.

The End

Have a Great Weekend